#5) Taylor 5.22

In[ ]:=
```
Clear["Global`*"]
```

In[ ]:=
```
eq1 = x''[t] + 2 β x'[t] + w0^2 x[t] == 0
```

Out[ ]= $w0^2\, x[t] + 2\,\beta\, x'[t] + x''[t] == 0$

In[ ]:=
```
dsol = DSolve[eq1, x[t], t][[1]]
```

Out[ ]= $\left\{x[t] \to e^{t\left(-\beta-\sqrt{-w0^2+\beta^2}\right)}\, c_1 + e^{t\left(-\beta+\sqrt{-w0^2+\beta^2}\right)}\, c_2\right\}$

In[ ]:=
```
bc = {x[0] == x0, x'[0] == v0}
```

Out[ ]= $\{x[0] == x0,\ x'[0] == v0\}$

In[ ]:=
```
eq2 = Join[{eq1}, bc]
```

Out[ ]= $\{w0^2\, x[t] + 2\,\beta\, x'[t] + x''[t] == 0,\ x[0] == x0,\ x'[0] == v0\}$

In[ ]:=
```
dsol = DSolve[eq2, x[t], t][[1]] // FullSimplify
```

Out[ ]= $\left\{x[t] \to e^{-t\beta}\left(x0\, \text{Cosh}\left[t\sqrt{-w0^2+\beta^2}\right] + \frac{(v0 + x0\,\beta)\, \text{Sinh}\left[t\sqrt{-w0^2+\beta^2}\right]}{\sqrt{-w0^2+\beta^2}}\right)\right\}$

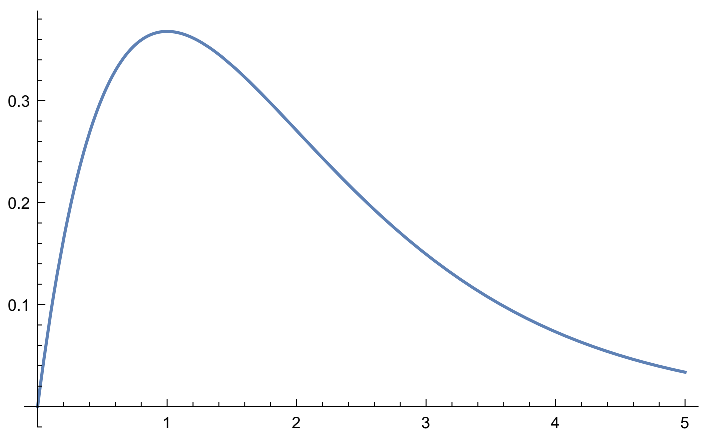
In[ ]:=
```
critical = {β → w0};
```

In[ ]:=
```
xCrit[t_] = Limit[x[t] /. dsol, critical]
```

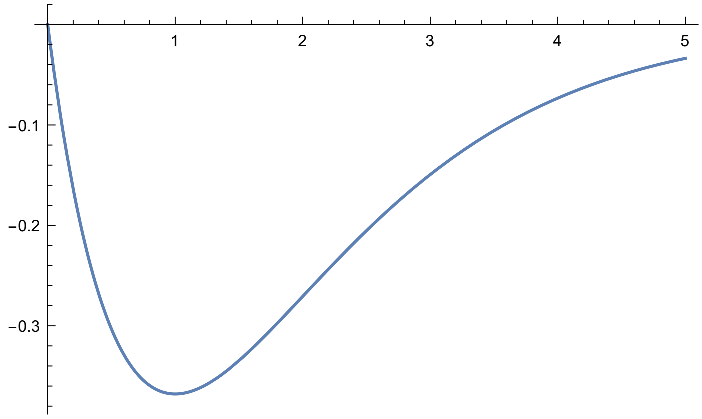
Out[ ]= $e^{-t\, w0}\,(x0 + t\,(v0 + w0\, x0))$

