In[1]:=
```
Lag[qi_,L_ ,Qi_,t_:t] :=
D[ D[L,(D[qi[t],t])],t]-D[L,qi[t]]==Qi;

Lag[qi_List,L_,Qi_List,t_:t] :=
(D[ D[L,(D[qi[[#]][t],t])],t]-D[L,qi[[#]][t]]==Qi[[#]]
)& /@ Range[1,Length[qi]];
Protect[Lag];
```

Problem 3: Sphere Rolling on a Fixed Sphere With 2 Lagrange Parameters $\{\lambda_1, \lambda_2\}$

Remarks and outline

Solution

In[6]:=
```
Clear["Global`*"]
```

Clear : Symbol Lag is Protected .

Part a

Start with some basics

In[7]:=
```
Tcm = 1/2 m (r'[t]^2 + r[t]^2 θ1'[t]^2);
```

In[8]:=
```
Trot = 1/2 I1 (θ1'[t] + θ2'[t])^2;
```

In[9]:=
```
T=Tcm+Trot;
```

In[10]:=
```
V= m g r[t] Cos[θ1[t]];
```

In[11]:=
```
iRule = {I1 → κ m a^2} (* Moment of inertia is some factor "κ" times m a^2 *)
```

Out[11]= $\{I1 \to a^2\, m\, \kappa\}$

In[12]:=
```
L=T-V /.iRule
```
Out[12]=
$$-g\, m\, \mathrm{Cos}[\theta 1[t]]\, r[t] + \frac{1}{2} m \left(r'[t]^2 + r[t]^2\, \theta 1'[t]^2\right) + \frac{1}{2} a^2\, m\, \kappa\, (\theta 1'[t] + \theta 2'[t])^2$$

In[13]:=
```
constraintsEq= {b θ1[t] - a θ2[t]==0, r[t]-a-b==0 }
```
Out[13]= {b θ1[t] − a θ2[t] == 0, −a − b + r[t] == 0}

In[14]:=
```
constraintRule = DSolve[constraintsEq , {θ2, r}, t]
```
Out[14]=
$$\left\{\left\{r \to \mathrm{Function}[\{t\},\ a+b],\ \theta 2 \to \mathrm{Function}\left[\{t\},\ \frac{b\,\theta 1[t]}{a}\right]\right\}\right\}$$

In[15]:=
```
vars = {r[t], θ1[t], θ2[t]};
```

In[16]:=
```
Qeq = Sum[λ[j] * D[constraintsEq [[j , 1]], vars[[#]]], {j , 1, 2}] & /@ {1, 2, 3}
```
Out[16]= {λ[2], b λ[1], −a λ[1]}

We’ll use the “Magic” function.
In general, I don’t like “black boxes”

In[17]:=
```
(eqMotion = Lag[{r, θ1 , θ2}, L, Qeq]) ;
eqMotion // TableForm
```
Out[18]//TableForm=

$$g\, m\, \mathrm{Cos}[\theta 1[t]] - m\, r[t]\, \theta 1'[t]^2 + m\, r''[t] == \lambda[2]$$
$$-g\, m\, r[t]\, \mathrm{Sin}[\theta 1[t]] + 2\, m\, r[t]\, r'[t]\, \theta 1'[t] + m\, r[t]^2\, \theta 1''[t] + a^2\, m\, \kappa\, (\theta 1''[t] + \theta 2''[t]) == b\, \lambda[1]$$
$$a^2\, m\, \kappa\, (\theta 1''[t] + \theta 2''[t]) == -a\, \lambda[1]$$

Let’s get the equations by hand:

The R equation

In[19]:=
```
part2 = D[L, r[t]]
```
Out[19]= $-g\, m\, \mathrm{Cos}[\theta 1[t]] + m\, r[t]\, \theta 1'[t]^2$

In[20]:=
```
D[L, r '[t]]
```
Out[20]= m r′[t]

In[21]:=
```
part1 = D[D[L, r '[t]], t]
```
Out[21]= m r″[t]

In[22]:=
```
constraintsEq
```
Out[22]= {b θ1[t] − a θ2[t] == 0, −a − b + r[t] == 0}

In[23]:=
```
constraints = First /@ constraintsEq
```
Out[23]= {b θ1[t] − a θ2[t], −a − b + r[t]}

