Homework 9:

NOTE: You may use Mathematica for all problems
EXCEPT for Problem 5); please do this by hand.

Rotational Motion of Rigid Bodies: Taylor Ch 10
1,2)Problems 35, 36

10.35 ★★ A rigid body consists of three masses fastened as follows: m at $(a, 0, 0)$, $2m$ at $(0, a, a)$, and $3m$ at $(0, a, -a)$. **(a)** Find the inertia tensor **I**. **(b)** Find the principal moments and a set of orthogonal principal axes.

10.36 ★★ A rigid body consists of three equal masses (m) fastened at the positions $(a, 0, 0)$, $(0, a, 2a)$, and $(0, 2a, a)$. **(a)** Find the inertia tensor **I**. **(b)** Find the principal moments and a set of orthogonal principal axes.

3) The matrix A=

$$\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \\ \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 \end{bmatrix}$$

represents a finite rotation about a certain axis.
Find the axis vector and the angle of rotation.
Also show the axis vector satisfied the eigenvector equation: m.v = λv.

4) The matrix M represents a rotation by an angle ϕ around some axis.
The eigenvalues of M are $\lambda_1 = +1$, $\lambda_2 = (\sqrt{3}+i)/2$. $\lambda_3 = (\sqrt{3}-i)/2$.
Find the angle ϕ.

NOTE: You may use Mathematica for all problems EXCEPT for Problem 5); please do this by hand.

5) Find the eigenvalues $\{\lambda_1,\lambda_2\}$ and eigenvectors $\{v_1,v_2\}$ of this matrix

$$m = \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}.$$

Now compute $m.v_1$ and sketch both this vector and v1 and comment. Do the same for v2 and m.v2.

Suppose I take an area that is 1 unit-squared spanned by the vectors x=(1,0) and y=(0,1). Compute the area $x\times y$, and then compute the area $x'\times y'$ where $x'=m.x$ and $y'=m.y$.

Could this matrix represent a rotation of a solid object?

6) Find the eigenvalues $\{\lambda_1,\lambda_2\}$ and eigenvectors $\{v_1,v_2\}$ of this matrix

$$m = \begin{bmatrix} 5/4 & -\sqrt{3}/4 \\ -\sqrt{3}/4 & 7/4 \end{bmatrix}.$$

Now compute $m.v_1$ and sketch both this vector and v1 and comment. Do the same for v2 and m.v2.

Could this matrix represent a rotation of a solid object?

7). For a tennis racket, what are the principle axes. Sort them in order of increasing moment of inertia, and identify the stable ones.
[Please make the drawing good enough so I can figure out your answer.]