1) 9.8 Coriolis and Centrifugal Forces

In[122]:=
```
Clear["Global`*"]
```

a) moving south near north pole

In[123]:=
```
(* a: moving south near north pole *)
Ω = Ω0 {0, 0, 1};
r = {0, Sin[θ], Cos[θ]};
v = v0 {0, +Cos[θ], -Sin[θ]};
```

In[126]:=
```
(* Direction : west: +x direction *)
coriolis = 2 m Cross[v, Ω]
```

Out[126]= {2 m v0 Ω0 Cos[θ], 0, 0}

In[127]:=
```
(* Direction : up and south: +y direction *)
centrifugal = m Cross[Cross[Ω, r], Ω]
```

Out[127]= $\{0,\ m\,\Omega 0^2\,\mathrm{Sin}[\theta],\ 0\}$

b) moving east near equator

In[128]:=
```
(* a: moving south near north pole *)
Ω = Ω0 {0, 0, 1};
r = {0, Sin[θ], Cos[θ]};
v = v0 {-1, 0, 0};
```

In[131]:=
```
(* Direction : up: +y direction *)
coriolis = 2 m Cross[v, Ω]
```

Out[131]= {0, 2 m v0 Ω0, 0}

In[132]:=
```
(* Direction : up: +y direction *)
centrifugal = m Cross[Cross[Ω, r], Ω]
```

Out[132]= $\{0,\ m\,\Omega 0^2\,\mathrm{Sin}[\theta],\ 0\}$

In[133]:=
```
centrifugal /. {θ → π / 2}
```

Out[133]= $\{0,\ m\,\Omega 0^2,\ 0\}$

c) moving south near equator

In[134]:=
```
(* a: moving south near north pole *)
Ω = Ω0 {0, 0, 1};
r = {0, Sin[θ], Cos[θ]};
v = v0 {0, 0, -1};
```

In[137]:=
```
(* Direction : up: +y direction *)
coriolis = 2 m Cross[v, Ω]
```

Out[137]= {0, 0, 0}

In[138]:=
```
(* Direction : up: +y direction *)
centrifugal = m Cross[Cross[Ω, r], Ω]
```

Out[138]= $\{0, m\,\Omega 0^2\,\mathrm{Sin}[\theta], 0\}$

In[139]:=
```
centrifugal /. {θ → π / 2}
```

Out[139]= $\{0, m\,\Omega 0^2, 0\}$

---

2) 9.9 Bullet problem

In[140]:=
```
Clear["Global`*"]
```

In[141]:=
```
Ω = Ω0 {0, 0, 1};
r = {0, Sin[θ], Cos[θ]};
v = v0 {0, -Cos[θ], Sin[θ]};
```

In[144]:=
```
(* Direction :east:-x direction *)
coriolis = 2 m Cross[v, Ω]
```

Out[144]= {-2 m v0 Ω0 Cos[θ], 0, 0}

In[145]:= values = $\left\{\theta \to 40\ \mathrm{Degree},\ v0 \to 1000,\ \Omega 0 \to \frac{2\pi}{86400},\ g \to 9.8\right\}$;

In[146]:=
```
tmp1 = coriolis[[1]] /. values
```

Out[146]= $-\frac{5}{108}\, m\,\pi\,\mathrm{Cos}[40\,°]$

In[147]:= $\frac{\mathrm{tmp1}}{m\,g}$ //. values

Out[147]= -0.011369

3) 9.13 Plumb line problem

In[148]:=
```
Clear["Global`*"]
```

In[149]:=
```
Ω = Ω0 {0, 0, 1};
r = re {0, Sin[θ], Cos[θ]};
```

In[151]:=
```
centrifugal = m Cross[Cross[Ω, r], Ω]
```

Out[151]= $\{0, m\, re\, \Omega 0^2 \, Sin[\theta], 0\}$

In[152]:=
```
(* Take part perpendicular to gravity:Cos[θ] *)
tmp1 = centrifugal[[2]] Cos[θ]
```

