Homework #4: Phys 3344: Prof. Olness Fall 2020

Due 16 September2020

Consider the equation:

$$x'' + 2\,\beta\, x' + \omega_0^2\, x = Q_0 \operatorname{Cos}(\omega_D\, t) \sim Q_0 \operatorname{Exp}(i\, \omega_D\, t)$$

The term $2\,\beta\, x'$ is the friction (dissipative) term.
The term $Q_0 \operatorname{Cos}(\omega_D\, t) \sim Q_0 \operatorname{Exp}(i\, \omega_D\, t)$ is the driving term.

1) Case: $x'' + 0 + \omega_0^2\, x = 0.$
2) Case: $x'' + 2\,\beta\, x' + \omega_0^2\, x = 0$
3) Case: $x'' + 0 + \omega_0^2\, x = Q_0 \operatorname{Exp}(i\, \omega_D\, t)$
4) Case: $x'' + 2\,\beta\, x' + \omega_0^2\, x = Q_0 \operatorname{Exp}(i\, \omega_D\, t)$

Note, if you prefer (personally, I do) you may also replace $Q_0 \operatorname{Cos}(\omega_D\, t)$ by $Q_0 \operatorname{Exp}(i\, \omega_D\, t)$; your choice.
Note also: the factor of 2 I've inserted in front of β above is not a universal standard, but it should be as it make the math much easier. [It does match the Taylor textbook.]

Goals: We are trying to obtain the general solution for each case, and characterize the solution in terms of the physical expectations. Later we'll verify these numerically.

Note, you may find it convenient to define: $\omega_1^2 = \omega_0^2 - \beta^2$.

Your mission: Solve each of the 4 above equations BY HAND. You can refer to your old texts and other reference material from previous courses, and other students. The important point is that you UNDERSTAND the physical properties of the solutions.

Here are a few observations I expect you to make. This is an example—I expect you to come up with more.

- For case 2, display a) under-damped, b) critically damped, and c) over-damped, and sketch these curves on the same plot for a selection of parameters. Comment.
- For case 3, vary β and ω_D, and comments. What happens when β is small?
  When ω_D,is close to ω_0?

Problem #5)

5.22 ★ **(a)** Consider a cart on a spring which is critically damped. At time $t = 0$, it is sitting at its equilibrium position and is kicked in the positive direction with velocity v_o. Find its position $x(t)$ for all subsequent times and sketch your answer. **(b)** Do the same for the case that it is released from rest at position $x = x_o$. In this latter case, how far is the cart from equilibrium after a time equal to $\tau_o = 2\pi/\omega_o$, the period in the absence of any damping?

Problem #6)

5.41 ★ We know that if the driving frequency ω is varied, the maximum response (A^2) of a driven damped oscillator occurs at $\omega \approx \omega_o$ (if the natural frequency is ω_o and the damping constant $\beta \ll \omega_o$). Show that A^2 is equal to half its maximum value when $\omega \approx \omega_o \pm \beta$, so that the full width at half maximum is just 2β. [*Hint:* Be careful with your approximations. For instance, it's fine to say $\omega + \omega_o \approx 2\omega_o$, but you certainly mustn't say $\omega - \omega_o \approx 0$.]

Problem #7)

1) Consider an RLC circuit with R=10 ohms, C=20uF, L=200mH, and V=200 volts. Useful info:

$$X_R = R, \quad X_C = 1/(\omega C), \quad X_L = \omega L \quad Z = \sqrt{X_R^2 + (X_L - X_C)^2}$$

a) Find the maximum value of the current.

b) Also, find the values of w where the current is ½ the maximum value.

(You may do this approximately, but I want a number.)

c) If I increase R to 20 ohms, compute the new max current.

d) Make a sketch of current I vs w for both values of R.