10.35 ★★ A rigid body consists of three masses fastened as follows: m at $(a, 0, 0)$, $2m$ at $(0, a, a)$, and $3m$ at $(0, a, -a)$. **(a)** Find the inertia tensor **I**. **(b)** Find the principal moments and a set of orthogonal principal axes.

10.36 ★★ A rigid body consists of three equal masses (m) fastened at the positions $(a, 0, 0)$, $(0, a, 2a)$, and $(0, 2a, a)$. **(a)** Find the inertia tensor **I**. **(b)** Find the principal moments and a set of orthogonal principal axes.

Problem 1:

10.35 ★★ A rigid body consists of three masses fastened as follows: m at $(a, 0, 0)$, $2m$ at $(0, a, a)$, and $3m$ at $(0, a, -a)$. **(a)** Find the inertia tensor **I**. **(b)** Find the principal moments and a set of orthogonal principal axes.

10.36 ★★ A rigid body consists of three equal masses (m) fastened at the positions $(a, 0, 0)$, $(0, a, 2a)$, and $(0, 2a, a)$. **(a)** Find the inertia tensor **I**. **(b)** Find the principal moments and a set of orthogonal principal axes.

```
Clear["Global`*"]
```

Three equal mass points are located at (a,0,0), (0, a, 2a) and (0, 2a, a). Find the principal moments of inertia about the origin and a set of principal axes.

```
location = {
    {a, 0, 0},
    {0, a, a},
    {0, a, -a}};
mass = {1, 2, 3};
v = {vx, vy, vz} = {{1, 0, 0}, {0, 1, 0}, {0, 0, 1}};
nMax = 3 (* Number of masses *)
```

Out[ ]= 3

Note, we program diagonal and off - diagonal with different formulas below!!!

```
Clear[term]
term[n_, i_, i_] := +mass[[n]] (location[[n]].location[[n]] - (location[[n]].v[[i]])^2)
term[n_, i_, j_] := -mass[[n]] (location[[n]].v[[i]] * location[[n]].v[[j]])
```

In[ ]:=
```
mat = Table[ Sum[ term[n, i, j], {n, 1, nMax}], {i, 1, 3}, {j, 1, 3}];
mat // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} 10\,a^2 & 0 & 0 \\ 0 & 6\,a^2 & a^2 \\ 0 & a^2 & 6\,a^2 \end{pmatrix}$$

In[ ]:=
```
evec = (Normalize /@ Eigenvectors [mat]) // Simplify // Transpose ;
evec // MatrixForm (* Column form *)
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

In[ ]:=
```
evec[[2]]
```

Out[ ]= $\left\{0, \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right\}$

In[ ]:=
```
eval = Eigenvalues [mat] ;
eval // Simplify // TableForm
```

Out[ ]//TableForm=

$10\,a^2$

$7\,a^2$

$5\,a^2$

Eigenvalues by hand

In[ ]:=
```
one = DiagonalMatrix [{1, 1, 1}];
eq = Det[mat - λ one] == 0
```

Out[ ]= $350\,a^6 - 155\,a^4\,\lambda + 22\,a^2\,\lambda^2 - \lambda^3 == 0$

In[ ]:=
```
sol = Solve[eq, λ]
```

Out[ ]= $\{\{\lambda \to 5\,a^2\}, \{\lambda \to 7\,a^2\}, \{\lambda \to 10\,a^2\}\}$

Eigenvectors by hand

In[ ]:=
```
eqs = (mat - λ one).{x, y, z} == 0 // Thread
```

Out[ ]= $\{x\,(10\,a^2 - \lambda) == 0,\ a^2\,z + y\,(6\,a^2 - \lambda) == 0,\ a^2\,y + z\,(6\,a^2 - \lambda) == 0\}$

In[ ]:=
```
eqs /. sol[[1]]
```

Out[ ]= $\{5\,a^2\,x == 0,\ a^2\,y + a^2\,z == 0,\ a^2\,y + a^2\,z == 0\}$

Eigenvector for λ1

In[ ]:=
```
Solve[eqs /. sol[[1]], {x, y, z}]
```

Solve : Equations may not give solutions for all "solve" variables.