In[24]:=
```
D[constraints , r[t]]
```
Out[24]=
```
{0, 1}
```
In[25]:=
```
part3 = D[constraints , r[t]] .{λ[1], λ[2]}
```
Out[25]=
```
λ[2]
```
In[26]:=
```
eqR = part1 - part2 == part3
```
Out[26]= $g\, m\, \mathrm{Cos}[\theta 1[t]] - m\, r[t]\, \theta 1'[t]^2 + m\, r''[t] == \lambda[2]$

In[27]:=
```
eqMotion [[1]]
```
Out[27]= $g\, m\, \mathrm{Cos}[\theta 1[t]] - m\, r[t]\, \theta 1'[t]^2 + m\, r''[t] == \lambda[2]$

In[28]:=
```
eqR == eqMotion [[1]]
```
Out[28]=
```
True
```

The θ1 equation

In[29]:=
```
part2 = D[L, θ1[t]]
```
Out[29]= $g\, m\, r[t]\, \mathrm{Sin}[\theta 1[t]]$

In[30]:=
```
D[L, θ1 '[t]]
```
Out[30]= $m\, r[t]^2\, \theta 1'[t] + a^2\, m\, \kappa\, (\theta 1'[t] + \theta 2'[t])$

In[31]:=
```
part1 = D[D[L, θ1 '[t]], t]
```
Out[31]= $2\, m\, r[t]\, r'[t]\, \theta 1'[t] + m\, r[t]^2\, \theta 1''[t] + a^2\, m\, \kappa\, (\theta 1''[t] + \theta 2''[t])$

In[32]:=
```
constraintsEq
```
Out[32]= $\{b\, \theta 1[t] - a\, \theta 2[t] == 0,\ -a - b + r[t] == 0\}$

In[33]:=
```
constraints = First /@ constraintsEq
```
Out[33]= $\{b\, \theta 1[t] - a\, \theta 2[t],\ -a - b + r[t]\}$

In[34]:=
```
D[constraints , θ1[t]]
```
Out[34]=
```
{b, 0}
```
In[35]:=
```
part3 = D[constraints , θ1[t]] .{λ[1], λ[2]}
```
Out[35]= $b\, \lambda[1]$

In[36]:=
```
eqθ1 = part1 - part2 == part3
```
Out[36]= $-g\, m\, r[t]\, \mathrm{Sin}[\theta 1[t]] + 2\, m\, r[t]\, r'[t]\, \theta 1'[t] + m\, r[t]^2\, \theta 1''[t] + a^2\, m\, \kappa\, (\theta 1''[t] + \theta 2''[t]) == b\, \lambda[1]$

In[37]:=
```
eqMotion [[2]]
```
Out[37]= $-g\, m\, r[t]\, \mathrm{Sin}[\theta 1[t]] + 2\, m\, r[t]\, r'[t]\, \theta 1'[t] + m\, r[t]^2\, \theta 1''[t] + a^2\, m\, \kappa\, (\theta 1''[t] + \theta 2''[t]) == b\, \lambda[1]$

In[38]:=
```
eqθ1 == eqMotion [[2]]
```

Out[38]= True

The θ2 equation

In[39]:= `part2 = D[L, θ2[t]]`

Out[39]= 0

In[40]:= `D[L, θ2'[t]]`

Out[40]= $a^2 m \kappa (\theta 1'[t] + \theta 2'[t])$

In[41]:= `part1 = D[D[L, θ2'[t]], t]`

Out[41]= $a^2 m \kappa (\theta 1''[t] + \theta 2''[t])$

In[42]:= `constraintsEq`

Out[42]= {b θ1[t] – a θ2[t] == 0, – a – b + r[t] == 0}

In[43]:= `constraints = First /@ constraintsEq`

Out[43]= {b θ1[t] – a θ2[t], – a – b + r[t]}

In[44]:= `D[constraints, θ2[t]]`

Out[44]= {– a, 0}

In[45]:= `part3 = D[constraints, θ2[t]].{λ[1], λ[2]}`

Out[45]= – a λ[1]

In[46]:= `eqθ2 = part1 – part2 == part3`

Out[46]= $a^2 m \kappa (\theta 1''[t] + \theta 2''[t]) == - a \lambda[1]$

In[47]:= `eqMotion[[3]]`

Out[47]= $a^2 m \kappa (\theta 1''[t] + \theta 2''[t]) == - a \lambda[1]$

In[48]:= `eqθ2 == eqMotion[[3]]`

Out[48]= True

Part b

In[61]:= `abRule = {b → r – a};`

NEED TO FIX ORDER HERE

In[66]:=
```
({θSol , λ1Sol , λ2Sol} =
    Flatten[Solve[Flatten[eqMotion //.constraintRule ], {θ1''[t], λ[1], λ[2]}] /.Rule → Equal]) ;
{θSol , λ1Sol , λ2Sol}  // TableForm[#, TableHeadings  → {{"θSol", "λ1Sol", "λ2Sol"}, {}}] &
```

