1) Sun & Earth

Part a)

In[7]:=
```
Clear["Global`*"]
```

In[8]:=
```
(* The answers *)
```
$mSunAnswer = 2 \times 10^{30};$

$mEarthAnswer = 6 \times 10^{24};$

In[1]:=

$earthOrbit = 1.49 \times 10^{8}\ km;$

$moonOrbit = 3.8 \times 10^{5}\ km;\ m$

Out[2]= m

In[12]:=

$$eq1 = \left\{\frac{G\ m1\ m2}{r^2} == m1\ \frac{v^2}{r},\ v == \frac{2\ \pi\ r}{period}\right\};$$

In[13]:=
```
sol = Solve[eq1, {m2, v}][[1]]
```

Out[13]=

$$\left\{m2 \to \frac{4\ \pi^2\ r^3}{G\ period^2},\ v \to \frac{2\ \pi\ r}{period}\right\}$$

In[14]:=
```
mass = m2 /. sol
```

Out[14]=

$$\frac{4\ \pi^2\ r^3}{G\ period^2}$$

In[15]:=
```
mSun = mass /. {r → earthOrbit, period → 365.25 days}
```

Out[15]=

$$\frac{9.78899 \times 10^{20}\ km^3}{days^2\ G}$$

In[16]:=
```
mEarth = mass /. {r → moonOrbit, period → 27.3 days}
```

Out[16]=

$$\frac{2.9066 \times 10^{15}\ km^3}{days^2\ G}$$

In[17]:=

$$\frac{mSun}{mEarth}\ //\ ScientificForm$$

Out[17]//ScientificForm=

3.36785×10^{5}

In[18]:=

$$\frac{mSunAnswer}{mEarthAnswer}\ //\ N\ //\ ScientificForm$$

Out[18]//ScientificForm=

3.33333×10^{5}

Part b) Measure relative to the center of the sun

In[28]:=
```
Clear["Global`*"]
```

In[37]:=
```
mSun = 2.0 × 10^30;
mEarth = 6.0 × 10^24;
dEarthSun = 149. × 10^9;
radiusSun = 696 × 10^6;
```

In[42]:=
$$\text{reducedMass} = \frac{\text{mSun mEarth}}{\text{mSun} + \text{mEarth}}$$

Out[42]= 5.99998×10^{24}

In[43]:=
$$\frac{\text{reducedMass}}{\text{mSun}}$$

Out[43]= 2.99999×10^{-6}

Part c) Measure relative to the center of the sun

In[28]:=
```
Clear["Global`*"]
```

In[37]:=
```
mSun = 2.0 × 10^30;
mEarth = 6.0 × 10^24;
dEarthSun = 149. × 10^9;
radiusSun = 696 × 10^6;
```

In[36]:=
$$\text{rcms} = \frac{\text{mSun} * 0 + \text{mEarth dEarthSun}}{\text{mSun} + \text{mEarth}}$$

Out[36]= 446 999.

In[41]:=
$$\frac{\text{rcms}}{\text{radiusSun}}$$

Out[41]= 0.000642239

2) Orbits

In[55]:=
```
Clear["Global`*"]
```

In[56]:=
```
? PolarPlot
```

Out[56]=

Symbol

PolarPlot [r, {θ, θ_{min}, θ_{max}}] generates a polar plot of a curve with radius r as a function of angle θ.

PolarPlot [{r_1, r_2, …}, {θ, θ_{min}, θ_{max}}] makes a polar plot of curves with radius functions r_1, r_2, ….

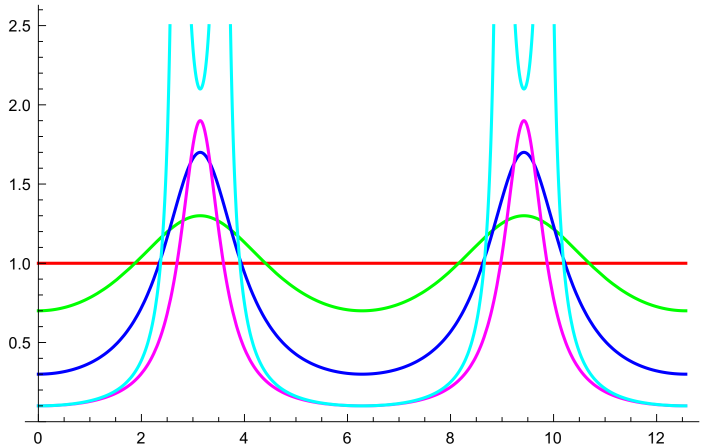
In[57]:=
```
r[θ_, e_ : 0, a_ : 1] = a (1 - e^2) / (1 + e Cos[θ])
```