Out[ ]= {{x → 0, z → -y}}

In[ ]:=
```
v1 = {0, 1, -1};
```

In[ ]:=
```
mat.v1 == λ v1
```

Out[ ]= $\{0, 5 a^2, -5 a^2\} == \{0, \lambda, -\lambda\}$

In[ ]:=
```
mat.v1 == λ v1 /. sol[[1]]
```

Out[ ]= True

Eigenvector for λ4

In[ ]:=
```
Solve[eqs /. sol[[2]], {x, y, z}]
```

Solve : Equations may not give solutions for all "solve" variables.

Out[ ]= {{x → 0, z → y}}

In[ ]:=
```
v2 = {0, 1, 1};
```

In[ ]:=
```
mat.v2 == λ v2
```

Out[ ]= $\{0, 7 a^2, 7 a^2\} == \{0, \lambda, \lambda\}$

In[ ]:=
```
mat.v2 == λ v2 /. sol[[2]]
```

Out[ ]= True

Eigenvector for λ3

In[ ]:=
```
Solve[eqs /. sol[[3]], {x, y, z}]
```

Solve : Equations may not give solutions for all "solve" variables.

Out[ ]= {{y → 0, z → 0}}

In[ ]:=
```
v3 = {1, 0, 0};
```

In[ ]:=
```
mat.v3 == λ v3
```

Out[ ]= $\{10 a^2, 0, 0\} == \{\lambda, 0, 0\}$

In[ ]:=
```
mat.v3 == λ v3 /. sol[[3]]
```

Out[ ]= True

Use Eigenvectors to diagonalize the matrix

In[ ]:=
```
Transpose[evec].mat.evec // Simplify // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} 10\,a^2 & 0 & 0 \\ 0 & 7\,a^2 & 0 \\ 0 & 0 & 5\,a^2 \end{pmatrix}$$

In[ ]:=
```
Transpose[evec].evec // Simplify // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Use Rotation Matrix to diagonalize the matrix

In[ ]:=
```
mx[θ_] = {{1, 0, 0}, {0, Cos[θ], -Sin[θ]}, {0, +Sin[θ], Cos[θ]}};
my[θ_] = {{Cos[θ], 0, +Sin[θ]}, {0, 1, 0}, {-Sin[θ], 0, Cos[θ]}};
mz[θ_] = {{Cos[θ], -Sin[θ], 0}, {+Sin[θ], Cos[θ], 0}, {0, 0, 1}};
mx[θ] // MatrixForm
my[θ] // MatrixForm
mz[θ] // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \text{Cos}[\theta] & -\text{Sin}[\theta] \\ 0 & \text{Sin}[\theta] & \text{Cos}[\theta] \end{pmatrix}$$

Out[ ]//MatrixForm=

$$\begin{pmatrix} \text{Cos}[\theta] & 0 & \text{Sin}[\theta] \\ 0 & 1 & 0 \\ -\text{Sin}[\theta] & 0 & \text{Cos}[\theta] \end{pmatrix}$$

Out[ ]//MatrixForm=

$$\begin{pmatrix} \text{Cos}[\theta] & -\text{Sin}[\theta] & 0 \\ \text{Sin}[\theta] & \text{Cos}[\theta] & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

In[ ]:=
```
mx[π / 4] // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