Out[152]= $m\, re\, \Omega 0^2\, Cos[\theta]\, Sin[\theta]$

In[153]:=
```
tanα = tmp1 / (m g)
```

Out[153]= $\frac{re\, \Omega 0^2\, Cos[\theta]\, Sin[\theta]}{g}$

In[154]:=
```
tanα // TrigReduce
```

Out[154]= $\frac{re\, \Omega 0^2\, Sin[2\, \theta]}{2\, g}$

5) Foucault Pendulum

In[155]:=
```
Clear["Global`*"]
```

In[156]:=
```
Ωz = Ω Cos[θ]
```

Out[156]= $\Omega\, Cos[\theta]$

In[157]:=
```
eta = amp Exp[-I Ωz t] Cos[ω0 t]
```

Out[157]= $amp\, e^{-i\, t\, \Omega\, Cos[\theta]}\, Cos[t\, \omega 0]$

In[158]:=
```
values = {ω0 → 16 Ω, amp → 1, Ω → 2 π}
```

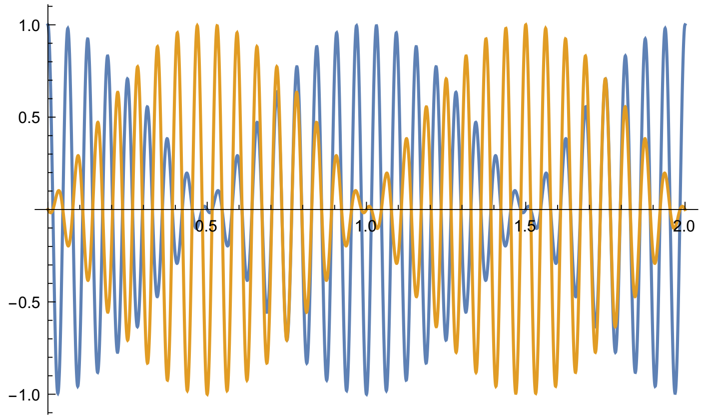
Out[158]= $\{\omega 0 \to 16\, \Omega, amp \to 1, \Omega \to 2\, \pi\}$

In[159]:=
```
f[t_, θ_] = eta //. values
```

Out[159]= $e^{-2\, i\, \pi\, t\, Cos[\theta]}\, Cos[32\, \pi\, t]$

In[160]:= Plot[{f[t, π / 3] // Re, f[t, π / 3] // Im}, {t, 0, 2}]

Out[160]=



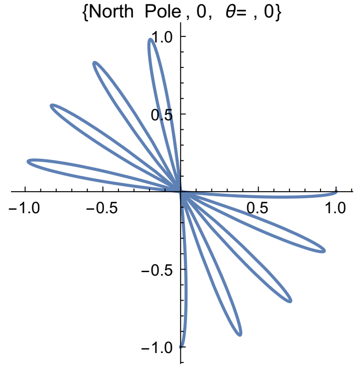
In[161]:= titles = {"North Pole", "Dallas", "Equator", "South America", "South Pole"};

In[162]:= ff[x_] := ParametricPlot[{f[t, x $\frac{\pi}{4}$] // Re, f[t, x $\frac{\pi}{4}$] // Im},
{t, 0, 1 / 4},
PlotLabel → {titles[[x + 1]], x, " θ= ", x $\frac{\pi}{4}$}]

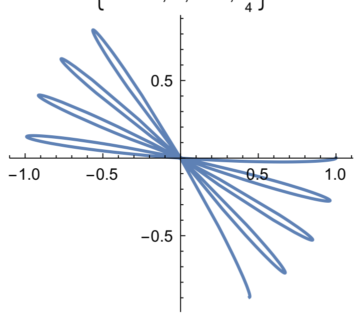
In[163]:= Table[ff[i], {i, 0, 4, 1}] // TableForm

