Phys 3344: Tuesday 13 Oct

Office Hours: Wed 5:00-6:00

Exam #2: this week Ch. 6-9 + Ch5. Fourier Transforms

Ch 9

Eigenvalues

Homework #8:

	2020 FALL PHYS 3344				
#	**DAY**	**LECTURE:**	**NOTES:**	**Chpt**	**TOPIC**
1	TUE	08/25/20	First Class	1	Newtons Laws
2	THUR	08/27/20		2	Projectiles
3	TUE	09/01/20		3	Momentum & Angular Momentum
4	THUR	09/03/20		4	Energy
5	TUE	09/08/20		5	Oscillations
6	THUR	09/10/20			
7	TUE	09/15/20			
8	THUR	09/17/20			**EXAM 1**
9	TUE	09/22/20		6	Calculus of Variations
10	THUR	09/24/20		7	Lagrange's Equation
11	TUE	09/29/20			
12	THUR	10/01/20		8	Two Body Problems
13	TUE	10/06/20			
14	THUR	10/08/20		9	Non-Inertial Frames
	TUE	10/13/20	~~**Fall Break**~~	10	Rotational Motion
15	THUR	10/15/20			**EXAM 2**
16	TUE	10/20/20		10	Rotational Motion
17	THUR	10/22/20			
18	TUE	10/27/20		11	Coupled Oscillations
19	THUR	10/29/20			
20	TUE	11/03/20		13	Hamiltonian Mechanics
21	THUR	11/05/20	Drop Date		
22	TUE	11/10/20			
23	THUR	11/12/20			**EXAM 3**
24	TUE	11/17/20		14	Collision Theory
25	THUR	11/19/20			
26	TUE	11/24/20		15	Special relativity
27	THUR	11/26/20	**Thanksgiving**		No Class
28	TUE	12/01/20			No Class
29	THUR	12/03/20	**Last Class**		Review
	WED	**Dec 16**	**FINAL EXAM**	**Wednesday Dec. 16,2020, 11:30am - 2:30**	
	Adjustments may be made depending on student interests/needs and unplanned events				

$$S[q, q'] = \int L[q, q']dq$$

$$L = T - U$$

$$\delta S[q, q'] = 0$$

$$\mathbf{F}_{\text{cor}} = 2m\dot{\mathbf{r}} \times \mathbf{\Omega} \quad \text{and} \quad \mathbf{F}_{\text{cf}} = m(\mathbf{\Omega} \times \mathbf{r}) \times \mathbf{\Omega}. \quad \text{[Eqs. (9.35) \& (9.36)]}$$

$$\frac{d}{dt}\frac{dL}{dq'} - \frac{dL}{dq} = \lambda\frac{df}{dq}$$

f is constraint equation

- the Coriolis force

$$\mathbf{F}_{\text{Coriolis}} = -2m\mathbf{\Omega} \times \mathbf{v}_{\text{r}}$$

- the centrifugal force

$$\mathbf{F}_{\text{centrifugal}} = -m\mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{r})$$

- and the Euler force

$$\mathbf{F}_{\text{Euler}} = -m\frac{\mathrm{d}\mathbf{\Omega}}{\mathrm{d}t} \times \mathbf{r}$$