Out[57]=

$$\frac{a\left(1-e^2\right)}{1+e\,\text{Cos}[\theta]}$$

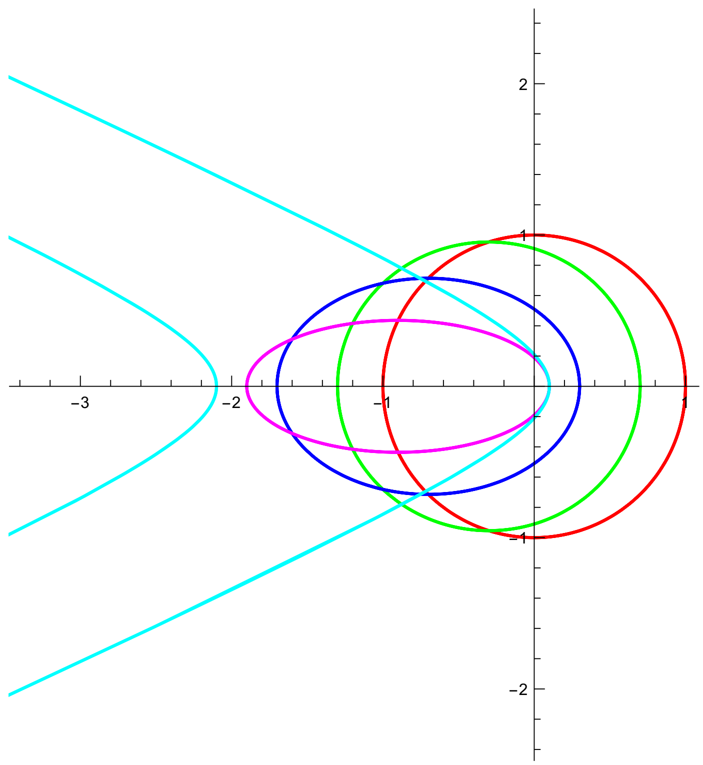
In[72]:=
```
Plot[{r[θ, 0.0], r[θ, 0.3], r[θ, 0.7], r[θ, 0.9], r[θ, 1.1]} // Abs // Evaluate, {θ, 0, 4 π},
 PlotStyle → {Red, Green, Blue, Magenta, Cyan}, PlotRange → {0, Automatic}]
```

Out[72]=



In[75]:=
```
PolarPlot[{r[θ, 0.0], r[θ, 0.3], r[θ, 0.7], r[θ, 0.9], r[θ, 1.1]} // Abs // Evaluate,
 {θ, 0, 4 π}, PlotStyle → {Red, Green, Blue, Magenta, Cyan}, PlotRange → Automatic]
```

Out[75]=



In[77]:=
```
eq1 = D[r[θ, e, a], θ] == 0
```
Out[77]=
$$\frac{a\, e\, (1 - e^2)\, \text{Sin}[\theta]}{(1 + e\, \text{Cos}[\theta])^2} == 0$$

In[79]:=
```
Solve[eq1, θ] /. {C[_] :→ 0}
```
Out[79]= $\{\{\theta \to 0\}, \{\theta \to \pi\}\}$

3) Small Meteroid

In[3]:=
```
Clear["Global`*"]
```

Part a)

In[10]:=
$$\text{eq1} = \frac{G\, M\, m}{r^2} == m\, \frac{v^2}{r};$$
```
vSol = Solve[eq1, v][[2]]
```
Out[11]=
$$\left\{v \to \frac{\sqrt{G}\, \sqrt{M}}{\sqrt{r}}\right\}$$

In[12]:=
```
angMom = m r v /. vSol
```
Out[12]= $\sqrt{G}\, m\, \sqrt{M}\, \sqrt{r}$

In[14]:=
$$\text{energy0} = \frac{1}{2} m\, v^2 - \frac{G\, M\, m}{r} \;/.\; \text{vSol}$$
Out[14]=
$$-\frac{G\, m\, M}{2\, r}$$

Part b) After explosing, v is the same, the mass of the Sun “M” reduces from M -> κ M, where we will determin κ. If it is just barely bound, the new energy will be zero.