Out[67]//TableForm=

θSol	$\theta 1''[t] == \frac{g\,\mathrm{Sin}[\theta 1[t]]}{(a+b)\,(1+\kappa)}$
λ1Sol	$\lambda[1] == -\frac{g\,m\,\kappa\,\mathrm{Sin}[\theta 1[t]]}{1+\kappa}$
λ2Sol	$\lambda[2] == g\,m\,\mathrm{Cos}[\theta 1[t]] - a\,m\,\theta 1'[t]^2 - b\,m\,\theta 1'[t]^2$

In[71]:=
```
{θSol , λ1Sol , λ2Sol}  = {θSol , λ1Sol , λ2Sol}  /. abRule  // Simplify  // Factor
```

Out[71]=
$$\left\{\frac{g\,\mathrm{Sin}[\theta 1[t]]}{r\,(1+\kappa)} == \theta 1''[t],\ \lambda[1] == -\frac{g\,m\,\kappa\,\mathrm{Sin}[\theta 1[t]]}{1+\kappa},\ g\,m\,\mathrm{Cos}[\theta 1[t]] == \lambda[2] + m\,r\,\theta 1'[t]^2\right\}$$

In[72]:=
```
eqθ = ( θSol[[1]] - θSol[[2]])
```

Out[72]=
$$\frac{g\,\mathrm{Sin}[\theta 1[t]]}{r\,(1+\kappa)} - \theta 1''[t]$$

DO INDEFINITE INTEGRAL HERE, AND AFTER SUBSTITUTE IN LIMITS

In[73]:=
```
intθ[t_] = Integrate[eqθ  θ1'[t], t]
```

Out[73]=
$$-\frac{g\,\mathrm{Cos}[\theta 1[t]]}{r\,(1+\kappa)} - \frac{1}{2}\,\theta 1'[t]^2$$

In[74]:=
```
defIntegral  = intθ[t] - intθ[0]
```

Out[74]=
$$\frac{g\,\mathrm{Cos}[\theta 1[0]]}{r\,(1+\kappa)} - \frac{g\,\mathrm{Cos}[\theta 1[t]]}{r\,(1+\kappa)} + \frac{1}{2}\,\theta 1'[0]^2 - \frac{1}{2}\,\theta 1'[t]^2$$

In[75]:=
```
defIntegral  = defIntegral   /. {θ1[0] → 0, θ1'[0] → 0}  // Simplify
```

Out[75]=
$$\frac{g - g\,\mathrm{Cos}[\theta 1[t]]}{r + r\,\kappa} - \frac{1}{2}\,\theta 1'[t]^2$$

In[76]:=
```
solθ = Solve[defIntegral  == 0, {θ1'[t]}][[1]] /. √(-1 + Cos[x_]) → ⅈ √(1 - Cos[x])
```

Out[76]=
$$\left\{\theta 1'[t] \to -\frac{\sqrt{2}\,\sqrt{g - g\,\mathrm{Cos}[\theta 1[t]]}}{\sqrt{r + r\,\kappa}}\right\}$$

Part c

In[77]:=
```
eq1 = λ2Sol /. λ[2] → 0 //. solθ  // Simplify
```

Out[77]=
$$\frac{g\,m\,(-2 + (3+\kappa)\,\mathrm{Cos}[\theta 1[t]])}{1+\kappa} == 0$$

In[78]:=
```
sol2=Solve[eq1,Cos[θ1[t]]]   //Flatten
```

Out[78]=
$$\left\{\mathrm{Cos}[\theta 1[t]] \to \frac{2}{3+\kappa}\right\}$$

```
In[79]:= sol3=Solve[eq1,θ1[t]] /.{C[_]:→0}//Flatten
```

Out[79]= $\left\{\theta1[t] \to -\text{ArcCos}\left[\frac{2}{3+\kappa}\right],\ \theta1[t] \to \text{ArcCos}\left[\frac{2}{3+\kappa}\right]\right\}$

In[80]:= $\theta1[t]$ /. sol3[[2]] /. $\left\{\kappa \to \left\{0, 1, \frac{1}{2}, \frac{2}{5}\right\}\right\}$