$$\mu = \frac{m_1 m_2}{m_1 + m_2}$$

$$R_{CM} = \frac{r_1 m_1 + r_2 m_2}{m_1 + m_2}$$

$$r(\phi) = \frac{c}{1 + \epsilon\cos\phi}$$

$$\sin\theta^2 + \cos\theta^2 = 1$$

MECHANICS

$v_x = v_{x0} + a_x t$

$x = x_0 + v_{x0}t + \frac{1}{2}a_x t^2$

$v_x^2 = v_{x0}^2 + 2a_x(x - x_0)$

$\vec{a} = \frac{\sum \vec{F}}{m} = \frac{\vec{F}_{net}}{m}$

$|\vec{F}_f| \le \mu |\vec{F}_n|$

$a_c = \frac{v^2}{r}$

$\vec{p} = m\vec{v}$

$\Delta\vec{p} = \vec{F}\,\Delta t$

$K = \frac{1}{2}mv^2$

$\Delta E = W = F_{\parallel}d = Fd\cos\theta$

$P = \frac{\Delta E}{\Delta t}$

$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$

$\omega = \omega_0 + \alpha t$

$x = A\cos(2\pi ft)$

$\vec{\alpha} = \frac{\sum \vec{\tau}}{I} = \frac{\vec{\tau}_{net}}{I}$

$\tau = r_{\perp}F = rF\sin\theta$

$L = I\omega$

$\Delta L = \tau\,\Delta t$

$K = \frac{1}{2}I\omega^2$

$|\vec{F}_s| = k|\vec{x}|$

$U_s = \frac{1}{2}kx^2$

$\rho = \frac{m}{V}$

a = acceleration
A = amplitude
d = distance
E = energy
f = frequency
F = force
I = rotational inertia
K = kinetic energy
k = spring constant
L = angular momentum
ℓ = length
m = mass
P = power
p = momentum
r = radius or separation
T = period
t = time
U = potential energy
V = volume
v = speed
W = work done on a system
x = position
y = height
α = angular acceleration
μ = coefficient of friction
θ = angle
ρ = density
τ = torque
ω = angular speed

$\Delta U_g = mg\,\Delta y$

$T = \frac{2\pi}{\omega} = \frac{1}{f}$

$T_s = 2\pi\sqrt{\frac{m}{k}}$

$T_p = 2\pi\sqrt{\frac{\ell}{g}}$

$|\vec{F}_g| = G\frac{m_1 m_2}{r^2}$

$\vec{g} = \frac{\vec{F}_g}{m}$

$U_G = -\frac{Gm_1 m_2}{r}$

ELECTRICITY

$|\vec{F}_E| = k\left|\frac{q_1 q_2}{r^2}\right|$

$I = \frac{\Delta q}{\Delta t}$

$R = \frac{\rho\ell}{A}$

$I = \frac{\Delta V}{R}$

$P = I\,\Delta V$

$R_s = \sum_i R_i$

$\frac{1}{R_p} = \sum_i \frac{1}{R_i}$

A = area
F = force
I = current
ℓ = length
P = power
q = charge
R = resistance
r = separation
t = time
V = electric potential
ρ = resistivity

