

Phys 3344: Thursday 08 Oct

Office Hours: Wed 5:00-6:00

Exam #2: next week Ch. 6-9

Homework #7:

Ch 9

Homework #8:

2020 FALL PHYS 3344					
#	DAY	LECTURE:	NOTES:	Chpt	TOPIC
1	TUE	08/25/20	First Class	1	Newtons Laws
2	THUR	08/27/20		2	Projectiles
3	TUE	09/01/20		3	Momentum & Angular Momentum
4	THUR	09/03/20		4	Energy
5	TUE	09/08/20		5	Oscillations
6	THUR	09/10/20			
7	TUE	09/15/20			
8	THUR	09/17/20			EXAM 1
9	TUE	09/22/20		6	Calculus of Variations
10	THUR	09/24/20		7	Lagrange's Equation
11	TUE	09/29/20			
12	THUR	10/01/20		8	Two Body Problems
13	TUE	10/06/20			
14	THUR	10/08/20		9	Non-Inertial Frames
	TUE	10/13/20	Fall-Break	10	Rotational Motion
15	THUR	10/15/20			EXAM 2
16	TUE	10/20/20		10	Rotational Motion
17	THUR	10/22/20			
18	TUE	10/27/20		11	Coupled Oscillations
19	THUR	10/29/20			
20	TUE	11/03/20		13	Hamiltonian Mechanics
21	THUR	11/05/20	Drop Date		
22	TUE	11/10/20			
23	THUR	11/12/20			EXAM 3
24	TUE	11/17/20		14	Collision Theory
25	THUR	11/19/20			
26	TUE	11/24/20		15	Special relativity
27	THUR	11/26/20	Thanksgiving		No Class
28	TUE	12/01/20			No Class
29	THUR	12/03/20	Last Class		Review
	WED	Dec 16	FINAL EXAM	Wednesday Dec. 16,2020, 11:30am - 2:30	
<i>Adjustments may be made depending on student interests/needs and unplanned events</i>					

CHAPTER 6 Calculus of Variations 215

- 6.1 Two Examples 216
- 6.2 The Euler–Lagrange Equation 218
- 6.3 Applications of the Euler–Lagrange Equation 221
- 6.4 More than Two Variables 226
- Principal Definitions and Equations of Chapter 6 230
- Problems for Chapter 6 230

CHAPTER 7 Lagrange's Equations 237

- 7.1 Lagrange's Equations for Unconstrained Motion 238
- 7.2 Constrained Systems; an Example 245
- 7.3 Constrained Systems in General 247
- 7.4 Proof of Lagrange's Equations with Constraints 250
- 7.5 Examples of Lagrange's Equations 254
- 7.6 Generalized Momenta and Ignorable Coordinates 266
- 7.7 Conclusion 267
- 7.8 More about Conservation Laws* 268
- 7.9 Lagrange's Equations for Magnetic Forces* 272
- 7.10 Lagrange Multipliers and Constraint Forces* 275
- Principal Definitions and Equations of Chapter 7 280
- Problems for Chapter 7 281

CHAPTER 8 Two-Body Central-Force Problems 293

- 8.1 The Problem 293
- 8.2 CM and Relative Coordinates; Reduced Mass 295
- 8.3 The Equations of Motion 297
- 8.4 The Equivalent One-Dimensional Problem 300
- 8.5 The Equation of the Orbit 305
- 8.6 The Kepler Orbits 308
- 8.7 The Unbounded Kepler Orbits 313
- 8.8 Changes of Orbit 315
- Principal Definitions and Equations of Chapter 8 319
- Problems for Chapter 8 320

CHAPTER 9 Mechanics in Noninertial Frames 327

- 9.1 Acceleration without Rotation 327
- 9.2 The Tides 330
- 9.3 The Angular Velocity Vector 336
- 9.4 Time Derivatives in a Rotating Frame 339

- 9.5 Newton's Second Law in a Rotating Frame 342
- 9.6 The Centrifugal Force 344
- 9.7 The Coriolis Force 348
- 9.8 Free Fall and the Coriolis Force 351
- 9.9 The Foucault Pendulum 354
- 9.10 Coriolis Force and Coriolis Acceleration 358
- Principal Definitions and Equations of Chapter 9 359
- Problems for Chapter 9 360

