

Phys 3344: Tuesday 29 September

Office Hours: Wed 5:00-6:00

Schedule:

Lagrange equations

Homework #6:

Hints

Ch 6 & 7

brachistochrone problem

<https://youtu.be/skvnj67YGmw>

<https://youtu.be/Cld0p3a43fU>

<b>2020 FALL      PHYS 3344</b>					
#	DAY	LECTURE:	NOTES:	Chpt	TOPIC
1	TUE	08/25/20	First Class	1	Newtons Laws
2	THUR	08/27/20		2	Projectiles
3	TUE	09/01/20		3	Momentum & Angular Momentum
4	THUR	09/03/20		4	Energy
5	TUE	09/08/20		5	Oscillations
6	THUR	09/10/20			
7	TUE	09/15/20			
8	THUR	09/17/20			<b>EXAM 1</b>
9	TUE	09/22/20		6	Calculus of Variations
10	THUR	09/24/20		7	Lagrange's Equation
11	TUE	09/29/20			
12	THUR	10/01/20		8	Two Body Problems
13	TUE	10/06/20			
14	THUR	10/08/20		9	Non-Inertial Frames
	TUE	10/13/20	<b>Fall-Break</b>	10	Rotational Motion
15	THUR	10/15/20			<b>EXAM 2</b>
16	TUE	10/20/20		10	Rotational Motion
17	THUR	10/22/20			
18	TUE	10/27/20		11	Coupled Oscillations
19	THUR	10/29/20			
20	TUE	11/03/20		13	Hamiltonian Mechanics
21	THUR	11/05/20	Drop Date		
22	TUE	11/10/20			
23	THUR	11/12/20			<b>EXAM 3</b>
24	TUE	11/17/20		14	Collision Theory
25	THUR	11/19/20			
26	TUE	11/24/20		15	Special relativity
27	THUR	11/26/20	<b>Thanksgiving</b>		No Class
28	TUE	12/01/20			No Class
29	THUR	12/03/20	<b>Last Class</b>		Review
	WED	<b>Dec 16</b>	<b>FINAL EXAM</b>	<b>Wednesday Dec. 16,2020, 11:30am - 2:30</b>	
<i>Adjustments may be made depending on student interests/needs and unplanned events</i>					

# Contents

Preface xi

## PART I Essentials 1

### CHAPTER 1 Newton's Laws of Motion 3

- 1.1 Classical Mechanics 3
- 1.2 Space and Time 4
- 1.3 Mass and Force 9
- 1.4 Newton's First and Second Laws; Inertial Frames 13
- 1.5 The Third Law and Conservation of Momentum 17
- 1.6 Newton's Second Law in Cartesian Coordinates 23
- 1.7 Two-Dimensional Polar Coordinates 26
- Principal Definitions and Equations of Chapter 1 33
- Problems for Chapter 1 34

### CHAPTER 2 Projectiles and Charged Particles 43

- 2.1 Air Resistance 43
- 2.2 Linear Air Resistance 46
- 2.3 Trajectory and Range in a Linear Medium 54
- 2.4 Quadratic Air Resistance 57
- 2.5 Motion of a Charge in a Uniform Magnetic Field 65
- 2.6 Complex Exponentials 68
- 2.7 Solution for the Charge in a B Field 70
- Principal Definitions and Equations of Chapter 2 71
- Problems for Chapter 2 72

### CHAPTER 3 Momentum and Angular Momentum 83

- 3.1 Conservation of Momentum 83
- 3.2 Rockets 85
- 3.3 The Center of Mass 87
- 3.4 Angular Momentum for a Single Particle 90
- 3.5 Angular Momentum for Several Particles 93
- Principal Definitions and Equations of Chapter 3 98
- Problems for Chapter 3 99

### CHAPTER 4 Energy 105

- 4.1 Kinetic Energy and Work 105
- 4.2 Potential Energy and Conservative Forces 109
- 4.3 Force as the Gradient of Potential Energy 116
- 4.4 The Second Condition that F be Conservative 118
- 4.5 Time-Dependent Potential Energy 121
- 4.6 Energy for Linear One-Dimensional Systems 123
- 4.7 Curvilinear One-Dimensional Systems 129
- 4.8 Central Forces 133
- 4.9 Energy of Interaction of Two Particles 138
- 4.10 The Energy of a Multiparticle System 144
- Principal Definitions and Equations of Chapter 4 148
- Problems for Chapter 4 150