WAVES

$\lambda = \frac{v}{f}$

f = frequency
v = speed
λ = wavelength

GEOMETRY AND TRIGONOMETRY

Rectangle
$A = bh$

Triangle
$A = \frac{1}{2}bh$

Circle
$A = \pi r^2$
$C = 2\pi r$

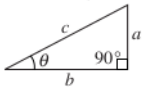
Rectangular solid
$V = \ell wh$

Cylinder
$V = \pi r^2 \ell$
$S = 2\pi r\ell + 2\pi r^2$

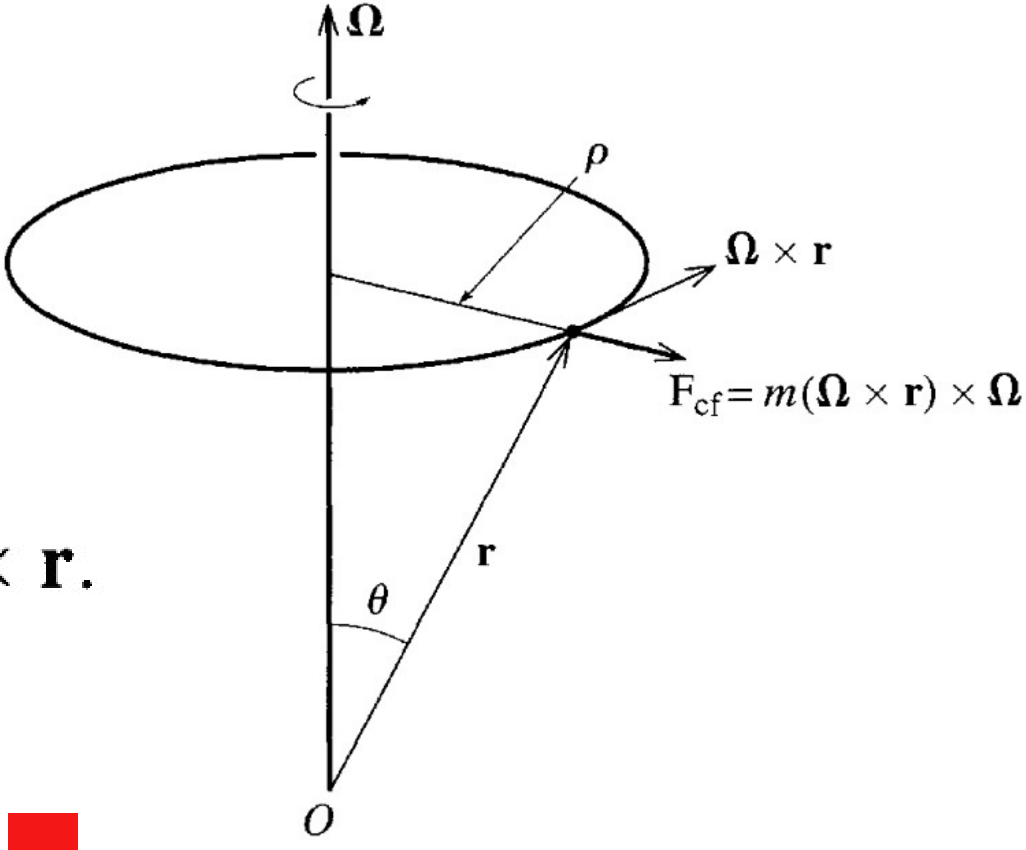
Sphere
$V = \frac{4}{3}\pi r^3$
$S = 4\pi r^2$

A = area
C = circumference
V = volume
S = surface area
b = base
h = height
ℓ = length
w = width
r = radius

Right triangle
$c^2 = a^2 + b^2$
$\sin\theta = \frac{a}{c}$
$\cos\theta = \frac{b}{c}$
$\tan\theta = \frac{a}{b}$



$$\left(\frac{d\mathbf{Q}}{dt}\right)_{\mathcal{S}_o} = \left(\frac{d\mathbf{Q}}{dt}\right)_{\mathcal{S}} + \mathbf{\Omega} \times \mathbf{Q}.$$



$$\left(\frac{d\mathbf{r}}{dt}\right)_{\mathcal{S}_o} = \left(\frac{d\mathbf{r}}{dt}\right)_{\mathcal{S}} + \mathbf{\Omega} \times \mathbf{r}.$$

$$\left(\frac{d\,\square}{dt}\right)_{\mathcal{S}_o} = \left(\frac{d\,\square}{dt}\right)_{\mathcal{S}} + \mathbf{\Omega} \times \square$$

The vector $\mathbf{\Omega} \times \mathbf{r}$ is the velocity of an object as it is dragged eastward with speed $\Omega\rho$ by the earth's rotation. Therefore, the centrifugal force, $m(\mathbf{\Omega} \times \mathbf{r}) \times \mathbf{\Omega}$, points radially outward from the axis and has magnitude $m\Omega^2\rho$.

Relation between velocities in the two frames [edit]

A velocity of an object is the time-derivative of the object's position, or

$$\mathbf{v} \stackrel{\text{def}}{=} \frac{\mathrm{d}\mathbf{r}}{\mathrm{d}t}$$

The time derivative of a position $\boldsymbol{r}(t)$ in a rotating reference frame has two components, one from the explicit time dependence due to motion of the particle itself, and another from the frame's own rotation. Applying the result of the previous subsection to the displacement $\boldsymbol{r}(t)$, the velocities in the two reference frames are related by the equation

$$\mathbf{v_i} \stackrel{\text{def}}{=} \frac{\mathrm{d}\mathbf{r}}{\mathrm{d}t} = \left(\frac{\mathrm{d}\mathbf{r}}{\mathrm{d}t}\right)_{\mathrm{r}} + \boldsymbol{\Omega} \times \mathbf{r} = \mathbf{v}_{\mathrm{r}} + \boldsymbol{\Omega} \times \mathbf{r} \ ,$$

where subscript *i* means the inertial frame of reference, and *r* means the rotating frame of reference.

