

Eigen values + Eigen vectors [see wiki]

For matrix A we want to find v

$$\text{where } Av = \lambda v \Rightarrow (A - \lambda I)v = 0$$

$$\circ \circ \quad |A - \lambda I| = 0 \quad \text{where "}||" is the determinant.}$$

Example: $A = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$

$$|A - \lambda I| = \left| \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right| = \begin{vmatrix} 2-\lambda & 1 \\ 1 & 2-\lambda \end{vmatrix}$$

$$= \lambda^2 - 4\lambda + 3 = 0 = (\lambda - 1)(\lambda - 3) = 0$$

$$\lambda = 1, 3$$

Eigen vectors

For $\lambda = 1$ solve $Av = \lambda v$

$$\text{where } v = \begin{pmatrix} a \\ b \end{pmatrix}$$

Find a, b

$$\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = 1 \cdot \begin{pmatrix} a \\ b \end{pmatrix}$$

$$\left. \begin{array}{l} 2a + b = a \\ a + 2b = b \end{array} \right\} \Rightarrow a = -b \Rightarrow v = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

or we can normalize

$$v = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

Eigen vector For $\lambda = 3$

$$\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = 3 \begin{pmatrix} a \\ b \end{pmatrix} \Rightarrow$$

$$\begin{array}{l} 2a + b = 3a \\ a + 2b = 3b \end{array}$$

$$\Rightarrow a = b$$

$$v = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\text{or normalized } v = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$