CHAPTER 10 Rotational Motion of Rigid Bodies 367

- 10.1 Properties of the Center of Mass 367
- 10.2 Rotation about a Fixed Axis 372
- 10.3 Rotation about Any Axis; the Inertia Tensor 378
- 10.4 Principal Axes of Inertia 387
- 10.5 Finding the Principal Axes; Eigenvalue Equations 389
- 10.6 Precession of a Top due to a Weak Torque 392
- 10.7 Euler's Equations 394
- 10.8 Euler's Equations with Zero Torque 397
- 10.9 Euler Angles* 401
- 10.10 Motion of a Spinning Top* 403
- Principal Definitions and Equations of Chapter 10 407
- Problems for Chapter 10 408

CHAPTER 11 Coupled Oscillators and Normal Modes 417

- 11.1 Two Masses and Three Springs 417
- 11.2 Identical Springs and Equal Masses 421
- 11.3 Two Weakly Coupled Oscillators 426
- 11.4 Lagrangian Approach: The Double Pendulum 430
- 11.5 The General Case 436
- 11.6 Three Coupled Pendulums 441
- 11.7 Normal Coordinates* 444
- Principal Definitions and Equations of Chapter 11 447
- Problems for Chapter 11 448

PART II Further Topics 455**CHAPTER 12** Nonlinear Mechanics and Chaos 457

- 12.1 Linearity and Nonlinearity 458
- 12.2 The Driven Damped Pendulum DDP 462
- 12.3 Some Expected Features of the DDP 463

$$\left(\frac{d\mathbf{Q}}{dt}\right)_{S_0} = \left(\frac{d\mathbf{Q}}{dt}\right)_S + \boldsymbol{\Omega} \times \mathbf{Q}.$$

$$\left(\frac{d\mathbf{r}}{dt}\right)_{S_0} = \left(\frac{d\mathbf{r}}{dt}\right)_S + \boldsymbol{\Omega} \times \mathbf{r}.$$

$$\left(\frac{d\blacksquare}{dt}\right)_{S_0} = \left(\frac{d\blacksquare}{dt}\right)_S + \boldsymbol{\Omega} \times \blacksquare$$

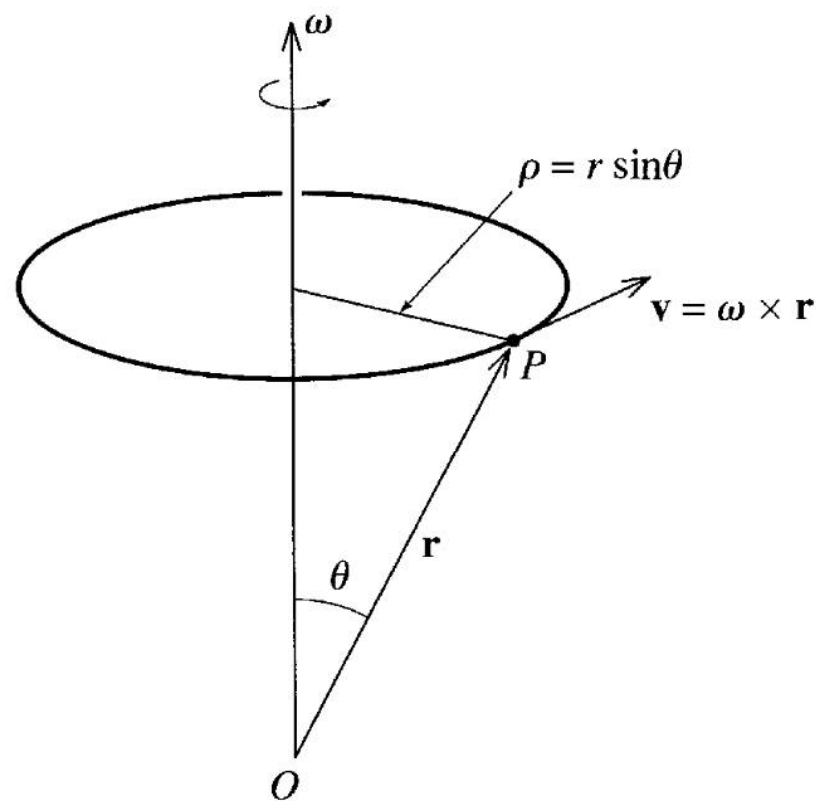


Figure 9.7 The earth's rotation drags the point P on the surface around a circle of latitude (radius $\rho = r \sin \theta$) with speed $v = \omega \rho = \omega r \sin \theta$ and hence velocity $\mathbf{v} = \boldsymbol{\omega} \times \mathbf{r}$.