Relation between accelerations in the two frames [edit]

Acceleration is the second time derivative of position, or the first time derivative of velocity

$$\mathbf{a}_{\mathrm{i}} \stackrel{\text{def}}{=} \left(\frac{\mathrm{d}^2\mathbf{r}}{\mathrm{d}t^2}\right)_{\mathrm{i}} = \left(\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t}\right)_{\mathrm{i}} = \left[\left(\frac{\mathrm{d}}{\mathrm{d}t}\right)_{\mathrm{r}} + \boldsymbol{\Omega}\times\right]\left[\left(\frac{\mathrm{d}\mathbf{r}}{\mathrm{d}t}\right)_{\mathrm{r}} + \boldsymbol{\Omega} \times \mathbf{r}\right] \ ,$$

where subscript *i* means the inertial frame of reference. Carrying out the differentiations and re-arranging some terms yields the acceleration *relative to the rotating* reference frame, $\mathbf{a}_{\mathrm{r}}$

$$\mathbf{a}_{\mathrm{r}} = \mathbf{a}_{\mathrm{i}} - 2\boldsymbol{\Omega} \times \mathbf{v}_{\mathrm{r}} - \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r}) - \frac{\mathrm{d}\boldsymbol{\Omega}}{\mathrm{d}t} \times \mathbf{r}$$

where $\mathbf{a}_{\mathrm{r}} \stackrel{\text{def}}{=} \left(\frac{\mathrm{d}^2\mathbf{r}}{\mathrm{d}t^2}\right)_{\mathrm{r}}$ is the apparent acceleration in the rotating reference frame, the term $-\boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r})$ represents centrifugal acceleration, and the term $-2\boldsymbol{\Omega} \times \mathbf{v}_{\mathrm{r}}$ is the Coriolis acceleration. The last term ($-\frac{\mathrm{d}\boldsymbol{\Omega}}{\mathrm{d}t} \times \mathbf{r}$) is the Euler acceleration and is zero in uniformly rotating frames.

Newton's second law in the two frames [edit]

When the expression for acceleration is multiplied by the mass of the particle, the three extra terms on the right-hand side result in fictitious forces in the rotating reference frame, that is, apparent forces that result from being in a non-inertial reference frame, rather than from any physical interaction between bodies.

Using Newton's second law of motion $\mathbf{F} = m\mathbf{a}$, we obtain:[1][2][3][5][6]

- the Coriolis force

$$\mathbf{F}_{\text{Coriolis}} = -2m\boldsymbol{\Omega} \times \mathbf{v}_{\text{r}}$$

- the centrifugal force

$$\mathbf{F}_{\text{centrifugal}} = -m\boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r})$$

- and the Euler force

$$\mathbf{F}_{\text{Euler}} = -m\frac{\mathrm{d}\boldsymbol{\Omega}}{\mathrm{d}t} \times \mathbf{r}$$

where m is the mass of the object being acted upon by these fictitious forces. Notice that all three forces vanish when the frame is not rotating, that is, when $\boldsymbol{\Omega} = 0$.

For completeness, the inertial acceleration $\mathbf{a}_{\text{i}}$ due to impressed external forces $\mathbf{F}_{\text{imp}}$ can be determined from the total physical force in the inertial (non-rotating) frame (for example, force from physical interactions such as electromagnetic forces) using Newton's second law in the inertial frame:

$$\mathbf{F}_{\text{imp}} = m\mathbf{a}_{\text{i}}$$

Newton's law in the rotating frame then becomes

$$\mathbf{F}_{\mathbf{r}} = \mathbf{F}_{\text{imp}} + \mathbf{F}_{\text{centrifugal}} + \mathbf{F}_{\text{Coriolis}} + \mathbf{F}_{\text{Euler}} = m\mathbf{a}_{\mathbf{r}} \ .$$

In other words, to handle the laws of motion in a rotating reference frame:[6][7][8]