Out[80]= $\left\{\text{ArcCos}\left[\frac{2}{3}\right],\ \frac{\pi}{3},\ \text{ArcCos}\left[\frac{4}{7}\right],\ \text{ArcCos}\left[\frac{10}{17}\right]\right\}$

In[81]:= $\frac{\theta1[t]}{\text{Degree}}$ /. sol3[[2]] /. $\left\{\kappa \to \left\{0, 1, \frac{1}{2}, \frac{2}{5}\right\}\right\}$ // N

Out[81]= {48.1897 , 60. , 55.1501 , 53.9681}

Problem 3: Sphere Rolling on a Fixed Sphere With 1 Lagrange Parameters $\{\lambda_1\}$

Remarks and outline

Solution

```
In[137]:= Clear["Global`*"]
```

••• Clear : Symbol Lag is Protected .

Part a

Start with some basics

In[138]:= $\text{Tcm} = \frac{1}{2}\,m\left(r'[t]^2 + r[t]^2\ \theta1'[t]^2\right)$;

In[139]:= $\text{Trot} = \frac{1}{2}\,\text{I1}\left(\theta1'[t] + \theta2'[t]\right)^2$;

```
In[140]:= T=Tcm+Trot;
In[141]:= V= m g r[t] Cos[θ1[t]];
```

In[142]:= iRule = $\{\text{I1} \to \kappa\, m\, a^2\}$ (* Moment of inertia is some factor "κ" times m a^2 *)

Out[142]= $\{\text{I1} \to a^2\, m\, \kappa\}$

In[143]:=
```
L=T-V /.iRule
```

Out[143]= $-g\, m \operatorname{Cos}[\theta 1[t]]\, r[t]+\frac{1}{2} m\left(r'[t]^2+r[t]^2\, \theta 1'[t]^2\right)+\frac{1}{2} a^2\, m\, \kappa\, (\theta 1'[t]+\theta 2'[t])^2$

In[144]:=
```
constraintsEq2= {b θ1[t] - a θ2[t]==0, r[t]-a-b==0 }
```

Out[144]= $\{b\, \theta 1[t]-a\, \theta 2[t]==0,\ -a-b+r[t]==0\}$

In[145]:=
```
constraintRuleθ2 = DSolve[constraintsEq2 [[1]], {θ2}, t][[1]]
```

Out[145]= $\left\{\theta 2 \to \operatorname{Function}\left[\{t\}, \frac{b\, \theta 1[t]}{a}\right]\right\}$

In[146]:=
```
L = L /. constraintRuleθ2
```

Out[146]= $-g\, m \operatorname{Cos}[\theta 1[t]]\, r[t]+\frac{1}{2} a^2\, m\, \kappa\left(\theta 1'[t]+\frac{b\, \theta 1'[t]}{a}\right)^2+\frac{1}{2} m\left(r'[t]^2+r[t]^2\, \theta 1'[t]^2\right)$

In[147]:=
```
constraintsEq= constraintsEq2[[2]] (* We only have 1 constraint equation left now *)
```

Out[147]= $-a-b+r[t]==0$

In[148]:=
```
constraintRule = DSolve[constraintsEq , {r}, t][[1]]
```

Out[148]= $\{r \to \operatorname{Function}[\{t\}, a+b]\}$

In[149]:=
```
vars = {r[t], θ1[t]};
```

In[150]:=
```
Qeq = λ[1] * D[constraintsEq [[1]], vars[[#]]] & /@ {1, 2}
```

Out[150]= $\{\lambda[1], 0\}$

We'll use the "Magic" function.
In general, I don't like "black boxes"

In[151]:=
```
(eqMotion = Lag[{r, θ1}, L, Qeq]) ;
eqMotion // TableForm
```

Out[152]//TableForm=

$g\, m \operatorname{Cos}[\theta 1[t]]-m\, r[t]\, \theta 1'[t]^2+m\, r''[t]==\lambda[1]$

$-g\, m\, r[t] \operatorname{Sin}[\theta 1[t]]+2\, m\, r[t]\, r'[t]\, \theta 1'[t]+m\, r[t]^2\, \theta 1''[t]+a^2\left(1+\frac{b}{a}\right) m\, \kappa\left(\theta 1''[t]+\frac{b\, \theta 1''[t]}{a}\right)==0$

Let's get the equations by hand:

The R equation

In[153]:=
```
part2 = D[L, r[t]]
```

Out[153]= $-g\, m \operatorname{Cos}[\theta 1[t]]+m\, r[t]\, \theta 1'[t]^2$

In[154]:=
```
D[L, r '[t]]
```

Out[154]= m r′[t]

```
In[155]:= part1 = D[D[L, r '[t]], t]
```

Out[155]= m r″[t]

```
In[156]:= constraintsEq
```

Out[156]= - a - b + r[t] == 0

```
In[157]:= constraints = constraintsEq [[1]]
```

Out[157]= - a - b + r[t]

```
In[158]:= D[constraints , r[t]]
```

Out[158]= 1

```
In[159]:= part3 = D[constraints , r[t]] × λ[1]
```

Out[159]= λ[1]

```
In[160]:= eqR = part1 - part2  ==  part3
```

Out[160]= $g\, m \,\mathrm{Cos}[\theta 1[t]] - m\, r[t]\, \theta 1'[t]^2 + m\, r''[t] == \lambda[1]$

```
In[161]:= eqMotion [[1]]
```

Out[161]= $g\, m \,\mathrm{Cos}[\theta 1[t]] - m\, r[t]\, \theta 1'[t]^2 + m\, r''[t] == \lambda[1]$

```
In[162]:= eqR == eqMotion [[1]] // Simplify
```

Out[162]= True

The θ1 equation

```
In[163]:= part2 = D[L, θ1[t]]
```

Out[163]= g m r[t] Sin[θ1[t]]

```
In[164]:= D[L, θ1 '[t]]
```

Out[164]= $m\, r[t]^2\, \theta 1'[t] + a^2 \left(1 + \frac{b}{a}\right) m\, \kappa \left(\theta 1'[t] + \frac{b\, \theta 1'[t]}{a}\right)$

```
In[165]:= part1 = D[D[L, θ1 '[t]], t]
```

Out[165]= $2\, m\, r[t]\, r'[t]\, \theta 1'[t] + m\, r[t]^2\, \theta 1''[t] + a^2 \left(1 + \frac{b}{a}\right) m\, \kappa \left(\theta 1''[t] + \frac{b\, \theta 1''[t]}{a}\right)$

```
In[166]:= constraintsEq
```

Out[166]= - a - b + r[t] == 0

```
In[167]:= constraints = constraintsEq [[1]]
```

Out[167]= - a - b + r[t]

```
In[168]:= D[constraints , θ1[t]]
```

Out[168]= 0

In[169]:= `part3 = D[constraints , θ1[t]] × λ[1]`

Out[169]= 0

In[170]:= `eqθ1 = part1 - part2 == part3`

Out[170]= $-g\, m\, r[t]\, Sin[\theta 1[t]] + 2\, m\, r[t]\, r'[t]\, \theta 1'[t] + m\, r[t]^2\, \theta 1''[t] + a^2 \left(1 + \frac{b}{a}\right) m\, \kappa \left(\theta 1''[t] + \frac{b\, \theta 1''[t]}{a}\right) == 0$

In[171]:= `eqMotion [[2]]`

Out[171]= $-g\, m\, r[t]\, Sin[\theta 1[t]] + 2\, m\, r[t]\, r'[t]\, \theta 1'[t] + m\, r[t]^2\, \theta 1''[t] + a^2 \left(1 + \frac{b}{a}\right) m\, \kappa \left(\theta 1''[t] + \frac{b\, \theta 1''[t]}{a}\right) == 0$

In[172]:= `eqθ1 == eqMotion [[2]]`

Out[172]= True

Part b

NEED TO FIX ORDER HERE

In[173]:=
```
({θSol , λ1Sol} =
    Flatten[Solve[Flatten[eqMotion //.constraintRule ], {θ1''[t], λ[1]}] /.Rule → Equal]);
{θSol , λ1Sol}  // TableForm[#, TableHeadings  → {{"θSol", "λ1Sol"}, {}}] &
```

Out[174]//TableForm=

θSol	$\theta 1''[t] == \frac{g\, Sin[\theta 1[t]]}{(a+b)\,(1+\kappa)}$
λ1Sol	$\lambda[1] == g\, m\, Cos[\theta 1[t]] - a\, m\, \theta 1'[t]^2 - b\, m\, \theta 1'[t]^2$

In[187]:= `abRule = {b → r - a};`

In[189]:= `{θSol , λ1Sol} = {θSol , λ1Sol}  /. abRule  // Simplify  // Factor`

Out[189]= $\left\{\frac{g\, Sin[\theta 1[t]]}{(1+\kappa)\, r[t]} == \theta 1''[t],\ g\, m\, Cos[\theta 1[t]] == \lambda[1] + m\, r[t]\, \theta 1'[t]^2\right\}$

