Homework #6: Phys 3344: Prof. Olness Fall 2020

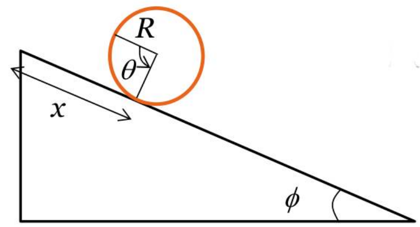
Due 30 September, 2020

1) Find the shortest distance between two points located on the surface of a cylinder.

2) Solve the brachistochrone as outline in the book. Add in all the intermediate steps.
(The book leaves out quite a bit. I want to see the trig substitutions and integration done by hand!)
Plot the resulting curves.

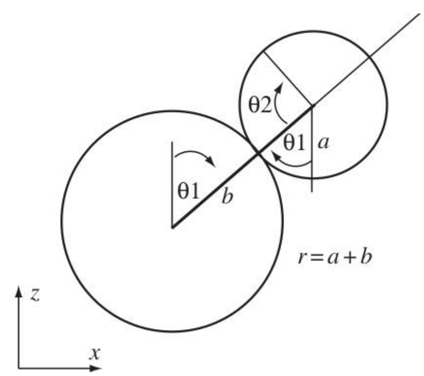
3) Consider light passing from one medium to another with indices of refraction of {n1,n2}. Use Fermat's principle to minimize the time and find the resulting law of refraction.

4) Consider a hoop of mass m and moment of inertia I=m R^2 sliding down an incline of angle ϕ a distance x (along the incline). The hoop rolls WITHOUT slipping.. <u>Do this using the Lagrange equations.</u> Find the acceleration and the Lagrange multiplier λ. Note, you will have a constraint equation: Rθ - x=0.

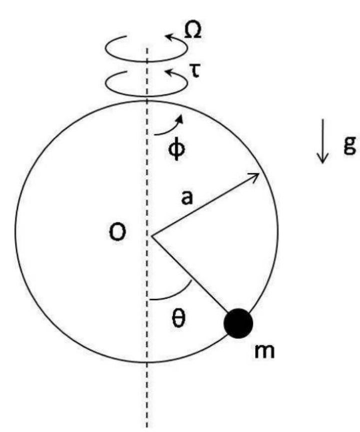


5) Atwood Machine: Consider an Atwood machine with masses m1 and m2 and lengths x1+x2=L. The pulley is massless and frictionless. Compute the acceleration of each mass.
a) Do this using either forces or energy. Also find the tension T in the string.
b) Do this using the Lagrange equations of x1, and substitute x2=L-x1.
c) Do this using the Lagrange equations of {x1,x2} and use a Lagrange multiplier λ.

6) A uniform hoop of mass m and radius a rolls without slipping on a fixed cylinder of radius b. You may take the moment of inertia of the hoop to be **I= ma^2**.The only external force is gravity. If the smaller cylinder starts rolling from rest on top of the larger cylinder,
find (using Lagrange multipliers) the point at which the hoop falls off the cylinder. [See my hints.]



7) A bead on a circular hoop is spinning at frequency w. Find all equilibrium points, and identify if they are stable or unstable. (It could be w dependent.) For the stable points, compute the frequency of small oscillations.



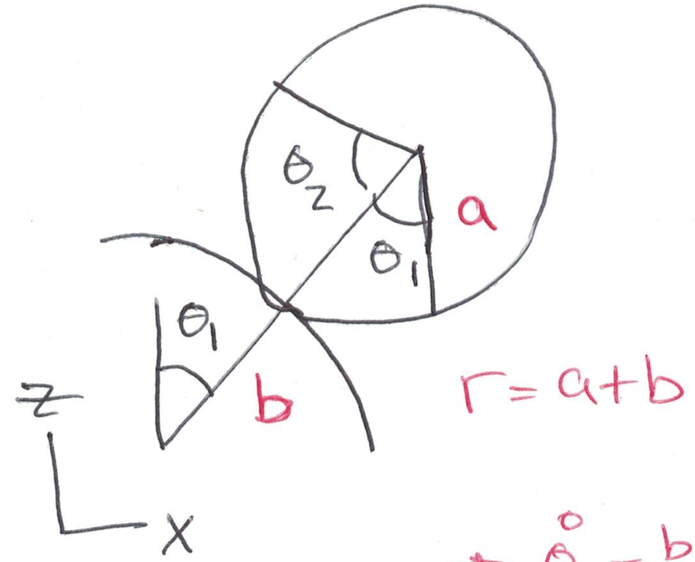
Homework #6

Constraint Equations

① $r = a + b$

② $b\theta_1 - a\theta_2 = 0$

use ② to eliminate θ_2



$$\theta_2 = \frac{b}{a}\theta_1$$

$$\dot{\theta}_2 = \frac{b}{a}\dot{\theta}_1$$

$$T = \frac{m}{2}\left[\dot{r}^2 + r^2\dot{\theta}_1^2\right] + \frac{I}{2}\left[\left(\dot{\theta}_1 + \dot{\theta}_2\right)^2\right]$$

Linear T — Rotational T

$$V = mgh = mgr\cos\theta_1$$

Equation of Constraint: $F \equiv r - a - b = 0$

$$\frac{\partial F}{\partial r} = 1 \qquad \frac{\partial F}{\partial \theta} = 0$$

Variables $\{r, \dot{r}, \theta_1, \dot{\theta}_1\}$ we have eliminated θ_2

$$\frac{\partial L}{\partial r} - \frac{d}{dt}\frac{\partial L}{\partial \dot{r}} = \lambda \frac{\partial F}{\partial r}$$

$\boxed{\text{Hint}}$ $\dot{r} = \ddot{r} = 0$

$$\frac{\partial L}{\partial \theta_1} - \frac{d}{dt}\frac{\partial L}{\partial \dot{\theta}_1} = \lambda \frac{\partial F}{\partial \theta_1}$$

Hint 2

Homework #6

Compute $\{R_{eq}, \theta_{eq}\}$

Use $\dot{r} = \ddot{r} = 0 \qquad r = a+b$

Also, lets assume $I = K m a^2$
where we will choose K at the end
This will simplify the algebra.

$\boxed{\theta_{eq}}$ This will be of the form

$$\ddot{\theta}_1 - [Z] \sin\theta_1 = 0$$

where $[Z]$ is a bunch of factors

We will integrate this.

$$\int d\theta_1 \left[\ddot{\theta}_1 - [Z]\sin\theta_1\right] = \left[\frac{1}{2}\dot{\theta}_1 + [Z]\cos\theta_1\right]$$

$\dot{\theta} = \theta = 0$

$$= \left[\frac{1}{2}\dot{\theta}_1 + [Z]\cos\theta\right] - \left[0 + [Z]\cos(0)\right]$$

$\hookrightarrow 1$

$$\Rightarrow \frac{1}{2}\dot{\theta}_1 + [Z](\cos\theta - 1) = 0$$

$$\dot{\theta}_1 = 2[Z](1 - \cos\theta_1)$$

Hint 3

$\boxed{Req}$ is of the form

$$[\ \ ]\cos\theta_1 = [\ \ ]\dot{\theta}_1^2 + \lambda$$

Substitute for $\dot{\theta}_1^2$

Set $\lambda = 0$

Solve for $\cos\theta_1$

$$[\ \ ]\cos\theta_1 + [\ \ ](1-\cos\theta_1) = 0$$

$$\cos\theta_1 = [\ \ ]$$

$$\theta_1 = \text{Arc}\cos[\ \ ] = \text{XXX degrees}$$