In[ ]:=
```
mx[π / 4].mat.Transpose[mx[π / 4]] // Simplify // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} 10\,a^2 & 0 & 0 \\ 0 & 5\,a^2 & 0 \\ 0 & 0 & 7\,a^2 \end{pmatrix}$$

Problem 2:

10.35 ★★ A rigid body consists of three masses fastened as follows: m at $(a, 0, 0)$, $2m$ at $(0, a, a)$, and $3m$ at $(0, a, -a)$. **(a)** Find the inertia tensor **I**. **(b)** Find the principal moments and a set of orthogonal principal axes.

10.36 ★★ A rigid body consists of three equal masses (m) fastened at the positions $(a, 0, 0)$, $(0, a, 2a)$, and $(0, 2a, a)$. **(a)** Find the inertia tensor **I**. **(b)** Find the principal moments and a set of orthogonal principal axes.

In[ ]:=
```
Clear["Global`*"]
```

Three equal mass points are located at (a,0,0), (0, a, 2a) and (0, 2a, a). Find the principal moments of inertia about the origin and a set of principal axes.

In[ ]:=
```
location = {
    {a, 0, 0},
    {0, a, 2 a},
    {0, 2 a, a}};
mass = {1, 1, 1};
v = {vx, vy, vz} = {{1, 0, 0}, {0, 1, 0}, {0, 0, 1}};
nMax = 3 (* Number of masses *)
```

Out[ ]= 3

Note, we program diagonal and off - diagonal with different formulas below!!!

In[ ]:=
```
Clear[term]
term[n_, i_, i_] := +mass[[n]] (location[[n]].location[[n]] - (location[[n]].v[[i]]) ^ 2)
term[n_, i_, j_] := -mass[[n]] (location[[n]].v[[i]] * location[[n]].v[[j]])
```

In[ ]:=
```
mat = Table[ Sum[term[n, i, j], {n, 1, nMax}], {i, 1, 3}, {j, 1, 3}];
mat // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} 10\,a^2 & 0 & 0 \\ 0 & 6\,a^2 & -4\,a^2 \\ 0 & -4\,a^2 & 6\,a^2 \end{pmatrix}$$

```
In[ ]:= evec = (Normalize /@ Eigenvectors[mat]) // Simplify // Transpose;
        evec // MatrixForm (* Column form *)
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} 0 & 1 & 0 \\ -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{pmatrix}$$

```
In[ ]:= evec[[2]]
```

Out[ ]= $\left\{-\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right\}$

```
In[ ]:= eval = Eigenvalues[mat];
        eval // Simplify // TableForm
```

Out[ ]//TableForm=

$10\,a^2$

$10\,a^2$

$2\,a^2$

```
In[ ]:= Transpose[evec].mat.evec // Simplify // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} 10\,a^2 & 0 & 0 \\ 0 & 10\,a^2 & 0 \\ 0 & 0 & 2\,a^2 \end{pmatrix}$$

```
In[ ]:= Transpose[evec].evec // Simplify // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Eigenvalues by hand

```
In[ ]:= one = DiagonalMatrix[{1, 1, 1}];
        eq = Det[mat - λ one] == 0
```

Out[ ]= $200\,a^6 - 140\,a^4\,\lambda + 22\,a^2\,\lambda^2 - \lambda^3 == 0$

```
In[ ]:= sol = Solve[eq, λ]
```

Out[ ]= $\{\{\lambda \to 2\,a^2\}, \{\lambda \to 10\,a^2\}, \{\lambda \to 10\,a^2\}\}$

Eigenvectors by hand

```
In[ ]:= eqs = (mat - λ one).{x, y, z} == 0 // Thread
```

Out[ ]= $\{x\,(10\,a^2 - \lambda) == 0, -4\,a^2\,z + y\,(6\,a^2 - \lambda) == 0, -4\,a^2\,y + z\,(6\,a^2 - \lambda) == 0\}$

```
In[ ]:= eqs /. sol[[1]]
Out[ ]= {8 a^2 x == 0, 4 a^2 y - 4 a^2 z == 0, -4 a^2 y + 4 a^2 z == 0}
```

Eigenvector for λ1

```
In[ ]:= Solve[eqs /. sol[[1]], {x, y, z}]
   Solve : Equations may not give solutions for all "solve" variables .
Out[ ]= {{x → 0, z → y}}

In[ ]:= v1 = {0, 1, 1};

In[ ]:= mat.v1 == λ v1
Out[ ]= {0, 2 a^2, 2 a^2} == {0, λ, λ}

In[ ]:= mat.v1 == λ v1 /. sol[[1]]
Out[ ]= True
```

Eigenvector for λ4

```
In[ ]:= Solve[eqs /. sol[[2]], {x, y, z}]
   Solve : Equations may not give solutions for all "solve" variables .
Out[ ]= {{z → -y}}

In[ ]:= v2 = {0, 1, -1};

In[ ]:= mat.v2 == λ v2
Out[ ]= {0, 10 a^2, -10 a^2} == {0, λ, -λ}

In[ ]:= mat.v2 == λ v2 /. sol[[2]]
Out[ ]= True
```

Eigenvector for λ3

```
In[ ]:= Solve[eqs /. sol[[3]], {x, y, z}]
   Solve : Equations may not give solutions for all "solve" variables .
Out[ ]= {{z → -y}}

