Air Resistance

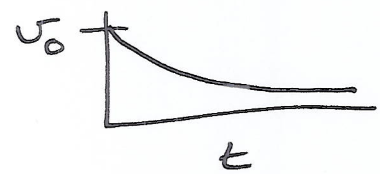
$$F = ma = m\dot{v}$$

$$F = bv + cv^2$$

$$-bv = m\dot{v} \qquad \text{Guess} \quad v = Ae^{\lambda t}$$

$$-bv = m\lambda v \quad \Leftarrow \quad \dot{v} = \lambda v$$

$$\lambda = -\frac{b}{m} \qquad A = v_0 \quad \Rightarrow \quad v = v_0 e^{-\frac{b}{m}t}$$



Add Gravity

$$mg - bv = m\dot{v} \qquad \dot{v} = 0 \Rightarrow \text{Terminal Velocity}$$

$$\Rightarrow \quad v_T = \frac{mg}{b}$$

$$\underbrace{\frac{mg}{b} - v}_{-u} = \frac{m}{b}\dot{v}$$

$$-u \Rightarrow \quad u = v - \frac{mg}{b} \qquad \dot{u} = \dot{v}$$

$$-u = \frac{m}{b}\dot{u} \quad \Rightarrow \quad u = u_0 e^{-\frac{b}{m}t}$$

$$v = u + \frac{mg}{b} = u + v_T = u_0 e^{-\frac{b}{m}t} + v_T$$

At $t=0$ $\quad v = v_0 = u_0 + v_T \Rightarrow u_0 = v_0 + v_T$

$$\therefore \quad v = (v_0 - v_T)e^{-\frac{b}{m}t} + v_T = v_0 e^{-\frac{b}{m}t} + v_T\left(1 - e^{-\frac{b}{m}t}\right)$$

Quadratic Air Resistance

$F = ma = m\dot{v}$ Horizontal: $g = 0$

Quadratic: $F = -cv^2$

Not Linear !!!

$-cv^2 = m\dot{v} = m\frac{dv}{dt}$

$\int_0^t -c\,dt = \int_{v_0}^{v} m\frac{dv}{v^2}$

$-ct = m\left(-\frac{1}{v} + \frac{1}{v_0}\right) \Rightarrow v(t) = \frac{v_0}{1 + t/\tau}$

where $\tau = \frac{m}{cv_0}$; $x(t) = v_0 \tau \ln(1 + t/\tau)$

Vertical: $F = m\dot{v}$; $F = mg - cv^2$

$m\dot{v} = mg - cv^2$

Terminal velocity: $\dot{v} = 0 \Rightarrow v^2 = \frac{mg}{c}$

or $v_{ter} = \sqrt{\frac{mg}{c}}$

Solution:

$$v = v_{ter} \tanh\left(\frac{gt}{v_{ter}}\right) \qquad (2.57)$$

$$y = \frac{v_{ter}^2}{g} \ln\left[\cosh\left(\frac{gt}{v_{ter}}\right)\right] \qquad (2.58)$$

Differential Eqs..

Horiz Motion w/ linear drag: (2.15)

$m\dot{v} = -bv$ Guess $v = Ae^{-kt}$

$-mkv = -bv$ $\therefore$ $\dot{v} = -kv$

$k = b/m$ Note: $\underset{\text{time}}{\tau = 1/k} = m/b$

$v = Ae^{-kt} = Ae^{-t/\tau}$ with $\tau = m/b$

Hookes Law: $F = -kx$

$m\ddot{x} = -kx$ Guess: $x = Ae^{\lambda t}$

$\dot{x} = \lambda x$

$m\lambda^2 x = -kx$ $\Leftarrow$ $\ddot{x} = \lambda^2 x$

$\lambda^2 = \dfrac{-k}{m}$ $\Rightarrow$ $\lambda = \pm i\sqrt{\dfrac{k}{m}} \equiv \pm i\omega$

$x = Ae^{\pm i\omega t}$

$e^{\pm i\theta} = \cos(\theta) \pm i\sin(\theta)$

$$F = ma = m\ddot{x} \Rightarrow a = \ddot{x} = \dot{v}$$

$$\dot{v} = a \qquad \frac{dv}{dt} = a \Rightarrow \int_{v_0}^{v} dv = \int_0^t a\,dt$$

$$v - v_0 = at \Rightarrow \boxed{v = v_0 + at}$$

$$v = \frac{dx}{dt} \Rightarrow \int_{x_0}^{x} dx = \int_0^t (v_0 + at)\,dt$$

$$x - x_0 = v_0 t + \frac{1}{2}at^2 \Rightarrow \boxed{x = x_0 + v_0 t + \frac{1}{2}at^2}$$

$v - v_0 = at \Rightarrow t = \dfrac{v - v_0}{a}$ Sub in above

$$\Rightarrow \boxed{2a\Delta x = v^2 - v_0^2}$$

Notes $F = ma = m\ddot{x}$ 2nd order DEQ

$\Rightarrow$ two constants of integration $\{x_0, v_0\}$

<u>Linear</u> $F = ma = m\ddot{x} = m\dfrac{d^2x}{dt^2}$

Rockets $F = \dfrac{d(p)}{dt} = \dfrac{d(mv)}{dt} = \dot{m}v + m\dot{v}$

important if mass is <u>not</u> constant

Typical: ~95% of rocket mass is Fuel.