### CHAPTER 5 Oscillations 161

- 5.1 Hooke's Law 161
- 5.2 Simple Harmonic Motion 163
- 5.3 Two-Dimensional Oscillations 170
- 5.4 Damped Oscillations 173
- 5.5 Driven Damped Oscillations 179
- 5.6 Resonance 187
- 5.7 Fourier Series\* 192
- 5.8 Fourier Series Solution for the Driven Oscillator\* 197
- 5.9 The RMS Displacement; Parseval's Theorem\* 203
- Principal Definitions and Equations of Chapter 5 205
- Problems for Chapter 5 207

\* Sections marked with an asterisk could be omitted on a first reading.

$$\frac{\partial \mathcal{L}}{\partial x} + \lambda \frac{\partial f}{\partial x} = \frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{x}} \quad \text{or} \quad m_1 g + \lambda = m_1 \ddot{x} \quad (7.125)$$

$$\frac{d}{dt} \frac{dL}{dq'} - \frac{dL}{dq} = \lambda \frac{df}{dq}$$

f is constraint equation

$$S[q, q'] = \int L[q, q'] dq$$

$$\delta S[q, q'] = 0$$

### Principal Definitions and Equations of Chapter 6

#### The Euler–Lagrange Equation

An integral of the form

$$S = \int_{x_1}^{x_2} f[y(x), y'(x), x] dx \quad [\text{Eq. (6.4)}]$$

taken along a path  $y = y(x)$  is stationary with respect to variations of that path if and only if  $y(x)$  satisfies the **Euler–Lagrange equation**

$$\frac{\partial f}{\partial y} - \frac{d}{dx} \frac{\partial f}{\partial y'} = 0. \quad [\text{Eq. (6.13)}]$$

Here, f is function, we will replace by Lagrangian L

### EXAMPLE 7.8 Atwood's Machine Using a Lagrange Multiplier

Analyze the Atwood machine of Figure 7.6 (shown again here as Figure 7.11) by the method of Lagrange multipliers and using the coordinates  $x$  and  $y$  of the two masses.

In terms of the given coordinates, the Lagrangian is

$$\mathcal{L} = T - U = \frac{1}{2}m_1\dot{x}^2 + \frac{1}{2}m_2\dot{y}^2 + m_1gx + m_2gy \quad (7.123)$$

and the constraint equation is

$$f(x, y) = x + y = \text{const.} \quad (7.124)$$

The modified Lagrange equation (7.118) for  $x$  reads

$$\frac{\partial \mathcal{L}}{\partial x} + \lambda \frac{\partial f}{\partial x} = \frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{x}} \quad \text{or} \quad m_1g + \lambda = m_1\ddot{x} \quad (7.125)$$

and that for  $y$  is

$$\frac{\partial \mathcal{L}}{\partial y} + \lambda \frac{\partial f}{\partial y} = \frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{y}} \quad \text{or} \quad m_2g + \lambda = m_2\ddot{y}. \quad (7.126)$$

These two equations, together with the constraint equation (7.124), are easily solved for the unknowns  $x(t)$ ,  $y(t)$ , and  $\lambda(t)$ . From (7.124) we see that  $\ddot{y} = -\ddot{x}$ , and then subtracting (7.126) from (7.125) we can eliminate  $\lambda$  and arrive at the same result as before,

$$\ddot{x} = (m_1 - m_2)g / (m_1 + m_2).$$

To better understand the two modified Lagrange equations (7.125) and (7.126), it is helpful to compare them with the two equations of the Newtonian solution. Newton's second law for  $m_1$  is

$$m_1g - F_t = m_1\ddot{x}$$

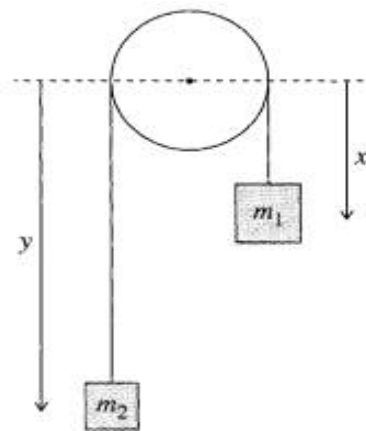


Figure 7.11 The Atwood machine again.

where  $F_t$  is the tension in the string, and that for  $m_2$  is

$$m_2g - F_t = m_2\ddot{y}.$$

These are precisely the two Lagrange equations (7.125) and (7.126), with the Lagrange multiplier identified as the constraint force

$$\lambda = -F_t.$$

[Two small comments: The minus sign occurs because both coordinates  $x$  and  $y$  were measured downward, whereas both tension forces are upward. In general, according to (7.122) the constraint force is  $\lambda \partial f / \partial x$ , but in this simple case,  $\partial f / \partial x = 1$ .]