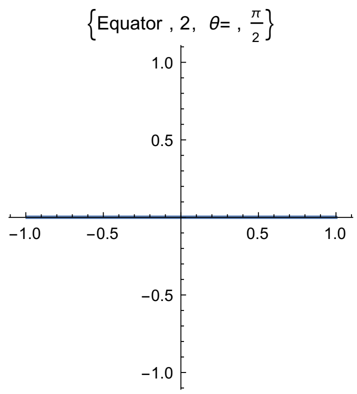
Out[163]//TableForm=

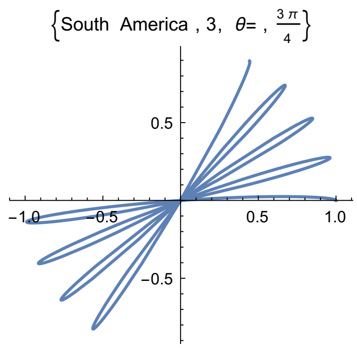
{North Pole, 0, θ= , 0}

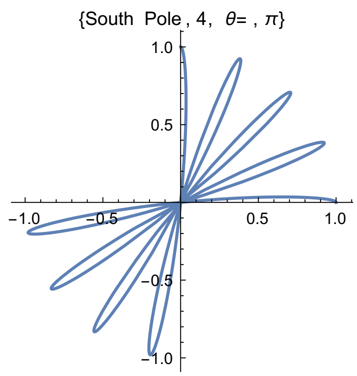


{Dallas, 1, θ= , $\frac{\pi}{4}$}









Problem 6

In[164]:=
```
Clear["Global`*"]
```

In[165]:=
```
m = {{Cos[θ], Sin[θ]}, {-Sin[θ], Cos[θ]}};
m // MatrixForm
```

Out[166]//MatrixForm=

$$\begin{pmatrix} \text{Cos}[\theta] & \text{Sin}[\theta] \\ -\text{Sin}[\theta] & \text{Cos}[\theta] \end{pmatrix}$$

In[167]:=
```
one = DiagonalMatrix [{1, 1}];
one // MatrixForm
```

Out[168]//MatrixForm=

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

In[169]:=
```
m - λ one // MatrixForm
```

Out[169]//MatrixForm=

$$\begin{pmatrix} -\lambda + \text{Cos}[\theta] & \text{Sin}[\theta] \\ -\text{Sin}[\theta] & -\lambda + \text{Cos}[\theta] \end{pmatrix}$$

In[170]:=
```
eq = Det[m - λ one] // Simplify
```

Out[170]= $1 + \lambda^2 - 2\,\lambda\,\text{Cos}[\theta]$

In[171]:=
```
sol = Solve[eq == 0, λ]
```

Out[171]= $\{\{\lambda \to \text{Cos}[\theta] - i\,\text{Sin}[\theta]\}, \{\lambda \to \text{Cos}[\theta] + i\,\text{Sin}[\theta]\}\}$

In[172]:=
```
sol // TrigToExp
```

Out[172]= $\{\{\lambda \to e^{-i\,\theta}\}, \{\lambda \to e^{i\,\theta}\}\}$

In[173]:=
```
λ1 = Exp[+I θ] // ExpToTrig
```

Out[173]= $\text{Cos}[\theta] + i\,\text{Sin}[\theta]$

In[174]:=
```
λ2 = Exp[-I θ] // ExpToTrig
```

Out[174]= $\text{Cos}[\theta] - i\,\text{Sin}[\theta]$

In[175]:=
```
vec = {a, b}
```

Out[175]= {a, b}

In[176]:=
```
eqs = m.vec == λ1 vec
```

Out[176]= $\{a\,\text{Cos}[\theta] + b\,\text{Sin}[\theta],\ b\,\text{Cos}[\theta] - a\,\text{Sin}[\theta]\} == \{a\,(\text{Cos}[\theta] + i\,\text{Sin}[\theta]),\ b\,(\text{Cos}[\theta] + i\,\text{Sin}[\theta])\}$

In[177]:=
```
Solve[eqs, {a, b}]
```

••• Solve : Equations may not give solutions for all "solve " variables .

