Phys 3344: Tuesday 06 Oct

Office Hours: Wed 5:00-6:00

Schedule:

Homework #7:

Ch 8

Homework #7:

	2020 FALL PHYS 3344				
#	**DAY**	**LECTURE:**	**NOTES:**	**Chpt**	**TOPIC**
1	TUE	08/25/20	First Class	1	Newtons Laws
2	THUR	08/27/20		2	Projectiles
3	TUE	09/01/20		3	Momentum & Angular Momentum
4	THUR	09/03/20		4	Energy
5	TUE	09/08/20		5	Oscillations
6	THUR	09/10/20			
7	TUE	09/15/20			
8	THUR	09/17/20			**EXAM 1**
9	TUE	09/22/20		6	Calculus of Variations
10	THUR	09/24/20		7	Lagrange's Equation
11	TUE	09/29/20			
12	THUR	10/01/20		8	Two Body Problems
13	TUE	10/06/20			
14	THUR	10/08/20		9	Non-Inertial Frames
	TUE	10/13/20	~~**Fall Break**~~	10	Rotational Motion
15	THUR	10/15/20			**EXAM 2**
16	TUE	10/20/20		10	Rotational Motion
17	THUR	10/22/20			
18	TUE	10/27/20		11	Coupled Oscillations
19	THUR	10/29/20			
20	TUE	11/03/20		13	Hamiltonian Mechanics
21	THUR	11/05/20	Drop Date		
22	TUE	11/10/20			
23	THUR	11/12/20			**EXAM 3**
24	TUE	11/17/20		14	Collision Theory
25	THUR	11/19/20			
26	TUE	11/24/20		15	Special relativity
27	THUR	11/26/20	**Thanksgiving**		No Class
28	TUE	12/01/20			No Class
29	THUR	12/03/20	**Last Class**		Review
	WED	**Dec 16**	**FINAL EXAM**	**Wednesday Dec. 16,2020, 11:30am - 2:30**	
	Adjustments may be made depending on student interests/needs and unplanned events				

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$$\left(\frac{d\mathbf{Q}}{dt}\right)_{\mathcal{S}_o} = \left(\frac{d\mathbf{Q}}{dt}\right)_{\mathcal{S}} + \boldsymbol{\Omega} \times \mathbf{Q}.$$

$$\left(\frac{d\mathbf{r}}{dt}\right)_{\mathcal{S}_o} = \left(\frac{d\mathbf{r}}{dt}\right)_{\mathcal{S}} + \boldsymbol{\Omega} \times \mathbf{r}.$$

$$\left(\frac{d\,\text{[redacted]}}{dt}\right)_{\mathcal{S}_o} = \left(\frac{d\,\text{[redacted]}}{dt}\right)_{\mathcal{S}} + \boldsymbol{\Omega} \times \text{[redacted]}$$