In[ ]:=
```
Plot[xCrit[t] /. {x0 → 0, v0 → 1, w0 → 1}, {t, 0, 5}]
```

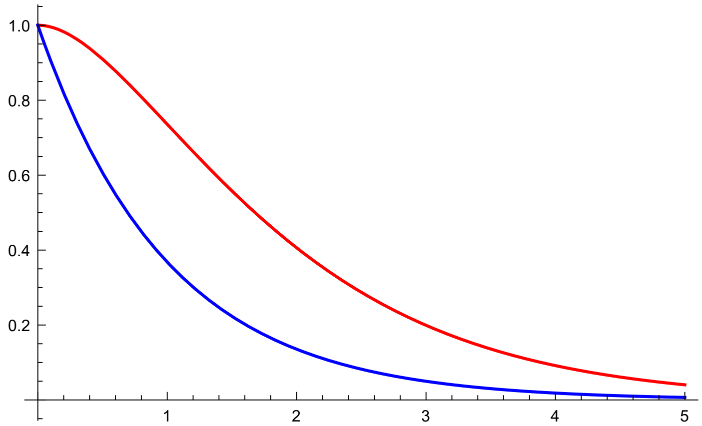
Out[ ]=



In[ ]:= Plot[xCrit[t] /. {x0 → 0, v0 → -1, w0 → 1}, {t, 0, 5}]

Out[ ]=


In[ ]:= Plot[{ xCrit[t], Exp[-t w0]} /. {x0 → 1, v0 → 0, w0 → 1} // Evaluate , {t, 0, 5},
PlotStyle → {Red, Blue, Green, Thick}]

Out[ ]=


In[ ]:= xCrit[t] /. {v0 → 0} // Simplify

Out[ ]= $e^{-t\,w0}\,(1 + t\,w0)\,x0$

In[ ]:= xCrit[t] /. $\left\{t \to \frac{2\pi}{w0},\ v0 \to 0\right\}$ // Simplify

Out[ ]= $e^{-2\pi}\,(1 + 2\pi)\,x0$

In[ ]:= xCrit[t] /. $\left\{t \to \frac{2\pi}{w0},\ v0 \to 0\right\}$ // Simplify // N

Out[ ]= 0.0136009 x0

In[ ]:= xCrit[t] /. {t → 0, v0 → 0} // Simplify // N

Out[ ]= x0

#5) Taylor 5.22 The Wrong way!

Also See p.177: Critical Damping!!!

In[ ]:=
```
Clear["Global`*"]
```

In[ ]:=
```
eq1 = x''[t] + 2 β x'[t] + w0^2 x[t] == 0
```

Out[ ]= $w0^2 x[t] + 2 \beta x'[t] + x''[t] == 0$

In[ ]:=
```
dsol = DSolve[eq1, x[t], t][[1]]
```

Out[ ]= $\left\{x[t] \to e^{t\left(-\beta-\sqrt{-w0^2+\beta^2}\right)} c_1 + e^{t\left(-\beta+\sqrt{-w0^2+\beta^2}\right)} c_2\right\}$

In[ ]:=
```
dsol0 = dsol /. {β → w0}
```

Out[ ]= $\{x[t] \to e^{-t\,w0} c_1 + e^{-t\,w0} c_2\}$

In[ ]:=
```
dsol0 = dsol0 /. {C[2] → 0}
```

Out[ ]= $\{x[t] \to e^{-t\,w0} c_1\}$

In[ ]:=
```
eqX = x[0] == x[t] /. dsol0
```

Out[ ]= $x[0] == e^{-t\,w0} c_1$

In[ ]:=
```
eqX /. t → 0
```

Out[ ]= $x[0] == c_1$

In[ ]:=
```
solX0 = Solve[eqX /. t → 0, C[1]][[1]]
```

Out[ ]= $\{c_1 \to x[0]\}$

In[ ]:=
```
eqV = v0 == D[x[t] /. dsol0, t]
```

Out[ ]= $v0 == -e^{-t\,w0}\, w0\, c_1$

In[ ]:=
```
solV0 = eqV /. t → 0
```

Out[ ]= $v0 == -w0\, c_1$

In[ ]:=
```
solV0 /. solX0
```

Out[ ]= $v0 == -w0\, x[0]$

Problem : if x[0] == 0, then v0 = 0.