Out[177]= $\{\{b \to i\,a\}\}$

In[178]:=
```
Solve[m.vec == λ2 vec, {a, b}]
```

••• Solve : Equations may not give solutions for all "solve " variables .

Out[178]= $\{\{b \to -i\,a\}\}$

In[179]:=
```
vec1 = {1, I};
vec2 = {1, -I};
```

In[181]:=
```
m.vec1 == λ1 vec1 // Simplify
```

Out[181]= True

In[182]:=
```
m.vec2 == λ2 vec2 // Simplify
```
Out[182]= True

In[183]:=
```
Eigensystem[m] // MatrixForm
```
Out[183]//MatrixForm=

$$\begin{pmatrix} \text{Cos}[\theta] - i\,\text{Sin}[\theta] & \text{Cos}[\theta] + i\,\text{Sin}[\theta] \\ \{i, 1\} & \{-i, 1\} \end{pmatrix}$$

Problem 7

In[184]:=
```
Clear["Global`*"]
```
In[185]:=
```
m = {{Cos[θ], Sin[θ], 0}, {-Sin[θ], Cos[θ], 0}, {0, 0, 1}};
m // MatrixForm
```
Out[186]//MatrixForm=

$$\begin{pmatrix} \text{Cos}[\theta] & \text{Sin}[\theta] & 0 \\ -\text{Sin}[\theta] & \text{Cos}[\theta] & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

In[187]:=
```
one = DiagonalMatrix [{1, 1, 1}];
one // MatrixForm
```
Out[188]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

In[189]:=
```
m - λ one // MatrixForm
```
Out[189]//MatrixForm=

$$\begin{pmatrix} -\lambda + \text{Cos}[\theta] & \text{Sin}[\theta] & 0 \\ -\text{Sin}[\theta] & -\lambda + \text{Cos}[\theta] & 0 \\ 0 & 0 & 1 - \lambda \end{pmatrix}$$

In[190]:=
```
eq = Det[m - λ one] // Simplify
```
Out[190]= $-(-1 + \lambda)\left(1 + \lambda^2 - 2\,\lambda\,\text{Cos}[\theta]\right)$

In[191]:=
```
sol = Solve[eq == 0, λ]
```
Out[191]= $\{\{\lambda \to 1\}, \{\lambda \to \text{Cos}[\theta] - i\,\text{Sin}[\theta]\}, \{\lambda \to \text{Cos}[\theta] + i\,\text{Sin}[\theta]\}\}$

In[192]:=
```
sol // TrigToExp
```
Out[192]= $\{\{\lambda \to 1\}, \{\lambda \to e^{-i\,\theta}\}, \{\lambda \to e^{i\,\theta}\}\}$

In[193]:=
```
{λ3, λ2, λ1} = λ /. sol
```
Out[193]= $\{1, \text{Cos}[\theta] - i\,\text{Sin}[\theta], \text{Cos}[\theta] + i\,\text{Sin}[\theta]\}$

In[194]:=
```
vec = {a, b, c};
eq = m.vec == λ vec
```
Out[195]= {a Cos[θ] + b Sin[θ], b Cos[θ] - a Sin[θ], c} == {a λ, b λ, c λ}

In[196]:=
```
Solve[eq /. sol[[1]], {a, b, c}]
```
Solve : Equations may not give solutions for all "solve " variables .

Out[196]= {{a → 0, b → 0}}

In[197]:=
```
Solve[eq /. sol[[2]], {a, b, c}]
```
Solve : Equations may not give solutions for all "solve " variables .

Out[197]= {{b → -ⅈ a, c → 0}}

In[198]:=
```
Solve[eq /. sol[[3]], {a, b, c}]
```
Solve : Equations may not give solutions for all "solve " variables .