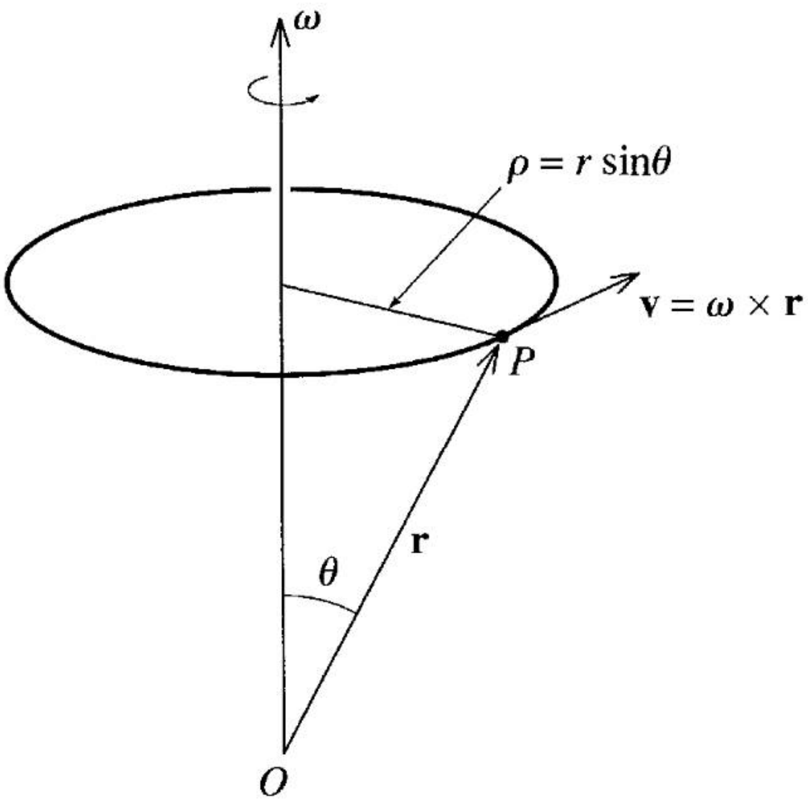

Figure 9.7 The earth's rotation drags the point P on the surface around a circle of latitude (radius $\rho = r\sin\theta$) with speed $v = \omega\rho = \omega r\sin\theta$ and hence velocity $\mathbf{v} = \boldsymbol{\omega} \times \mathbf{r}$.

on a rotating Earth, because the rightward motion of the ball is faster than that of the tower.

Formula [edit]

See also: Fictitious force

In Newtonian mechanics, the equation of motion for an object in an inertial reference frame is

$$\boldsymbol{F} = m\boldsymbol{a}$$

where $\boldsymbol{F}$ is the vector sum of the physical forces acting on the object, m is the mass of the object, and $\boldsymbol{a}$ is the acceleration of the object relative to the inertial reference frame.

Transforming this equation to a reference frame rotating about a fixed axis through the origin with rotation vector $\boldsymbol{\Omega}$ having variable rotation rate, the equation takes the form

$$\boldsymbol{F'} - m\frac{\mathrm{d}\boldsymbol{\Omega}}{\mathrm{d}t} \times \boldsymbol{r'} - 2m\boldsymbol{\Omega} \times \boldsymbol{v'} - m\boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \boldsymbol{r'}) = m\boldsymbol{a'}$$

where

$\boldsymbol{F'}$ is the vector sum of the physical forces acting on the object relative to the rotating reference frame

$\boldsymbol{\Omega}$ is the rotation vector, with magnitude ω, of the rotating reference frame relative to the inertial frame

$\boldsymbol{v'}$ is the velocity relative to the rotating reference frame

$\boldsymbol{r'}$ is the position vector of the object relative to the rotating reference frame

$\boldsymbol{a'}$ is the acceleration relative to the rotating reference frame

The fictitious forces as they are perceived in the rotating frame act as additional forces that contribute to the apparent acceleration just like the real external forces.[25][26] The fictitious force terms of the equation are, reading from left to right:[27]

- Euler force $-m\frac{\mathrm{d}\boldsymbol{\Omega}}{\mathrm{d}t} \times \boldsymbol{r'}$
- Coriolis force $-2m\boldsymbol{\Omega} \times \boldsymbol{v'}$
- centrifugal force $-m\boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \boldsymbol{r'})$

Notice the Euler and centrifugal forces depend on the position vector $\boldsymbol{r'}$ of the object, while the Coriolis force depends on the object's velocity $\boldsymbol{v'}$ as measured in the rotating reference frame. As expected, for a non-rotating inertial frame of reference $(\boldsymbol{\Omega} = 0)$ the Coriolis force and all other fictitious forces disappear.[28] The forces also disappear for zero mass $(m = 0)$.

