Phys 3344: Tuesday 29 September

Office Hours: Wed 5:00-6:00

Schedule:

Lagrange equations

Homework #6:

Hints

Ch 6 & 7

brachistochrone problem

https://youtu.be/skvnj67YGmw

https://youtu.be/Cld0p3a43fU

	2020 FALL PHYS 3344				
#	**DAY**	**LECTURE:**	**NOTES:**	**Chpt**	**TOPIC**
1	TUE	08/25/20	First Class	1	Newtons Laws
2	THUR	08/27/20		2	Projectiles
3	TUE	09/01/20		3	Momentum & Angular Momentum
4	THUR	09/03/20		4	Energy
5	TUE	09/08/20		5	Oscillations
6	THUR	09/10/20			
7	TUE	09/15/20			
8	THUR	09/17/20			**EXAM 1**
9	TUE	09/22/20		6	Calculus of Variations
10	THUR	09/24/20		7	Lagrange's Equation
11	TUE	09/29/20			
12	THUR	10/01/20		8	Two Body Problems
13	TUE	10/06/20			
14	THUR	10/08/20		9	Non-Inertial Frames
	TUE	10/13/20	~~**Fall Break**~~	10	Rotational Motion
15	THUR	10/15/20			**EXAM 2**
16	TUE	10/20/20		10	Rotational Motion
17	THUR	10/22/20			
18	TUE	10/27/20		11	Coupled Oscillations
19	THUR	10/29/20			
20	TUE	11/03/20		13	Hamiltonian Mechanics
21	THUR	11/05/20	Drop Date		
22	TUE	11/10/20			
23	THUR	11/12/20			**EXAM 3**
24	TUE	11/17/20		14	Collision Theory
25	THUR	11/19/20			
26	TUE	11/24/20		15	Special relativity
27	THUR	11/26/20	**Thanksgiving**		No Class
28	TUE	12/01/20			No Class
29	THUR	12/03/20	**Last Class**		Review
	WED	**Dec 16**	**FINAL EXAM**	**Wednesday Dec. 16,2020, 11:30am - 2:30**	
	Adjustments may be made depending on student interests/needs and unplanned events				

Contents

* Sections marked with an asterisk could be omitted on a first reading.

$$\frac{\partial \mathcal{L}}{\partial x} + \lambda \frac{\partial f}{\partial x} = \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{x}} \quad \text{or} \quad m_1 g + \lambda = m_1 \ddot{x} \qquad (7.125)$$

$$\frac{d}{dt}\frac{dL}{dq'} - \frac{dL}{dq} = \lambda \frac{df}{dq}$$

f is constraint equation

$$S[q, q'] = \int L[q, q'] dq$$

$$\delta S[q, q'] = 0$$

Principal Definitions and Equations of Chapter 6

The Euler–Lagrange Equation

An integral of the form

$$S = \int_{x_1}^{x_2} f[y(x), y'(x), x]\, dx \qquad \text{[Eq. (6.4)]}$$

taken along a path $y = y(x)$ is stationary with respect to variations of that path if and only if $y(x)$ satisfies the **Euler–Lagrange equation**

$$\frac{\partial f}{\partial y} - \frac{d}{dx}\frac{\partial f}{\partial y'} = 0. \qquad \text{[Eq. (6.13)]}$$

Here, f is function, we will replace by Lagrangian L

EXAMPLE 7.8 Atwood's Machine Using a Lagrange Multiplier

Analyze the Atwood machine of Figure 7.6 (shown again here as Figure 7.11) by the method of Lagrange multipliers and using the coordinates x and y of the two masses.

In terms of the given coordinates, the Lagrangian is

$$\mathcal{L} = T - U = \tfrac{1}{2}m_1\dot{x}^2 + \tfrac{1}{2}m_2\dot{y}^2 + m_1 g x + m_2 g y \tag{7.123}$$

and the constraint equation is

$$f(x, y) = x + y = \text{const.} \tag{7.124}$$

The modified Lagrange equation (7.118) for x reads

$$\frac{\partial\mathcal{L}}{\partial x} + \lambda\frac{\partial f}{\partial x} = \frac{d}{dt}\frac{\partial\mathcal{L}}{\partial\dot{x}} \qquad \text{or} \qquad m_1 g + \lambda = m_1\ddot{x} \tag{7.125}$$

and that for y is

$$\frac{\partial\mathcal{L}}{\partial y} + \lambda\frac{\partial f}{\partial y} = \frac{d}{dt}\frac{\partial\mathcal{L}}{\partial\dot{y}} \qquad \text{or} \qquad m_2 g + \lambda = m_2\ddot{y}. \tag{7.126}$$

These two equations, together with the constraint equation (7.124), are easily solved for the unknowns $x(t)$, $y(t)$, and $\lambda(t)$. From (7.124) we see that $\ddot{y} = -\ddot{x}$, and then subtracting (7.126) from (7.125) we can eliminate λ and arrive at the same result as before,

$$\ddot{x} = (m_1 - m_2)g/(m_1 + m_2).$$

To better understand the two modified Lagrange equations (7.125) and (7.126), it is helpful to compare them with the two equations of the Newtonian solution. Newton's second law for m_1 is

$$m_1 g - F_t = m_1\ddot{x}$$

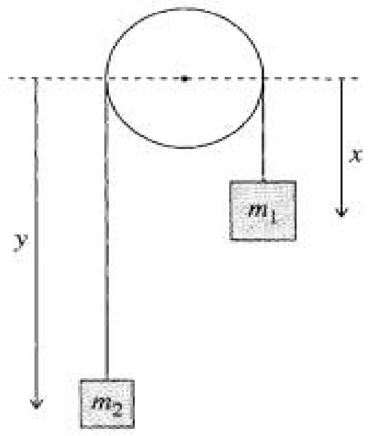

Figure 7.11 The Atwood machine again.

where F_t is the tension in the string, and that for m_2 is

$$m_2 g - F_t = m_2\ddot{y}.$$

These are precisely the two Lagrange equations (7.125) and (7.126), with the Lagrange multiplier identified as the constraint force

$$\lambda = -F_t.$$

[Two small comments: The minus sign occurs because both coordinates x and y were measured downward, whereas both tension forces are upward. In general, according to (7.122) the constraint force is $\lambda\partial f/\partial x$, but in this simple case, $\partial f/\partial x = 1$.]