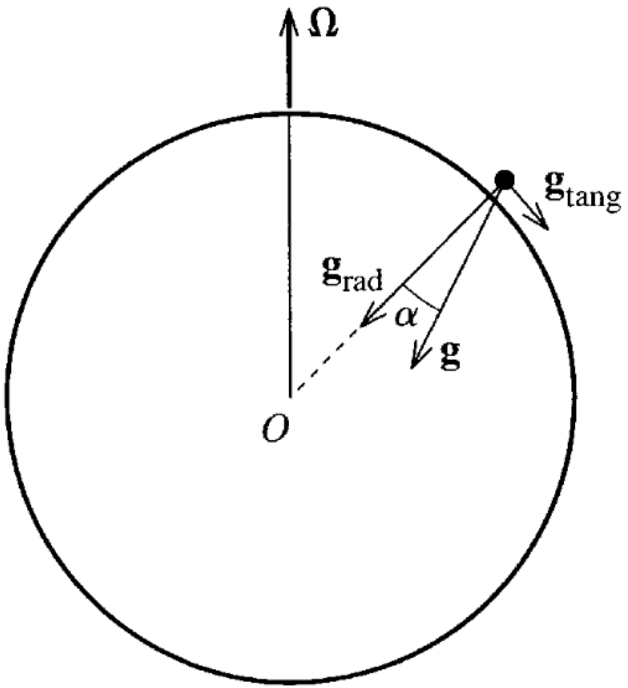


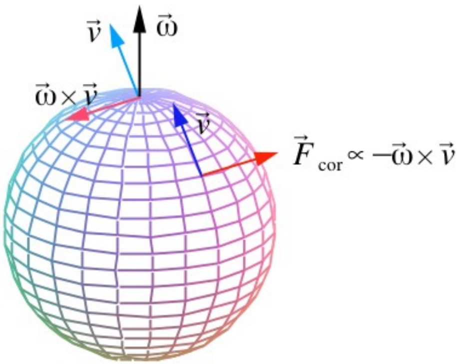
Figure 9.11 Because of the centrifugal force, the free-fall acceleration **g** has a nonzero tangential component (greatly exaggerated here) and **g** deviates from the radial direction by the small angle α.

Coriolis: about 3:00min

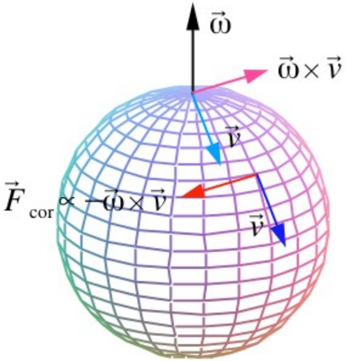
https://youtu.be/6L5UD240mCQ

https://youtu.be/f8IwL2ZtDTc

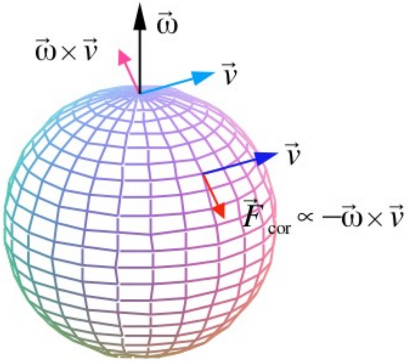
Body moving north (v_θ negative, other components zero); coriolis force eastward:



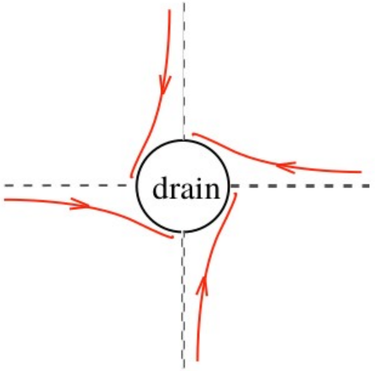
Body moving south (v_θ positive, other components zero); coriolis force westward:



Body moving east (v_φ positive, other components zero); coriolis force has component in southward direction:

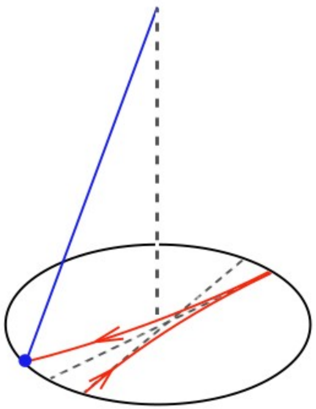


We see that for any body moving on the surface of the earth in the northern hemisphere, the coriolis force deflects it to the *right*. This is responsible for the counterclockwise rotation of the bath water as it drains from your tub, as viewed from above:



The same applies to the direction of air flow around an area of low atmospheric pressure.