In[ ]:= v3 = Cross[v2, v1] // Normalize
Out[ ]= {1, 0, 0}

In[ ]:= mat.v3 == λ v3
Out[ ]= {10 a^2, 0, 0} == {λ, 0, 0}
```

In[ ]:=
```
mat.v3 == λ v3 /. sol[[3]]
```
Out[ ]=
```
True
```

Use Eigenvectors to diagonalize the matrix

In[ ]:=
```
Transpose[evec].mat.evec // Simplify // MatrixForm
```
Out[ ]//MatrixForm=

$$\begin{pmatrix} 10\,a^2 & 0 & 0 \\ 0 & 10\,a^2 & 0 \\ 0 & 0 & 2\,a^2 \end{pmatrix}$$

In[ ]:=
```
Transpose[evec].evec // Simplify // MatrixForm
```
Out[ ]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Use Rotation Matrix to diagonalize the matrix

In[ ]:=
```
mx[θ_] = {{1, 0, 0}, {0, Cos[θ], -Sin[θ]}, {0, +Sin[θ], Cos[θ]}};
my[θ_] = {{Cos[θ], 0, +Sin[θ]}, {0, 1, 0}, {-Sin[θ], 0, Cos[θ]}};
mz[θ_] = {{Cos[θ], -Sin[θ], 0}, {+Sin[θ], Cos[θ], 0}, {0, 0, 1}};
mx[θ] // MatrixForm
my[θ] // MatrixForm
mz[θ] // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \text{Cos}[\theta] & -\text{Sin}[\theta] \\ 0 & \text{Sin}[\theta] & \text{Cos}[\theta] \end{pmatrix}$$

Out[ ]//MatrixForm=

$$\begin{pmatrix} \text{Cos}[\theta] & 0 & \text{Sin}[\theta] \\ 0 & 1 & 0 \\ -\text{Sin}[\theta] & 0 & \text{Cos}[\theta] \end{pmatrix}$$

Out[ ]//MatrixForm=

$$\begin{pmatrix} \text{Cos}[\theta] & -\text{Sin}[\theta] & 0 \\ \text{Sin}[\theta] & \text{Cos}[\theta] & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

In[ ]:=
```
mx[π / 4] // MatrixForm
```
Out[ ]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

In[ ]:=
```
mx[π / 4].mat.Transpose[mx[π / 4]]  // Simplify // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} 10\,a^2 & 0 & 0 \\ 0 & 10\,a^2 & 0 \\ 0 & 0 & 2\,a^2 \end{pmatrix}$$

Problem 3:

In[ ]:=
```
Clear["Global`*"]
```

In[ ]:=
```
m = {{1 / Sqrt[2], 1 / Sqrt[2], 0}, {0, 0, 1}, {1 / Sqrt[2], -1 / Sqrt[2], 0}};
m // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \end{pmatrix}$$

In[ ]:=
```
Det[m]
```

Out[ ]= 1

In[ ]:=
```
Tr[m]
```

Out[ ]= $\frac{1}{\sqrt{2}}$

In[ ]:=
```
eval = Eigenvalues[m] // Simplify
```

Out[ ]= $\left\{\frac{1}{4}\left(-2+\sqrt{2}+i\sqrt{10+4\sqrt{2}}\right),\ \frac{1}{4}\left(-2+\sqrt{2}-i\sqrt{10+4\sqrt{2}}\right),\ 1\right\}$

In[ ]:=
```
Eigenvectors[m] // Simplify
```

Out[ ]=

$$\left\{\left\{\frac{1}{2}\left(1-\sqrt{2}+i\sqrt{5+2\sqrt{2}}-\frac{8\,i}{2\,i-i\sqrt{2}+\sqrt{10+4\sqrt{2}}}\right),\ -\frac{4\,i}{-i\left(-2+\sqrt{2}\right)+\sqrt{10+4\sqrt{2}}},\ 1\right\},\right.$$

$$\left\{\frac{1}{2}\left(1-\sqrt{2}-i\sqrt{5+2\sqrt{2}}+\frac{8\,i}{i\left(-2+\sqrt{2}\right)+\sqrt{10+4\sqrt{2}}}\right),\ \frac{4\,i}{i\left(-2+\sqrt{2}\right)+\sqrt{10+4\sqrt{2}}},\ 1\right\},$$