Relation between velocities in the two frames [\[edit \]](#)

A velocity of an object is the time-derivative of the object's position, or

$$\mathbf{v} \stackrel{\text{def}}{=} \frac{d\mathbf{r}}{dt}$$

The time derivative of a position $\mathbf{r}(t)$ in a rotating reference frame has two components, one from the explicit time dependence due to motion of the particle itself, and another from the frame's own rotation. Applying the result of the previous subsection to the displacement $\mathbf{r}(t)$, the [velocities](#) in the two reference frames are related by the equation

$$\mathbf{v}_i \stackrel{\text{def}}{=} \frac{d\mathbf{r}}{dt} = \left(\frac{d\mathbf{r}}{dt} \right)_r + \boldsymbol{\Omega} \times \mathbf{r} = \mathbf{v}_r + \boldsymbol{\Omega} \times \mathbf{r} ,$$

where subscript i means the inertial frame of reference, and r means the rotating frame of reference.

Relation between accelerations in the two frames [\[edit \]](#)

Acceleration is the second time derivative of position, or the first time derivative of velocity

$$\mathbf{a}_i \stackrel{\text{def}}{=} \left(\frac{d^2\mathbf{r}}{dt^2} \right)_i = \left(\frac{d\mathbf{v}}{dt} \right)_i = \left[\left(\frac{d}{dt} \right)_r + \boldsymbol{\Omega} \times \right] \left[\left(\frac{d\mathbf{r}}{dt} \right)_r + \boldsymbol{\Omega} \times \mathbf{r} \right] ,$$

where subscript i means the inertial frame of reference. Carrying out the [differentiations](#) and re-arranging some terms yields the acceleration *relative to the rotating* reference frame, \mathbf{a}_r

$$\mathbf{a}_r = \mathbf{a}_i - 2\boldsymbol{\Omega} \times \mathbf{v}_r - \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r}) - \frac{d\boldsymbol{\Omega}}{dt} \times \mathbf{r}$$

where $\mathbf{a}_r \stackrel{\text{def}}{=} \left(\frac{d^2\mathbf{r}}{dt^2} \right)_r$ is the apparent acceleration in the rotating reference frame, the term $-\boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r})$ represents [centrifugal acceleration](#), and

the term $-2\boldsymbol{\Omega} \times \mathbf{v}_r$ is the [Coriolis acceleration](#). The last term $(-\frac{d\boldsymbol{\Omega}}{dt} \times \mathbf{r})$ is the [Euler acceleration](#) and is zero in uniformly rotating frames.

Newton's second law in the two frames [\[edit \]](#)

When the expression for acceleration is multiplied by the mass of the particle, the three extra terms on the right-hand side result in **fictitious forces** in the rotating reference frame, that is, apparent forces that result from being in a **non-inertial reference frame**, rather than from any physical interaction between bodies.

Using **Newton's second law of motion** $\mathbf{F} = m\mathbf{a}$, we obtain:^{[\[1\]](#)[\[2\]](#)[\[3\]](#)[\[5\]](#)[\[6\]](#)}

- the **Coriolis force**

$$\mathbf{F}_{\text{Coriolis}} = -2m\boldsymbol{\Omega} \times \mathbf{v}_r$$

- the **centrifugal force**

$$\mathbf{F}_{\text{centrifugal}} = -m\boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r})$$

- and the **Euler force**

$$\mathbf{F}_{\text{Euler}} = -m \frac{d\boldsymbol{\Omega}}{dt} \times \mathbf{r}$$

where m is the mass of the object being acted upon by these **fictitious forces**. Notice that all three forces vanish when the frame is not rotating, that is, when $\boldsymbol{\Omega} = 0$.