#6) Taylor 5.41

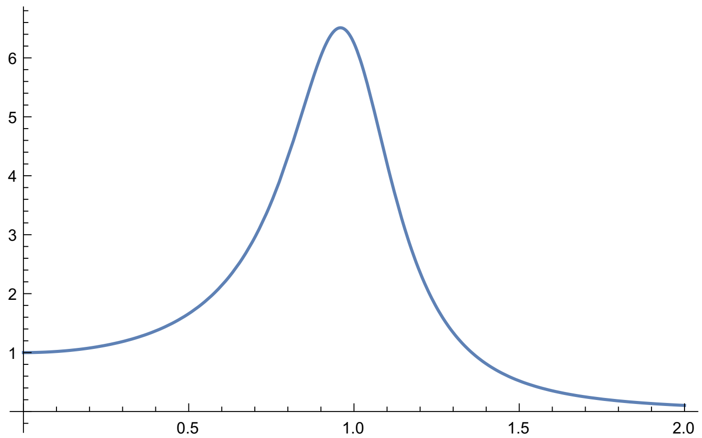
In[ ]:=
```
Clear["Global`*"]
```

In[ ]:= amp[w_] = $\frac{f0^2}{(w0^2 - w^2)^2 + 4\beta^2 w^2}$

Out[ ]= $\frac{f0^2}{(-w^2 + w0^2)^2 + 4 w^2 \beta^2}$

In[ ]:= values = {f0 → 1, w0 → 1, β → 0.2};

In[ ]:= Plot[amp[w] //. values // Abs , {w, 0, 2}]

Out[ ]=


In[ ]:= eq1 = $\frac{1}{2}$ amp[w0] == amp[w]

Out[ ]= $\frac{f0^2}{8 w0^2 \beta^2} == \frac{f0^2}{(-w^2 + w0^2)^2 + 4 w^2 \beta^2}$

In[ ]:= sol = Solve[eq1, w] // PowerExpand

Out[ ]= $\left\{\left\{w \to -\sqrt{w0^2 - 2\beta^2 - 2\beta\sqrt{w0^2 + \beta^2}}\right\}, \left\{w \to \sqrt{w0^2 - 2\beta^2 - 2\beta\sqrt{w0^2 + \beta^2}}\right\}, \left\{w \to -\sqrt{w0^2 - 2\beta^2 + 2\sqrt{w0^2\beta^2 + \beta^4}}\right\}, \left\{w \to \sqrt{w0^2 - 2\beta^2 + 2\sqrt{w0^2\beta^2 + \beta^4}}\right\}\right\}$

In[ ]:= Series[w /. sol , {β, 0, 1}] // Normal // PowerExpand // Simplify

Out[ ]= $\{-w0 + \beta, w0 - \beta, -w0 - \beta, w0 + \beta\}$

In[ ]:= w /. sol // PowerExpand // Simplify

Out[ ]= $\left\{-\sqrt{w0^2 - 2\beta\left(\beta + \sqrt{w0^2 + \beta^2}\right)}, \sqrt{w0^2 - 2\beta\left(\beta + \sqrt{w0^2 + \beta^2}\right)}, -\sqrt{w0^2 - 2\beta^2 + 2\sqrt{\beta^2(w0^2 + \beta^2)}}, \sqrt{w0^2 - 2\beta^2 + 2\sqrt{\beta^2(w0^2 + \beta^2)}}\right\}$

In[ ]:= ((w /. sol // PowerExpand // Simplify) /. {$w0^2 + \beta^2 \to w0^2$} // PowerExpand // ExpandAll) /. {$\beta^2 \to 0$}

Out[ ]= $\left\{-\sqrt{w0^2 - 2 w0 \beta}, \sqrt{w0^2 - 2 w0 \beta}, -\sqrt{w0^2 + 2 w0 \beta}, \sqrt{w0^2 + 2 w0 \beta}\right\}$

In[ ]:=
```
(Series[#, {β, 0, 1}] & /@ %) // PowerExpand // Normal // Simplify
```