$$\left.\left\{1+\sqrt{2},\ 1,\ 1\right\}\right\}$$

Find the Eigenvalues

In[ ]:=
```
one = DiagonalMatrix[{1, 1, 1}];
eq = Det[m - λ one] == 0
```

Out[ ]= $1-\frac{\lambda}{\sqrt{2}}+\frac{\lambda^2}{\sqrt{2}}-\lambda^3 == 0$

In[ ]:=
```
Solve[eq, λ]
```

Out[ ]= $\left\{\{\lambda\to 1\},\left\{\lambda\to\frac{1}{4}\left(-2+\sqrt{2}-i\sqrt{16-\left(-2+\sqrt{2}\right)^2}\right)\right\},\left\{\lambda\to\frac{1}{4}\left(-2+\sqrt{2}+i\sqrt{16-\left(-2+\sqrt{2}\right)^2}\right)\right\}\right\}$

Find the Eigenvalues

In[ ]:=
```
eqs = (m - λ one).{a, b, c} == 0 // Thread
```

Out[ ]= $\left\{\frac{b}{\sqrt{2}}+a\left(\frac{1}{\sqrt{2}}-\lambda\right) == 0,\ c-b\lambda == 0,\ \frac{a}{\sqrt{2}}-\frac{b}{\sqrt{2}}-c\lambda == 0\right\}$

In[ ]:=
```
sol = Solve[eqs /. {λ → 1}, {a, b, c}][[1]]
```

••• Solve : Equations may not give solutions for all "solve" variables .

Out[ ]= $\left\{b\to -\left(1-\sqrt{2}\right)a,\ c\to -\left(1-\sqrt{2}\right)a\right\}$

In[ ]:=
```
v1 = {a, b, c} /. sol /. {a → 1}
```

Out[ ]= $\left\{1,\ -1+\sqrt{2},\ -1+\sqrt{2}\right\}$

In[ ]:=
```
m.v1 == λ v1
```

Out[ ]= $\left\{\frac{1}{\sqrt{2}}+\frac{-1+\sqrt{2}}{\sqrt{2}},\ -1+\sqrt{2},\ \frac{1}{\sqrt{2}}-\frac{-1+\sqrt{2}}{\sqrt{2}}\right\} == \left\{\lambda,\ \left(-1+\sqrt{2}\right)\lambda,\ \left(-1+\sqrt{2}\right)\lambda\right\}$

In[ ]:=
```
m.v1 == λ v1 /. {λ → 1}
```

Out[ ]= True

Find the angle

In[ ]:=
```
eq = Tr[m] == 1 + 2 Cos[θ]
```

Out[ ]= $\frac{1}{\sqrt{2}} == 1+2\,\mathrm{Cos}[\theta]$

In[ ]:=
```
sol = Solve[eq, θ] /. {C[_] :→ 0} // Normal
```

Out[ ]= $\left\{\left\{\theta\to -\mathrm{ArcCos}\left[\frac{1}{4}\left(-2+\sqrt{2}\right)\right]\right\},\left\{\theta\to \mathrm{ArcCos}\left[\frac{1}{4}\left(-2+\sqrt{2}\right)\right]\right\}\right\}$

In[ ]:= `θ / Degree /. sol // N`

Out[ ]= {-98.4211, 98.4211}

In[ ]:= `ev3 = Eigenvectors[m][[3]]`

Out[ ]= $\left\{1+\sqrt{2},\ 1,\ 1\right\}$

Find the angle a different way: $\lambda = e^{i\theta}$

In[ ]:= `eqθ = Eigenvalues[m][[1]] == Cos[θ] + I Sin[θ]`

Out[ ]= $\frac{1}{4}\left(-2+\sqrt{2}+i\sqrt{16-\left(-2+\sqrt{2}\right)^2}\right) == \text{Cos}[\theta]+i\,\text{Sin}[\theta]$

In[ ]:= `solθ = Solve[eqθ, θ][[1]] /. {C[_] → 0}`

Out[ ]= $\left\{\theta \to \pi + \text{ArcTan}\left[\frac{\sqrt{2\left(5+2\sqrt{2}\right)}}{-2+\sqrt{2}}\right]\right\}$