In[190]:= `{θSol , λ1Sol}  = {θSol , λ1Sol}  /. {r'[t] → 0,  r''[t] → 0,  r[t] → r} // Simplify`

Out[190]= $\left\{\frac{g\, Sin[\theta 1[t]]}{r + r\, \kappa} == \theta 1''[t],\ g\, m\, Cos[\theta 1[t]] == \lambda[1] + m\, r\, \theta 1'[t]^2\right\}$

In[192]:= `{θSol , λ1Sol}  = {θSol , λ1Sol}  /. {(a + b) → r[t]} // Simplify  // Factor`

Out[192]= $\left\{\frac{g\, Sin[\theta 1[t]]}{r\,(1 + \kappa)} == \theta 1''[t],\ g\, m\, Cos[\theta 1[t]] == \lambda[1] + m\, r\, \theta 1'[t]^2\right\}$

In[193]:= `eqθ = ( θSol[[1]] - θSol[[2]])`

Out[193]= $\frac{g\, Sin[\theta 1[t]]}{r\,(1 + \kappa)} - \theta 1''[t]$

DO INDEFINITE INTEGRAL HERE, AND AFTER SUBSTITUTE IN LIMITS

In[194]:=
```
intθ[t_] = Integrate[eqθ θ1'[t], t]
```
Out[194]=
$$-\frac{g\,\mathrm{Cos}[\theta 1[t]]}{r\,(1+\kappa)}-\frac{1}{2}\,\theta 1'[t]^2$$

In[195]:=
```
defIntegral = intθ[t] - intθ[0]
```
Out[195]=
$$\frac{g\,\mathrm{Cos}[\theta 1[0]]}{r\,(1+\kappa)}-\frac{g\,\mathrm{Cos}[\theta 1[t]]}{r\,(1+\kappa)}+\frac{1}{2}\,\theta 1'[0]^2-\frac{1}{2}\,\theta 1'[t]^2$$

In[196]:=
```
defIntegral = defIntegral /. {θ1[0] → 0, θ1'[0] → 0} // Simplify
```
Out[196]=
$$\frac{g-g\,\mathrm{Cos}[\theta 1[t]]}{r+r\,\kappa}-\frac{1}{2}\,\theta 1'[t]^2$$

In[197]:=
```
solθ = Solve[defIntegral == 0, {θ1'[t]}][[1]] /. Sqrt[-1 + Cos[x_]] → ⅈ Sqrt[1 - Cos[x]]
```
Out[197]=
$$\left\{\theta 1'[t]\to-\frac{\sqrt{2}\,\sqrt{g-g\,\mathrm{Cos}[\theta 1[t]]}}{\sqrt{r+r\,\kappa}}\right\}$$

Part c

In[198]:=
```
eq1 = λ1Sol /. λ[1] → 0 //. solθ // Simplify
```
Out[198]=
$$\frac{g\,m\,(-2+(3+\kappa)\,\mathrm{Cos}[\theta 1[t]])}{1+\kappa}==0$$

In[199]:=
```
sol2=Solve[eq1,Cos[θ1[t]]] //Flatten
```
Out[199]=
$$\left\{\mathrm{Cos}[\theta 1[t]]\to\frac{2}{3+\kappa}\right\}$$

In[200]:=
```
sol3=Solve[eq1,θ1[t]] /.{C[_]:→0}//Flatten
```
Out[200]=
$$\left\{\theta 1[t]\to-\mathrm{ArcCos}\left[\frac{2}{3+\kappa}\right],\ \theta 1[t]\to\mathrm{ArcCos}\left[\frac{2}{3+\kappa}\right]\right\}$$

In[201]:=
```
θ1[t] /. sol3[[2]] /. {κ → {0, 1, 1/2, 2/5}}
```
Out[201]=
$$\left\{\mathrm{ArcCos}\left[\frac{2}{3}\right],\ \frac{\pi}{3},\ \mathrm{ArcCos}\left[\frac{4}{7}\right],\ \mathrm{ArcCos}\left[\frac{10}{17}\right]\right\}$$

In[202]:=
```
θ1[t]/Degree /. sol3[[2]] /. {κ → {0, 1, 1/2, 2/5}} // N
```
Out[202]=
```
{48.1897 , 60. , 55.1501 , 53.9681}
```