Out[ ]= $\{-w0+\beta, w0-\beta, -w0-\beta, w0+\beta\}$

In[ ]:=
```
Series[√(1 + ϵ), {ϵ, 0, 2}]
```

Out[ ]= $1+\frac{\epsilon}{2}-\frac{\epsilon^2}{8}+O[\epsilon]^3$

#7) RLC

In[ ]:=
```
Clear["Global`*"]
```

In[ ]:=
```
xR = R;
xC = 1/(w c);
xL = w L;
z = Sqrt[xR^2 + (xL - xC)^2]
```

Out[ ]= $\sqrt{R^2+\left(-\frac{1}{c\,w}+L\,w\right)^2}$

In[ ]:=
```
current = voltage/z
```

Out[ ]= $\frac{\text{voltage}}{\sqrt{R^2+\left(-\frac{1}{c\,w}+L\,w\right)^2}}$

In[ ]:=
```
sol1 = Solve[D[current, w] == 0, w]
```

Out[ ]= $\left\{\left\{w\to-\frac{1}{\sqrt{c}\,\sqrt{L}}\right\},\left\{w\to-\frac{i}{\sqrt{c}\,\sqrt{L}}\right\},\left\{w\to\frac{i}{\sqrt{c}\,\sqrt{L}}\right\},\left\{w\to\frac{1}{\sqrt{c}\,\sqrt{L}}\right\}\right\}$

In[ ]:=
```
maxCurrent = current /. {w → 1/(√c √L)} // PowerExpand
```

Out[ ]= $\frac{\text{voltage}}{R}$

In[ ]:=
```
eq1 = current == 1/2 maxCurrent
```

Out[ ]= $\frac{\text{voltage}}{\sqrt{R^2+\left(-\frac{1}{c\,w}+L\,w\right)^2}}==\frac{\text{voltage}}{2\,R}$

In[ ]:= `sol2 = Solve[eq1, w] // Simplify // PowerExpand`

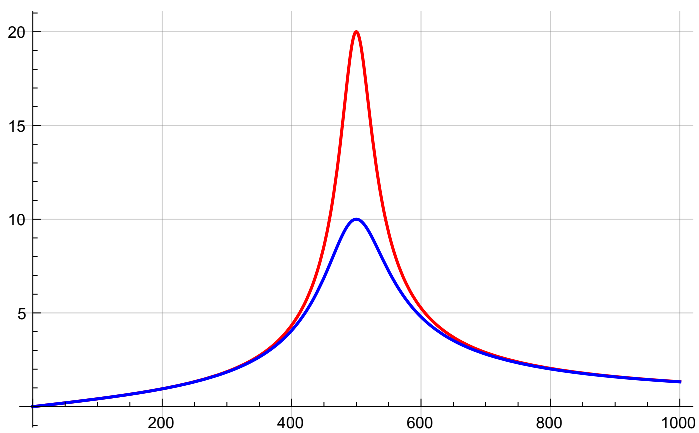
Out[ ]= $\left\{\left\{w \to -\frac{\sqrt{2\,L + 3\,c\,R^2 - \sqrt{3}\,\sqrt{c}\,\sqrt{4\,L\,R^2 + 3\,c\,R^4}}}{\sqrt{2}\,\sqrt{c}\,L}\right\}, \left\{w \to \frac{\sqrt{2\,L + 3\,c\,R^2 - \sqrt{3}\,\sqrt{c}\,\sqrt{4\,L\,R^2 + 3\,c\,R^4}}}{\sqrt{2}\,\sqrt{c}\,L}\right\}, \left\{w \to -\frac{\sqrt{2\,L + 3\,c\,R^2 + \sqrt{3}\,\sqrt{c}\,\sqrt{4\,L\,R^2 + 3\,c\,R^4}}}{\sqrt{2}\,\sqrt{c}\,L}\right\}, \left\{w \to \frac{\sqrt{2\,L + 3\,c\,R^2 + \sqrt{3}\,\sqrt{c}\,\sqrt{4\,L\,R^2 + 3\,c\,R^4}}}{\sqrt{2}\,\sqrt{c}\,L}\right\}\right\}$

