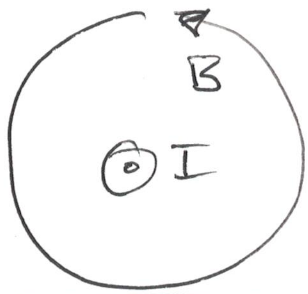
Line Integrals

$$\oint B \circ dL = \mu_0 I_{enc}$$

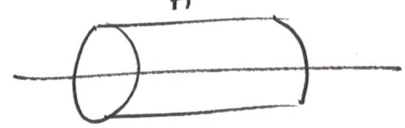


$$\int B \, 2\pi dr = \mu_0 I$$

$$B = \frac{\mu_0}{2\pi} \frac{I}{r}$$

Gauss Law

$$\int E \circ dA = \frac{Q}{\varepsilon_0}$$



$A = 2\pi r L$

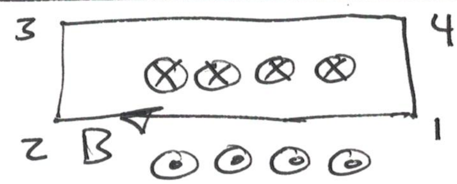
$$\lambda = \frac{Q}{L}$$

$$E A = \frac{Q}{\varepsilon_0}$$

$$E \, 2\pi r L = \frac{\lambda L}{\varepsilon_0} \Rightarrow E = \frac{1}{2\pi \varepsilon_0} \frac{\lambda}{r}$$

Solenoid

$$\oint B \circ dL = \mu_0 I_{enc}$$

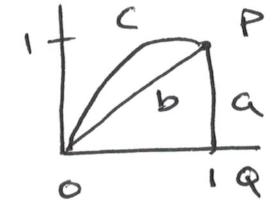


$\int_2^3 = 0 = \int_4^1$ because $B \perp L$

$\int_3^4 = 0$ because $B = 0$

$$\int_1^2 B \circ dL = B L = \mu_0 I_{enc} \Rightarrow B = \mu_0 \frac{I}{L}$$

Example 4.1 $\vec{F} = (y, 2x) = (y)\hat{x} + (2x)\hat{y}$



(a)

$$W = \int_a \vec{F} \circ \vec{dr} = \int (F_x \, dx + F_y \, dy)$$

$$= \int_0^Q + \int_Q^P = \int_0^1 dx \; F_x(x,0) + \int_0^1 dy \; F_y(1,y)$$

$$= \int_0^1 dx \quad 0 \quad + \int_0^1 dy \; (2) = 2$$

(b)

$$\int_b (F_x \, dx + F_y \, dy) = \int (y) \, dx + (2x) \, dy$$

but $y = x$ and $dy = dx$

$$\Rightarrow \int (x) \, dx + (2x) \, dx = \int_0^1 3x \, dx = \frac{3}{2} = 1.5$$

(c) $\{x, y\} = \{1 - \cos\theta, \sin\theta\}$

$\{dx, dy\} = \{\sin\theta \, d\theta, \cos\theta \, d\theta\}$

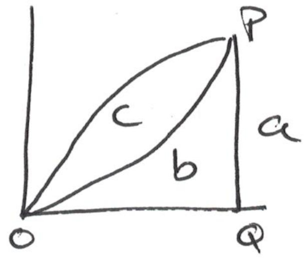
$$W = \int_c (F_x \, dx + F_y \, dy) = \int_c (y) \, dx + (2x) \, dy$$

$$= \int (\sin\theta)(\sin\theta \, d\theta) + (2(1 - \cos\theta))(\cos\theta \, d\theta)$$

$$= \int_0^{\pi/2} d\theta \left[\sin^2\theta + 2\cos\theta \, (1 - \cos\theta) \right]$$

$$= 2 - \frac{\pi}{4} \cong 1.21$$

Problem 4.2 $F = (x^2, 2xy) = (x^2)\hat{x} + (2xy)\hat{y}$



$$W = \int F\cdot dr = \int (F_x\,dx + F_y\,dy)$$

(a)

$$= \int_O^Q + \int_Q^P = \int_0^1 dx\; F_x(x,0) + \int_0^1 dy\; F_y(1,y)$$

$$= \int_0^1 dx\,(x^2) + \int_0^1 dy\,(2y) = \frac{x^3}{3}\Big]_0^1 + \frac{2y^2}{2}\Big]_0^1 = \frac{4}{3}$$

(b) $y = x^2 \quad dy = 2x\,dx$

$$W = \int F_x(x,\underset{\to x^2}{y})\,dx + F_y(x,\underset{\to x^2}{y})\,dy \to dy \to 2x\,dx$$

$$= \int (x^2)\,dx + (2x\cdot(x^2))\cdot(2x\,dx)$$

$$= \int_0^1 dx\,[x^2 + 4x^4] = \frac{x^3}{3} + \frac{4x^5}{5}\Big]_0^1 = \frac{17}{15}$$

(c) $x = t^3 \quad dx = 3t^2\,dt \qquad y = t^2 \quad dy = 2t\,dt$

$$W = \int F_x(t^3,t^2)\,dx + F_y(t^3,t^2)\,dy$$

$$= \int_0^1 dt\,[(t^3)^2\cdot(3t^2\,dt) + (2t^3t^2)(2t\,dt)]$$

$$= \int_0^1 dt\,[3t^8 + 4t^6] = \frac{3t^9}{9} + \frac{4t^7}{7}\Big]_0^1$$

$$= \frac{1}{3} + \frac{4}{7} = \frac{19}{21}$$