As the Coriolis force is proportional to a cross product of two vectors, it is perpendicular to both vectors, in this case the object's velocity and the frame's rotation vector. It therefore follows that:

- if the velocity is parallel to the rotation axis, the Coriolis force is zero. (For example, on Earth, this situation occurs for a body on the equator moving north or south relative to Earth's surface.)
- if the velocity is straight inward to the axis, the Coriolis force is in the direction of local rotation. (For example, on Earth, this situation occurs for a body on the equator falling downward, as in the Dechales illustration above, where the falling ball travels further to the east than does the tower.)
- if the velocity is straight outward from the axis, the Coriolis force is against the direction of local rotation. (In the tower example, a ball launched upward would move toward the west.)
- if the velocity is in the direction of rotation, the Coriolis force is outward from the axis. (For example, on Earth, this situation occurs for a body on the equator moving east relative to Earth's surface. It would move upward as seen by an observer on the surface. This effect (see Eötvös effect below) was discussed by Galileo Galilei in 1632 and by Riccioli in 1651.[29])
- if the velocity is against the direction of rotation, the Coriolis force is inward to the axis. (On Earth, this situation occurs for a body on the equator moving west, which would deflect downward as seen by an observer.)

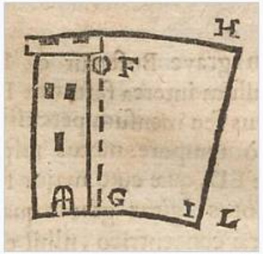

Image from *Cursus seu Mundus Mathematicus* (1674) of C.F.M. Dechales, showing how a ball should fall from a tower on a rotating Earth. The ball is released from *F*. The top of the tower moves faster than its base, so while the ball falls, the base of the tower moves to *I*, but the ball, which has the eastward speed of the tower's top, outruns the tower's base and lands further to the east at *L*.

Relation between velocities in the two frames [edit]

A velocity of an object is the time-derivative of the object's position, or

$$\mathbf{v} \stackrel{\text{def}}{=} \frac{\mathrm{d}\mathbf{r}}{\mathrm{d}t}$$

The time derivative of a position $\boldsymbol{r}(t)$ in a rotating reference frame has two components, one from the explicit time dependence due to motion of the particle itself, and another from the frame's own rotation. Applying the result of the previous subsection to the displacement $\boldsymbol{r}(t)$, the velocities in the two reference frames are related by the equation

$$\mathbf{v}_{\mathrm{i}} \stackrel{\text{def}}{=} \frac{\mathrm{d}\mathbf{r}}{\mathrm{d}t} = \left(\frac{\mathrm{d}\mathbf{r}}{\mathrm{d}t}\right)_{\mathrm{r}} + \boldsymbol{\Omega} \times \mathbf{r} = \mathbf{v}_{\mathrm{r}} + \boldsymbol{\Omega} \times \mathbf{r} \,,$$

where subscript *i* means the inertial frame of reference, and *r* means the rotating frame of reference.

Relation between accelerations in the two frames [edit]

Acceleration is the second time derivative of position, or the first time derivative of velocity

$$\mathbf{a}_{\mathrm{i}} \stackrel{\text{def}}{=} \left(\frac{\mathrm{d}^2\mathbf{r}}{\mathrm{d}t^2}\right)_{\mathrm{i}} = \left(\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t}\right)_{\mathrm{i}} = \left[\left(\frac{\mathrm{d}}{\mathrm{d}t}\right)_{\mathrm{r}} + \boldsymbol{\Omega}\times\right]\left[\left(\frac{\mathrm{d}\mathbf{r}}{\mathrm{d}t}\right)_{\mathrm{r}} + \boldsymbol{\Omega} \times \mathbf{r}\right] \,,$$

where subscript *i* means the inertial frame of reference. Carrying out the differentiations and re-arranging some terms yields the acceleration *relative to the rotating* reference frame, $\mathbf{a}_{\mathrm{r}}$