Out[198]= {{b → ⅈ a, c → 0}}

In[199]:=
```
vec1 = {1, I, 0};
vec2 = {1, -I, 0};
vec3 = {0, 0, 1};
```
In[202]:=
```
m.vec1 == λ1 vec1 // Simplify
```
Out[202]= True

In[203]:=
```
m.vec2 == λ2 vec2 // Simplify
```
Out[203]= True

In[204]:=
```
m.vec3 == λ3 vec3 // Simplify
```
Out[204]= True

In[205]:=
```
Eigensystem[m] // MatrixForm
```
Out[205]//MatrixForm=

$$\begin{pmatrix} 1 & \text{Cos}[\theta] - i\,\text{Sin}[\theta] & \text{Cos}[\theta] + i\,\text{Sin}[\theta] \\ \{0, 0, 1\} & \{i, 1, 0\} & \{-i, 1, 0\} \end{pmatrix}$$

Problem 8

In[206]:=
```
Clear["Global`*"]
```
In[207]:=
```
m = {{5, -√3}, {-√3, 7}} / 4;
m // MatrixForm
```
Out[208]//MatrixForm=

$$\begin{pmatrix} \frac{5}{4} & -\frac{\sqrt{3}}{4} \\ -\frac{\sqrt{3}}{4} & \frac{7}{4} \end{pmatrix}$$

In[209]:=
```
one = DiagonalMatrix [{1, 1}];
one // MatrixForm
```

Out[210]//MatrixForm=

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

In[211]:=
```
m - λ one // MatrixForm
```

Out[211]//MatrixForm=

$$\begin{pmatrix} \frac{5}{4} - \lambda & -\frac{\sqrt{3}}{4} \\ -\frac{\sqrt{3}}{4} & \frac{7}{4} - \lambda \end{pmatrix}$$

In[212]:=
```
eq = Det[m - λ one] // Simplify
```

Out[212]= $2 - 3\lambda + \lambda^2$

In[213]:=
```
sol = Solve[eq == 0, λ]
```

Out[213]= $\{\{\lambda \to 1\}, \{\lambda \to 2\}\}$

In[214]:=
```
sol // TrigToExp
```

Out[214]= $\{\{\lambda \to 1\}, \{\lambda \to 2\}\}$

In[215]:=
```
{λ1, λ2} = λ /. sol
```

Out[215]= $\{1, 2\}$

In[216]:=
```
vec = {a, b};
eq = m.vec == λ vec
```

Out[217]= $\left\{\frac{5\,a}{4} - \frac{\sqrt{3}\,b}{4}, -\frac{\sqrt{3}\,a}{4} + \frac{7\,b}{4}\right\} == \{a\,\lambda, b\,\lambda\}$

In[218]:=
```
Solve[eq /. sol[[1]], {a, b}]
```

Solve : Equations may not give solutions for all "solve " variables .

Out[218]= $\left\{\left\{b \to \frac{a}{\sqrt{3}}\right\}\right\}$

In[219]:=
```
Solve[eq /. sol[[2]], {a, b}]
```

Solve : Equations may not give solutions for all "solve " variables .

Out[219]= $\left\{\left\{b \to -\sqrt{3}\,a\right\}\right\}$

In[220]:=

$$\text{vec1} = \left\{1, \frac{1}{\sqrt{3}}\right\};$$

$$\text{vec2} = \left\{1, -\sqrt{3}\right\};$$

In[222]:=
```
m.vec1 == λ1 vec1 // Simplify
```

Out[222]= True

In[223]:=
```
m.vec2 == λ2 vec2 // Simplify
```
Out[223]= True

In[224]:=
```
Eigensystem[m] // MatrixForm
```
Out[224]//MatrixForm=

$$\begin{pmatrix} 2 & 1 \\ \left\{-\frac{1}{\sqrt{3}}, 1\right\} & \left\{\sqrt{3}, 1\right\} \end{pmatrix}$$

In[225]:=
$$\left\{\frac{\text{vec2}}{-\sqrt{3}}, \sqrt{3}\ \text{vec1}\right\}$$

Out[225]=
$$\left\{\left\{-\frac{1}{\sqrt{3}}, 1\right\}, \left\{\sqrt{3}, 1\right\}\right\}$$