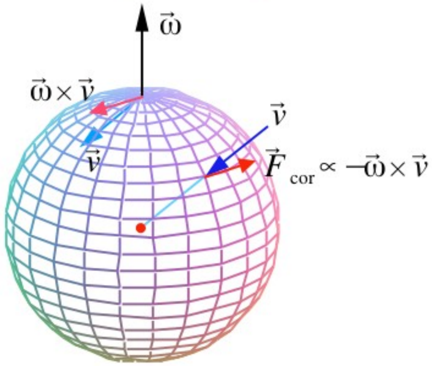
Another case where the coriolis force is important is the *Foucault pendulum* (pronounced "Foo-ko"). This is a very large pendulum which you sometimes see in the lobbies of big important buildings. The plane in which the pendulum swings back and forth *precesses*, or turns, slowly around in a clockwise direction as viewed from above. The following diagram shows why:



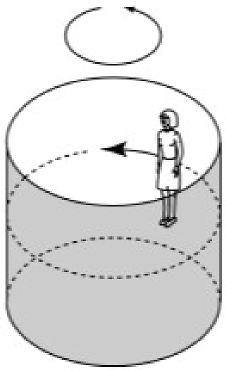
As the pendulum swings across, the coriolis force pushes it to the right. On the way back it is also pushed to the right, and this just rotates the plane of the pendulum further in the clockwise sense. The Foucault pendulum is a rather striking demonstration of the rotation of the earth. Here is a movie showing the Foucault pendulum from above.

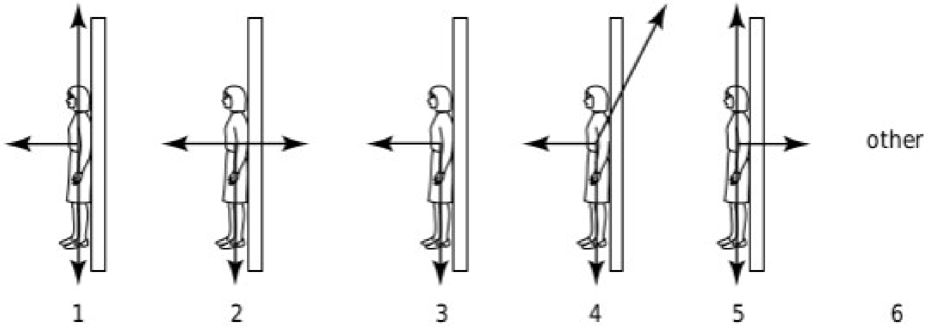
Last but not least, there is also an eastward force on a body falling vertically:

Body falling vertically (v_r negative, other components zero); coriolis force eastward:



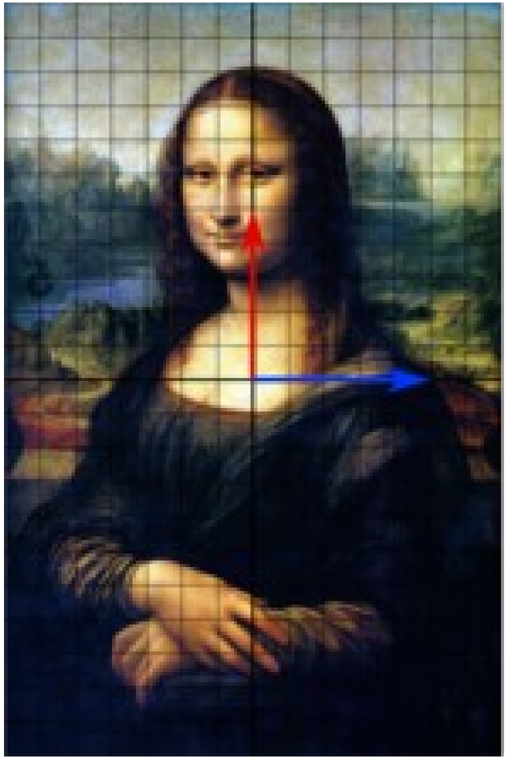
A rider in a “barrel of fun” finds herself stuck with her back to the wall. Which diagram co rectly shows the forces acting on her?