In[ ]:=
```
values1 = {R → 10, c → 20 × 10^-6, L → 200 × 10^-3, voltage → 200};
values2 = {R → 20, c → 20 × 10^-6, L → 200 × 10^-3, voltage → 200};
```

In[ ]:= `current /. {values1, values2} // Re`

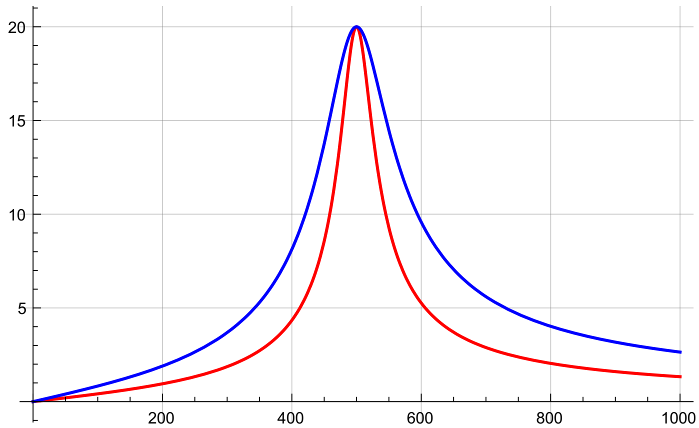
Out[ ]= $\left\{200\,\mathrm{Re}\left[\frac{1}{\sqrt{100 + \left(-\frac{50\,000}{w} + \frac{w}{5}\right)^2}}\right], 200\,\mathrm{Re}\left[\frac{1}{\sqrt{400 + \left(-\frac{50\,000}{w} + \frac{w}{5}\right)^2}}\right]\right\}$

In[ ]:=
```
Plot[current /. {values1, values2} // Re // Evaluate,
 {w, 0, 1000},
 PlotStyle → {Red, Blue},
 PlotRange → All,
 GridLines → Automatic]
```

Out[ ]=


In[ ]:=
```
Plot[{current /. values1, 2 current /. values2} // Re // Evaluate,
 {w, 0, 1000},
 PlotStyle → {Red, Blue},
 PlotRange → All,
 GridLines → Automatic]
```

Out[ ]=



In[ ]:=
```
current /. values1
```

Out[ ]= $$\frac{200}{\sqrt{100+\left(-\frac{50000}{w}+\frac{w}{5}\right)^2}}$$

In[ ]:=
```
eq1
```

Out[ ]= $$\frac{\text{voltage}}{\sqrt{R^2+\left(-\frac{1}{c\,w}+L\,w\right)^2}} == \frac{\text{voltage}}{2\,R}$$

In[ ]:=
```
NSolve[eq1 /. values1, w]
```

Out[ ]= {{w → 545.173}, {w → -545.173}, {w → -458.57}, {w → 458.57}}

In[ ]:=
```
545 - 458
```

Out[ ]= 87

In[ ]:=
```
NSolve[eq1 /. values2, w]
```

Out[ ]= {{w → -594.047}, {w → 594.047}, {w → 420.842}, {w → -420.842}}

In[ ]:=
```
594 - 420
```

Out[ ]= 174

In[ ]:=
```
174 / 87
```

Out[ ]= 2

quality factor

In[ ]:=
$$Q=\frac{\omega_r L}{R}=\frac{X_L}{R}=\frac{1}{\omega_r C R}=\frac{X_C}{R}=\frac{1}{R}\sqrt{\frac{L}{C}}$$

Out[ ]=
$$Q=\frac{\omega_r L}{R}=\frac{X_L}{R}=\frac{1}{\omega_r C R}=\frac{X_C}{R}=\frac{1}{R}\sqrt{\frac{L}{C}}$$

In[ ]:=
```
Sqrt[L / c]
----------- /. values1
     R
```

Out[ ]= 10

In[ ]:=
```
Sqrt[L / c]
----------- /. values2
     R
```

Out[ ]= 5