For completeness, the inertial acceleration \mathbf{a}_i due to impressed external forces \mathbf{F}_{imp} can be determined from the total physical force in the inertial (non-rotating) frame (for example, force from physical interactions such as **electromagnetic forces**) using **Newton's second law** in the inertial frame:

$$\mathbf{F}_{\text{imp}} = m\mathbf{a}_i$$

Newton's law in the rotating frame then becomes

$$\mathbf{F}_r = \mathbf{F}_{\text{imp}} + \mathbf{F}_{\text{centrifugal}} + \mathbf{F}_{\text{Coriolis}} + \mathbf{F}_{\text{Euler}} = m\mathbf{a}_r .$$

In other words, to handle the laws of motion in a rotating reference frame:^{[\[6\]](#)[\[7\]](#)[\[8\]](#)}

Formula [[edit](#)]

See also: *Fictitious force*

In [Newtonian mechanics](#), the equation of motion for an object in an inertial reference frame is

$$\mathbf{F} = m\mathbf{a}$$

where **F** is the vector sum of the physical forces acting on the object, **m** is the mass of the object, and **a** is the acceleration of the object relative to the inertial reference frame.

Transforming this equation to a reference frame rotating about a fixed axis through the origin with rotation vector **Ω** having variable rotation rate, the equation takes the form

$$\mathbf{F}' - m\frac{d\mathbf{\Omega}}{dt} \times \mathbf{r}' - 2m\mathbf{\Omega} \times \mathbf{v}' - m\mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{r}') = m\mathbf{a}'$$

where

F' is the vector sum of the physical forces acting on the object relative to the rotating reference frame

Ω is the [rotation vector](#), with magnitude *ω*, of the rotating reference frame relative to the inertial frame

v' is the velocity relative to the rotating reference frame

r' is the position vector of the object relative to the rotating reference frame

a' is the acceleration relative to the rotating reference frame

The fictitious forces as they are perceived in the rotating frame act as additional forces that contribute to the apparent acceleration just like the real external forces.^{[25][26]} The fictitious force terms of the equation are, reading from left to right:^[27]

- Euler force $-m\frac{d\mathbf{\Omega}}{dt} \times \mathbf{r}'$
- Coriolis force $-2m\mathbf{\Omega} \times \mathbf{v}'$
- centrifugal force $-m\mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{r}')$

Notice the Euler and centrifugal forces depend on the position vector **r'** of the object, while the Coriolis force depends on the object's velocity **v'** as measured in the rotating reference frame. As expected, for a non-rotating [inertial frame of reference](#) (**Ω** = 0) the Coriolis force and all other fictitious forces disappear.^[28] The forces also disappear for zero mass (**m** = 0).

As the Coriolis force is proportional to a [cross product](#) of two vectors, it is perpendicular to both vectors, in this case the object's velocity and the frame's rotation vector. It therefore follows that:

- if the velocity is parallel to the rotation axis, the Coriolis force is zero. (For example, on Earth, this situation occurs for a body on the equator moving north or south relative to Earth's surface.)
- if the velocity is straight inward to the axis, the Coriolis force is in the direction of local rotation. (For example, on Earth, this situation occurs for a body on the equator falling downward, as in the Dechales illustration above, where the falling ball travels further to the east than does the tower.)
- if the velocity is straight outward from the axis, the Coriolis force is against the direction of local rotation. (In the tower example, a ball launched upward would move toward the west.)
- if the velocity is in the direction of rotation, the Coriolis force is outward from the axis. (For example, on Earth, this situation occurs for a body on the equator moving east relative to Earth's surface. It would move upward as seen by an observer on the surface. This effect (see Eötvös effect below) was discussed by Galileo Galilei in 1632 and by Riccioli in 1651.^[29])
- if the velocity is against the direction of rotation, the Coriolis force is inward to the axis. (On Earth, this situation occurs for a body on the equator moving west, which would deflect downward as seen by an observer.)

On a rotating Earth, because the rightward motion of the ball is faster than that of the tower.

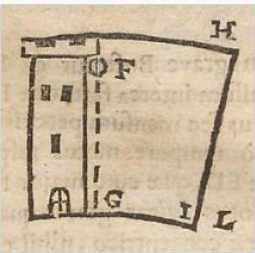


Image from *Cursus seu Mundus Mathematicus* (1674) of C.F.M. Dechales, showing how a ball should fall from a tower on a rotating Earth. The ball is released from *F*. The top of the tower moves faster than its base, so while the ball falls, the base of the tower moves to *I*, but the ball, which has the eastward speed of the tower's top, outruns the tower's base and lands further to the east at *L*.