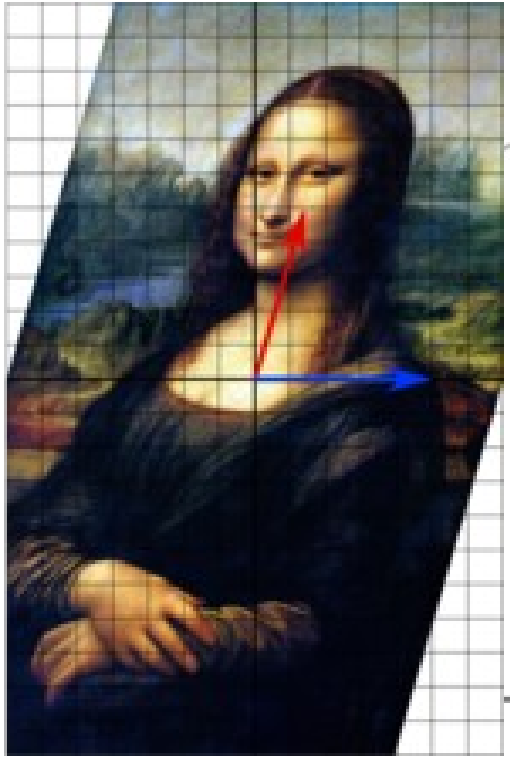


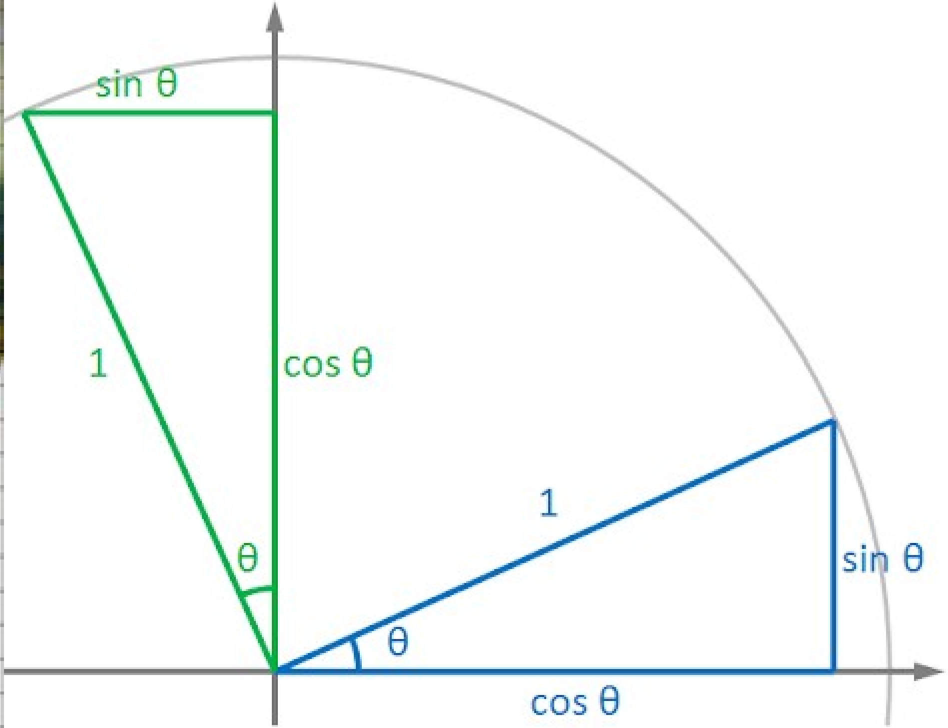


Consider two people on opposite sides of a rotating merry-go-round. One of them throws a ball toward the other. In which frame of reference is the path of the ball straight when viewed from above: (a) the frame of the merry-go-round or (b) that of Earth?

