

The Bra Ket Notation

$\langle \text{Bra} |$ $| \text{Ket} \rangle$

$$\langle u | = (y \ z) \quad | u \rangle = \begin{pmatrix} y \\ z \end{pmatrix}$$

Vectors

$$\langle u | u \rangle = u \cdot u = (y \ z) \cdot \begin{pmatrix} y \\ z \end{pmatrix} = y^2 + z^2$$

Scalar
Number

Unit Vectors: $\langle y | = (1 \ 0) \quad | y \rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$\hat{y} \quad \hat{z}$

$$\langle z | = (0 \ 1) \quad | z \rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\langle y | y \rangle = 1 = \langle z | z \rangle \quad \text{and} \quad \langle y | z \rangle = 0 = \langle z | y \rangle$$

$$| u \rangle \langle u | = \begin{pmatrix} y \\ z \end{pmatrix} (y \ z) = \begin{pmatrix} yy & yz \\ zy & zz \end{pmatrix} \quad \text{a matrix}$$

Vector: $\langle u |$ or $| u \rangle$ Number $\langle u | u \rangle$

Matrix $| u \rangle \langle u |$

$$\text{Unit Matrix} = \mathbb{1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = | y \rangle \langle y | + | z \rangle \langle z | = \sum_{\hat{z}} | x \rangle \langle x |$$

Project a vector:

$$\langle u | = \langle u | \mathbb{1} = \langle u | \{ | y \rangle \langle y | + | z \rangle \langle z | \}$$

$$= \underbrace{\langle u | y \rangle}_{\text{Project } u \text{ on } y} | y \rangle + \underbrace{\langle u | z \rangle}_{\text{Project } u \text{ on } z} | z \rangle$$

$$= u_y \hat{y} + u_z \hat{z}$$

Project
u on y

Project
u on z