In[ ]:= `θ0 = θ /. solθ`

Out[ ]= $\pi + \text{ArcTan}\left[\frac{\sqrt{2\left(5+2\sqrt{2}\right)}}{-2+\sqrt{2}}\right]$

In[ ]:= $\frac{\theta0}{\text{Degree}}$ // N

Out[ ]= 98.4211

Problem 4:

In[ ]:= `Clear["Global`*"]`

In[ ]:= `evals = {1, (Sqrt[3] + I) / 2, (Sqrt[3] - I) / 2}`

Out[ ]= $\left\{1,\ \frac{1}{2}\left(i+\sqrt{3}\right),\ \frac{1}{2}\left(-i+\sqrt{3}\right)\right\}$

In[ ]:= `Abs /@ evals`

Out[ ]= {1, 1, 1}

In[ ]:= `Solve[# == Exp[I θ], θ] & /@ evals /. {C[1] → 0} // Normal`

Out[ ]= $\left\{\{\{\theta \to 0\}\},\ \left\{\left\{\theta \to \frac{\pi}{6}\right\}\right\},\ \left\{\left\{\theta \to -\frac{\pi}{6}\right\}\right\}\right\}$

In[ ]:= `trace = Sum[evals[[i]], {i, 1, 3}] // Expand`

Out[ ]= $1+\sqrt{3}$

In[ ]:=
```
eq = trace == 1 + 2 Cos[θ]
```
Out[ ]=
$$1+\sqrt{3} == 1+2\,\text{Cos}[\theta]$$

In[ ]:=
```
sol = Solve[eq, θ] /. {C[_] :> 0} // Normal
```
Out[ ]=
$$\left\{\left\{\theta \to -\frac{\pi}{6}\right\}, \left\{\theta \to \frac{\pi}{6}\right\}\right\}$$

In[ ]:=
```
θ / Degree /. sol // N
```
Out[ ]=
```
{-30., 30.}
```

Problem 5:

In[ ]:=
```
Clear["Global`*"]
```

In[ ]:=
```
m = DiagonalMatrix[{3, 2}];
m // MatrixForm
```
Out[ ]//MatrixForm=
$$\begin{pmatrix} 3 & 0 \\ 0 & 2 \end{pmatrix}$$

In[ ]:=
```
Eigenvalues[m]
```
Out[ ]=
```
{3, 2}
```

In[ ]:=
```
Eigenvectors[m]
```
Out[ ]=
```
{{1, 0}, {0, 1}}
```

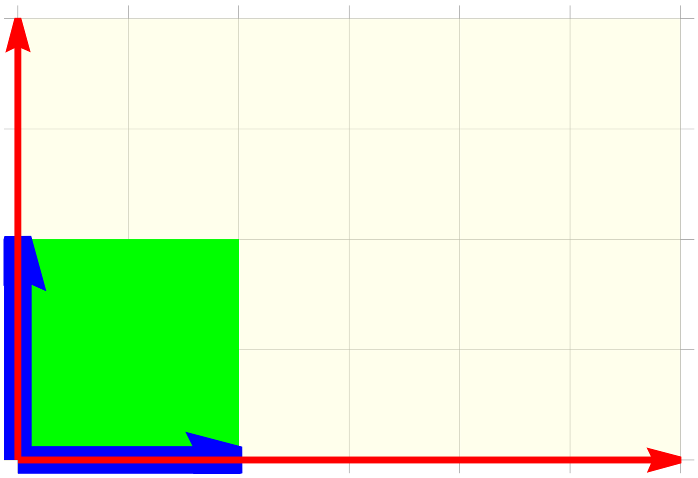
In[ ]:=
```
makeVec[v_] := Arrow[{{0, 0}, v}]
```

In[ ]:=
```
xVec = {1, 0};
yVec = {0, 1};
m.xVec // MatrixForm
m.yVec // MatrixForm
```
Out[ ]//MatrixForm=
$$\begin{pmatrix} 3 \\ 0 \end{pmatrix}$$