$$\frac{\partial \mathcal{L}}{\partial x} + \lambda \frac{\partial f}{\partial x} = \frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{x}} \quad \text{or} \quad m_1 g + \lambda = m_1 \ddot{x} \quad (7.125)$$

$$\frac{d}{dt} \frac{dL}{dq'} - \frac{dL}{dq} = \lambda \frac{df}{dq}$$

f is constraint equation

$$S[q, q'] = \int L[q, q'] dq$$

$$\delta S[q, q'] = 0$$

Principal Definitions and Equations of Chapter 6

The Euler–Lagrange Equation

An integral of the form

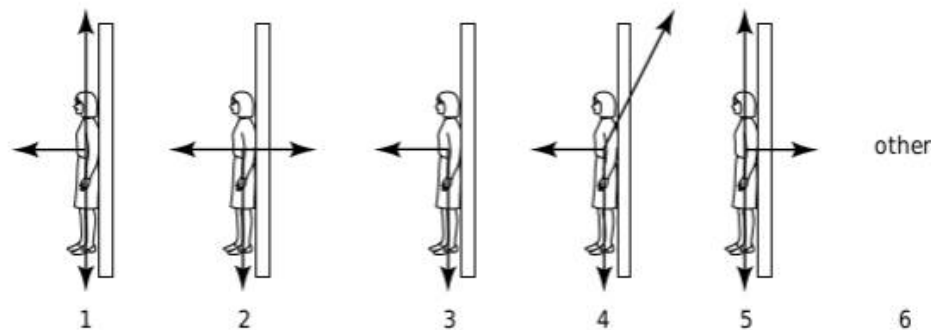
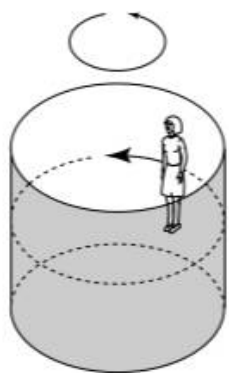
$$S = \int_{x_1}^{x_2} f[y(x), y'(x), x] dx \quad [\text{Eq. (6.4)}]$$

taken along a path $y = y(x)$ is stationary with respect to variations of that path if and only if $y(x)$ satisfies the **Euler–Lagrange equation**

$$\frac{\partial f}{\partial y} - \frac{d}{dx} \frac{\partial f}{\partial y'} = 0. \quad [\text{Eq. (6.13)}]$$

Here, f is function, we will replace by Lagrangian L

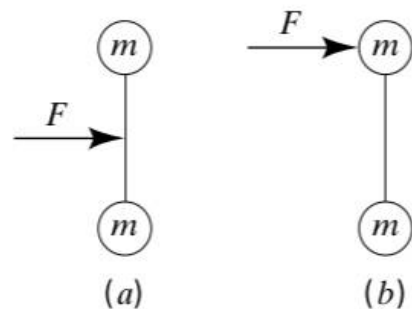
A rider in a “barrel of fun” finds herself stuck with her back to the wall. Which diagram correctly shows the forces acting on her?



Consider two people on opposite sides of a rotating merry-go-round. One of them throws a ball toward the other. In which frame of reference is the path of the ball straight when viewed from above: (a) the frame of the merry-go-round or (b) that of Earth?

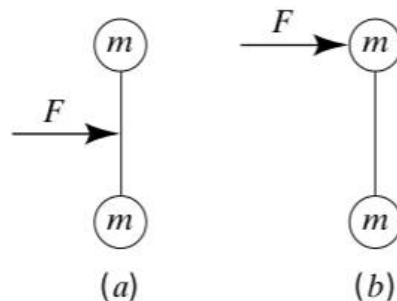
1. (a) only
2. (a) and (b)—although the paths appear to curve
3. (b) only
4. neither; because it's thrown while in circular motion, the ball travels along a curved path.

A force F is applied to a dumbbell for a time interval Δt , first as in (a) and then as in (b). In which case does the dumbbell acquire the greater center-of-mass speed?



1. (a)
2. (b)
3. no difference
4. The answer depends on the rotational inertia of the dumbbell.

A force F is applied to a dumbbell for a time interval Δt , first as in (a) and then as in (b). In which case does the dumbbell acquire the greater energy?



1. (a)
2. (b)
3. no difference
4. The answer depends on the rotational inertia of the dumbbell.