$$S = \int L(q\,\dot{q})\,dt$$

$$0 = \delta S = \int \frac{\partial L}{\partial q}\,dq + \frac{\partial L}{\partial \dot{q}}\,d\dot{q}$$

$$\delta S = \int \frac{\partial L}{\partial q}\,dq - \frac{d}{dt}\frac{\partial L}{\partial \dot{q}}\,dq$$

Integration By Parts:

$$\int d(uv) = \int (du)v + \int u(dv)$$

$$uv\Big]_0^1 = 0 \quad \text{Assume}$$

$$(du)v = -u(dv)$$

Lagrange Equations:

$$\frac{\partial L}{\partial q} - \frac{d}{dt}\frac{\partial L}{\partial \dot{q}} = 0$$

$$L = T - V$$

SHO: $T = \frac{1}{2}mv^2 = \frac{1}{2}m\dot{q}^2$

$V = \frac{1}{2}kx^2 = \frac{1}{2}kq^2$

$$\frac{\partial L}{\partial q} = -kq \qquad \frac{\partial L}{\partial \dot{q}} = m\dot{q}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) = m\ddot{q}$$

Lagrange Eqs $\Rightarrow$

$$-kq - m\ddot{q} = 0$$

$$m\ddot{q} + kq = 0$$

$$m\ddot{x} + kx = 0$$

$$\ddot{x} + \omega_0^2 x = 0$$