Out[ ]//MatrixForm=
$$\begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

In[ ]:=
```
Graphics[{
  Opacity[0.5], LightYellow , Rectangle[{0, 0}, {3, 2}],
  Opacity[1.0], Green, Rectangle[{0, 0}, {1, 1}], Thickness[0.04],
  Blue, makeVec[xVec], makeVec[yVec], Thickness[0.01],
  Red, makeVec[m.xVec], makeVec[m.yVec]},
 GridLines → Automatic]
```

Out[ ]=



Problem 6:

In[ ]:=
```
Clear["Global`*"]
```

In[ ]:=
```
m = {{5/4, -√3/4}, {-√3/4, 7/4}};
m // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} \frac{5}{4} & -\frac{\sqrt{3}}{4} \\ -\frac{\sqrt{3}}{4} & \frac{7}{4} \end{pmatrix}$$

In[ ]:=
```
Eigenvalues[m]
```

Out[ ]= {2, 1}

In[ ]:=
```
ev = Eigenvectors[m]
```

Out[ ]= $\left\{\left\{-\frac{1}{\sqrt{3}}, 1\right\}, \left\{\sqrt{3}, 1\right\}\right\}$

In[ ]:=
```
makeVec[v_] := Arrow[{{0, 0}, v}]
```

In[ ]:=
```
ev1 = ev[[1]] // Normalize
ev2 = ev[[2]] // Normalize

ev1p = m.ev[[1]] // Simplify
ev2p = m.ev[[2]]
```

Out[ ]= $\left\{-\frac{1}{2}, \frac{\sqrt{3}}{2}\right\}$

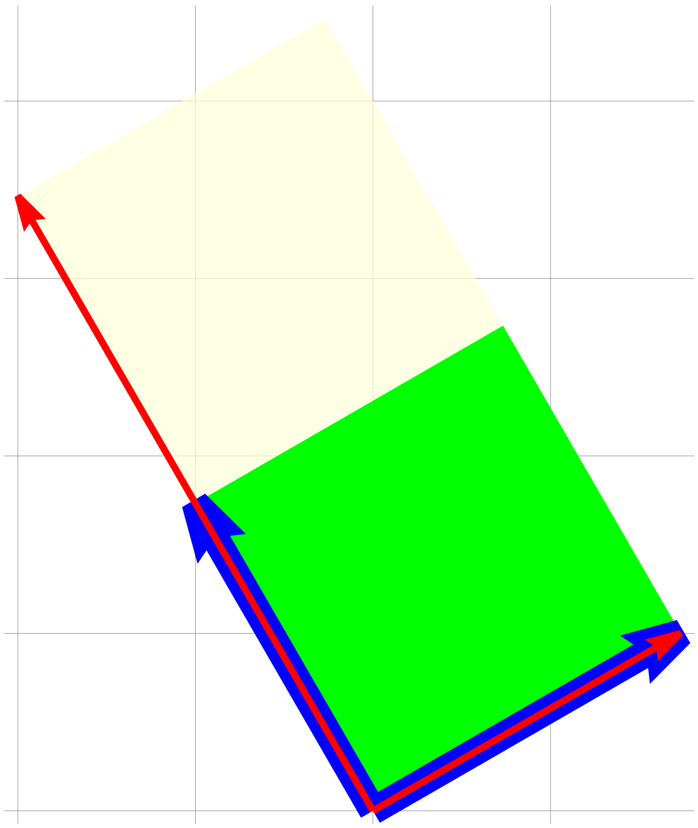
Out[ ]= $\left\{\frac{\sqrt{3}}{2}, \frac{1}{2}\right\}$

Out[ ]= $\left\{-\frac{2}{\sqrt{3}}, 2\right\}$

Out[ ]= $\left\{\sqrt{3}, 1\right\}$

In[ ]:=
```
Graphics[{
  Opacity[0.7], LightYellow, Rotate[Rectangle[{0, 0}, {1, 2}], π / 6, {0, 0}],
  Opacity[1.0], Green, Rotate[Rectangle[{0, 0}, {1, 1}], π / 6, {0, 0}],
  GridLines → Automatic, Thickness[0.04],
  Blue, makeVec[ev1], makeVec[ev2], Thickness[0.01],
  Red, makeVec[m.ev1], makeVec[m.ev2]},
 GridLines → Automatic]
```

Out[ ]=



Problem 7: Tennis Racquet

https://youtu.be/1VPfZ_XzisU

See about 3:00 mark