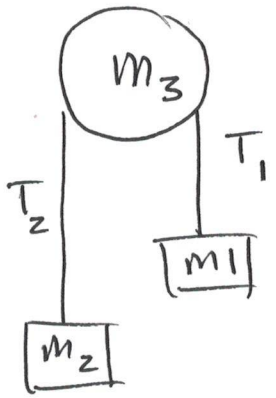
Atwood Machine

$F = ma$ $\quad \tau = I\alpha$ $\quad \tau = r \times F$ $\quad a = r\alpha$ $\quad I_3 = \frac{1}{2} m_3 r^2$



(#1) $m_1 g - T_1 = m_1 a_1$

(#2) $m_2 g - T_2 = m_2 a_2$

(#3) $T_1 r - T_2 r = I_3 \alpha$

Unknowns: T_1 T_2 a_1 a_2 α

(#4) $a_1 = -a_2$ $\qquad$ (#5) $a_1 = r\alpha$

$$a_1 = g \frac{m_1 - m_2}{m_1 + m_2 + \frac{m_3}{2}} \qquad a_2 = g \frac{m_2 - m_1}{m_1 + m_2 + \frac{m_3}{2}}$$

$$T_1 = m_1 g \frac{\left(\frac{m_3}{2} + 2m_2\right)}{m_1 + m_2 + \frac{m_3}{2}} \qquad T_2 = m_2 g \frac{\left(\frac{m_3}{2} + 2m_1\right)}{m_1 + m_2 + \frac{m_3}{2}}$$

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eqs = {
  m1 g - t1 == m1 a1,
  m2 g - t2 == m2 a2,
  t1 r - t2 r == i3 α,
  a1 == -a2,
  a1 == r α}
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{g m1 - t1 == a1 m1, g m2 - t2 == a2 m2, r t1 - r t2 == i3 α, a1 == -a2, a1 == r α}
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Solve[eqs, {a1, a2, α, t1, t2}] /. {i3 → 1/2 m3 r^2} // Simplify
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$$\left\{\left\{a1 \to \frac{2\,g\,(m1 - m2)}{2\,m1 + 2\,m2 + m3},\ a2 \to \frac{2\,g\,(-m1 + m2)}{2\,m1 + 2\,m2 + m3},\ \alpha \to \frac{2\,g\,(m1 - m2)}{(2\,m1 + 2\,m2 + m3)\,r},\ t1 \to \frac{g\,m1\,(4\,m2 + m3)}{2\,m1 + 2\,m2 + m3},\ t2 \to \frac{g\,m2\,(4\,m1 + m3)}{2\,m1 + 2\,m2 + m3}\right\}\right\}$$