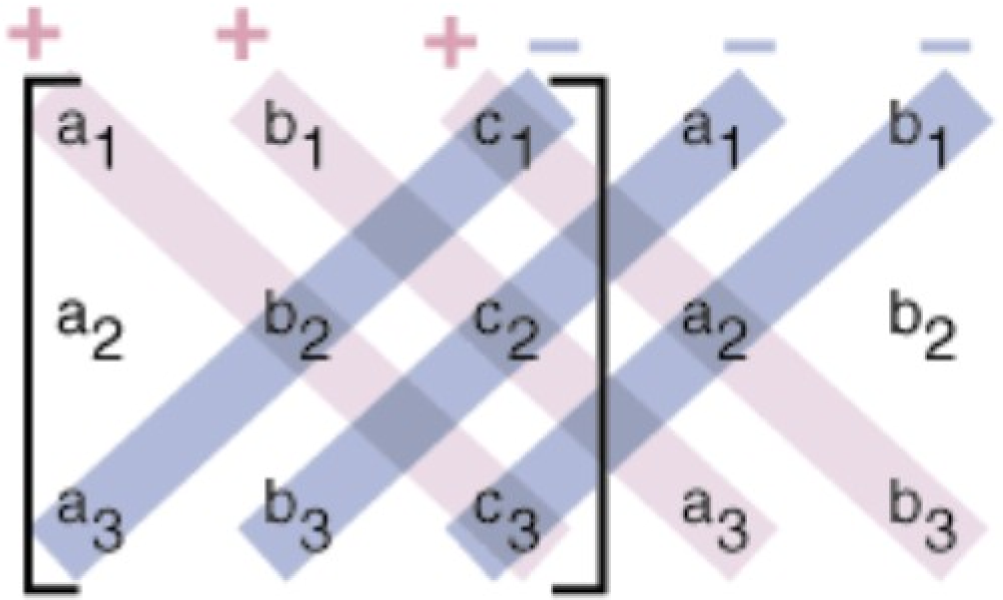
1. (a) only
2. (a) and (b)— although the paths appear to curve
3. (b) only
4. neither; because it’s thrown while in circular motion, the ball travels along a curved path.





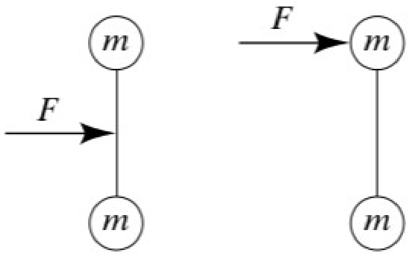

sin θ
1
cos θ
θ
1
sin θ
θ
cos θ

$$\det \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$



$$\det A = (a_1 b_2 c_3 + b_1 c_2 a_3 + c_1 a_2 b_3) - (a_3 b_2 c_1 + b_3 c_2 a_1 + c_3 a_2 b_1)$$

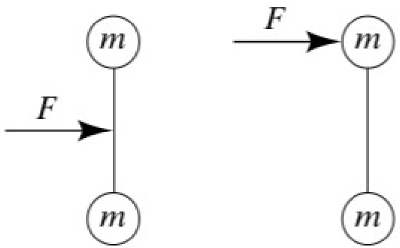
A force F is applied to a dumbbell for a time interval Δt, first as in (a) and then as in (b). In which case does the dumbbell acquire the greater center-of-mass speed?



(a) (b)

1. (a)
2. (b)
3. no difference
4. The answer depends on the rotational inertia of the dumbbell.

A force F is applied to a dumbbell for a time interval Δt, first as in (a) and then as in (b). In which case does the dumbbell acquire the greater energy?



(a) (b)

1. (a)
2. (b)
3. no difference
4. The answer depends on the rotational inertia of the dumbbell.