$$\mathbf{a}_{\mathrm{r}} = \mathbf{a}_{\mathrm{i}} - 2\boldsymbol{\Omega} \times \mathbf{v}_{\mathrm{r}} - \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r}) - \frac{\mathrm{d}\boldsymbol{\Omega}}{\mathrm{d}t} \times \mathbf{r}$$

where $\mathbf{a}_{\mathrm{r}} \stackrel{\text{def}}{=} \left(\frac{\mathrm{d}^2\mathbf{r}}{\mathrm{d}t^2}\right)_{\mathrm{r}}$ is the apparent acceleration in the rotating reference frame, the term $-\boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r})$ represents centrifugal acceleration, and the term $-2\boldsymbol{\Omega} \times \mathbf{v}_{\mathrm{r}}$ is the Coriolis acceleration. The last term ($-\frac{\mathrm{d}\boldsymbol{\Omega}}{\mathrm{d}t} \times \mathbf{r}$) is the Euler acceleration and is zero in uniformly rotating frames.

Newton's second law in the two frames [edit]

When the expression for acceleration is multiplied by the mass of the particle, the three extra terms on the right-hand side result in fictitious forces in the rotating reference frame, that is, apparent forces that result from being in a non-inertial reference frame, rather than from any physical interaction between bodies.

Using Newton's second law of motion $\mathbf{F} = m\mathbf{a}$, we obtain:[1][2][3][5][6]

- the Coriolis force

  $$\mathbf{F}_{\text{Coriolis}} = -2m\boldsymbol{\Omega} \times \mathbf{v}_{\text{r}}$$

- the centrifugal force

  $$\mathbf{F}_{\text{centrifugal}} = -m\boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r})$$

- and the Euler force

  $$\mathbf{F}_{\text{Euler}} = -m\frac{\mathrm{d}\boldsymbol{\Omega}}{\mathrm{d}t} \times \mathbf{r}$$

where m is the mass of the object being acted upon by these fictitious forces. Notice that all three forces vanish when the frame is not rotating, that is, when $\boldsymbol{\Omega} = 0$.

For completeness, the inertial acceleration $\mathbf{a}_{\text{i}}$ due to impressed external forces $\mathbf{F}_{\text{imp}}$ can be determined from the total physical force in the inertial (non-rotating) frame (for example, force from physical interactions such as electromagnetic forces) using Newton's second law in the inertial frame:

$$\mathbf{F}_{\text{imp}} = m\mathbf{a}_{\text{i}}$$

Newton's law in the rotating frame then becomes

$$\mathbf{F}_{\mathbf{r}} = \mathbf{F}_{\text{imp}} + \mathbf{F}_{\text{centrifugal}} + \mathbf{F}_{\text{Coriolis}} + \mathbf{F}_{\text{Euler}} = m\mathbf{a}_{\mathbf{r}} \ .$$

In other words, to handle the laws of motion in a rotating reference frame:[6][7][8]

$$\frac{\partial \mathcal{L}}{\partial x} + \lambda \frac{\partial f}{\partial x} = \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{x}} \qquad \text{or} \qquad m_1 g + \lambda = m_1 \ddot{x} \qquad (7.125)$$

$$\frac{d}{dt}\frac{dL}{dq'} - \frac{dL}{dq} = \lambda \frac{df}{dq}$$

f is constraint equation

$$S[q, q'] = \int L[q, q']dq$$

$$\delta S[q, q'] = 0$$

Principal Definitions and Equations of Chapter 6

The Euler–Lagrange Equation

An integral of the form

$$S = \int_{x_1}^{x_2} f[y(x), y'(x), x]\,dx \qquad \text{[Eq. (6.4)]}$$

taken along a path $y = y(x)$ is stationary with respect to variations of that path if and only if $y(x)$ satisfies the **Euler–Lagrange equation**

$$\frac{\partial f}{\partial y} - \frac{d}{dx}\frac{\partial f}{\partial y'} = 0. \qquad \text{[Eq. (6.13)]}$$

Here, f is function, we will replace by Lagrangian L