In[15]:=
$$\text{energy1} = \frac{1}{2} m\, v^2 - \frac{G\, \kappa\, M\, m}{r} \;/.\; \text{vSol}$$
Out[15]=
$$\frac{G\, m\, M}{2\, r} - \frac{G\, m\, M\, \kappa}{r}$$

In[18]:=
```
Solve[energy1 == 0, κ][[1]]
```
Out[18]=
$$\left\{\kappa \to \frac{1}{2}\right\}$$

Part 3) We will conserve energy and angular Momentum

In[45]:=
```
energy2 = energy1 /. {κ → 3/4}
```
Out[45]= $-\frac{G\, m\, M}{4\, r}$

In[38]:=
```
angMom == m rMax vMax
```
Out[38]= $\sqrt{G}\, m\, \sqrt{M}\, \sqrt{r} == m\, rMax\, vMax$

In[46]:=
```
kineticEnergyMax = 1/2 m vMax^2
```
Out[46]= $\frac{m\, vMax^2}{2}$

In[47]:=
```
potentialEnergyMax = -(G κ M m)/rMax /. {κ → 3/4}
```
Out[47]= $-\frac{3\, G\, m\, M}{4\, rMax}$

In[49]:=
```
totalEnergy = kineticEnergyMax + potentialEnergyMax
```
Out[49]= $-\frac{3\, G\, m\, M}{4\, rMax} + \frac{m\, vMax^2}{2}$

In[50]:=
```
eqs = {angMom == m rMax vMax, energy2 == totalEnergy}
```
Out[50]= $\left\{\sqrt{G}\, m\, \sqrt{M}\, \sqrt{r} == m\, rMax\, vMax,\ -\frac{G\, m\, M}{4\, r} == -\frac{3\, G\, m\, M}{4\, rMax} + \frac{m\, vMax^2}{2}\right\}$

In[52]:=
```
Solve[eqs, {rMax, vMax}][[1]]
```
Out[52]= $\left\{rMax \to 2\, r,\ vMax \to \frac{\sqrt{G}\, \sqrt{M}}{2\, \sqrt{r}}\right\}$

---

4) Haleys Comet

In[200]:=
```
Clear["Global`*"]
```

In[201]:=
```
kepler = period^2 == α r^3
```
Out[201]= $period^2 == r^3\, \alpha$

In[202]:=
```
eqEarth = kepler /. {period → 1 year, r → 1 au}
```
Out[202]= $year^2 == au^3\, \alpha$

In[203]:= eqComet = kepler /. $\left\{\text{period} \to 75.6\ \text{year},\ r \to \frac{(0.570 + \text{rMax})}{2}\ \text{au}\right\}$

Out[203]= $5715.36\ \text{year}^2 == \frac{1}{8}\ \text{au}^3\ (0.57 + \text{rMax})^3\ \alpha$

In[204]:= eqs = {eqEarth , eqComet}

Out[204]= $\left\{\text{year}^2 == \text{au}^3\ \alpha,\ 5715.36\ \text{year}^2 == \frac{1}{8}\ \text{au}^3\ (0.57 + \text{rMax})^3\ \alpha\right\}$

In[205]:= Solve[eqs, {α, rMax}]

Out[205]= $\left\{\left\{\alpha \to \frac{\text{year}^2}{\text{au}^3},\ \text{rMax} \to -18.4492 - 30.9677\ i\right\},\ \left\{\alpha \to \frac{\text{year}^2}{\text{au}^3},\ \text{rMax} \to -18.4492 + 30.9677\ i\right\},\ \left\{\alpha \to \frac{\text{year}^2}{\text{au}^3},\ \text{rMax} \to 35.1884\right\}\right\}$

5) Escape Velocity

In[180]:= Clear["Global`*"]

In[181]:= mSun = 2×10^{30};
mEarth = 6×10^{24};
rEarth = 6000×10^{3};

In[184]:= eq1 = $\frac{-G\,M\,m}{r} + \frac{1}{2}\,m\,v^2$

Out[184]= $-\frac{G\,m\,M}{r} + \frac{m\,v^2}{2}$

In[185]:= sol = Solve[eq1 == 0, v][[2]]

Out[185]= $\left\{v \to \frac{\sqrt{2}\ \sqrt{G}\ \sqrt{M}}{\sqrt{r}}\right\}$

In[196]:= values = {G → 6.67×10^{-11}, c → 3.0×10^{8}}

Out[196]= {G → 6.67×10^{-11}, c → $3. \times 10^{8}$}

In[187]:= v /. sol /. values /. {r → rEarth , M -> mEarth}

Out[187]= 11 549.9

In[189]:= eq2 = c == v /. sol

Out[189]= $c == \frac{\sqrt{2}\ \sqrt{G}\ \sqrt{M}}{\sqrt{r}}$

In[192]:=
```
cSol = Solve[eq2, r][[1]]
```

Out[192]=

$$\left\{r \to \frac{2\,G\,M}{c^2}\right\}$$

In[198]:=
```
r /. cSol /. values /. {M -> mEarth}
```

Out[198]= 0.00889333

In[199]:=
```
r /. cSol /. values /. {M → mSun}
```

Out[199]= 2964.44