Phys 3344: Tuesday 20 Oct

Office Hours: Wed 5:00-6:00

Exam 2: graded; solutions recorded and on website

Grades: make up homework promptly

Ch 10

Homework #9:

	2020 FALL PHYS 3344				
#	DAY	LECTURE:	NOTES:	Chpt	TOPIC
1	TUE	08/25/20	First Class	1	Newtons Laws
2	THUR	08/27/20		2	Projectiles
3	TUE	09/01/20		3	Momentum & Angular Momentum
4	THUR	09/03/20		4	Energy
5	TUE	09/08/20		5	Oscillations
6	THUR	09/10/20			
7	TUE	09/15/20			
8	THUR	09/17/20			**EXAM 1**
9	TUE	09/22/20		6	Calculus of Variations
10	THUR	09/24/20		7	Lagrange's Equation
11	TUE	09/29/20			
12	THUR	10/01/20		8	Two Body Problems
13	TUE	10/06/20			
14	THUR	10/08/20		9	Non-Inertial Frames
	TUE	10/13/20	~~Fall Break~~	10	Rotational Motion
15	THUR	10/15/20			**EXAM 2**
16	TUE	10/20/20		10	Rotational Motion
17	THUR	10/22/20			
18	TUE	10/27/20		11	Coupled Oscillations
19	THUR	10/29/20			
20	TUE	11/03/20		13	Hamiltonian Mechanics
21	THUR	11/05/20	Drop Date		
22	TUE	11/10/20			
23	THUR	11/12/20			**EXAM 3**
24	TUE	11/17/20		14	Collision Theory
25	THUR	11/19/20			
26	TUE	11/24/20		15	Special relativity
27	THUR	11/26/20	**Thanksgiving**		No Class
28	TUE	12/01/20			No Class
29	THUR	12/03/20	**Last Class**		Review
	WED	**Dec 16**	**FINAL EXAM**	**Wednesday Dec. 16,2020, 11:30am – 2:30**	
	Adjustments may be made depending on student interests/needs and unplanned events				

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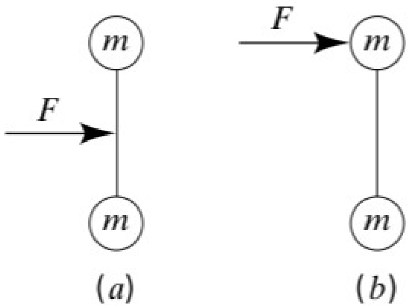
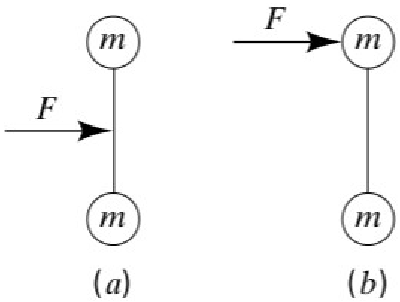
Chapter 10 Rotations

A force F is applied to a dumbbell for a time interval □ t, first as in (a) and then as in (b). In which case does the dumbbell acquire the greater center-of-mass speed?



1. (a)
2. (b)
3. no difference
4. The answer depends on the rotational inertia of the dumbbell.

A force F is applied to a dumbbell for a time interval □ t, first as in (a) and then as in (b). In which case does the dumbbell acquire the greater energy?



1. (a)
2. (b)
3. no difference
4. The answer depends on the rotational inertia of the dumbbell.

Source of the Coriolis effect: https://youtu.be/QfDQeKAyVag

Rotating bodies https://youtu.be/BPMjcN-sBJ4

textbook spinning

How many DOF???

Intermediate axis theorem https://youtu.be/1VPfZ_XzisU

Principal Definitions and Equations of Chapter 10

CM and Relative Motions

$$\mathbf{L} = \mathbf{L}(\text{motion of CM}) + \mathbf{L}\,(\text{motion relative to CM}). \qquad \text{[Eq. (10.9)]}$$

and

$$T = T(\text{motion of CM}) + T(\text{motion relative to CM}). \qquad \text{[Eq. (10.16)]}$$

The Moment of Inertia Tensor

The angular momentum $\mathbf{L}$ and angular velocity $\boldsymbol{\omega}$ of a rigid body are related by

$$\mathbf{L} = \mathbf{I}\boldsymbol{\omega} \qquad \text{[Eq.(10.42)]}$$

where $\mathbf{L}$ and $\boldsymbol{\omega}$ must be seen as 3×1 columns and $\mathbf{I}$ is the 3×3 **moment of inertia tensor**, whose diagonal and off-diagonal elements are defined as

$$I_{xx} = \sum_\alpha m_\alpha (y_\alpha^2 + z_\alpha^2), \text{ etc.} \quad \text{and} \quad I_{xy} = -\sum_\alpha m_\alpha x_\alpha y_\alpha, \text{ etc.}$$

Principal Axes

A **principal axis** of a body (about a point O) is any axis through O with the property that if $\boldsymbol{\omega}$ points along the axis, then $\mathbf{L}$ is parallel to $\boldsymbol{\omega}$; that is,

$$\mathbf{L} = \lambda\boldsymbol{\omega} \qquad \text{[Eq. (10.65)]}$$

angular momentum (L)

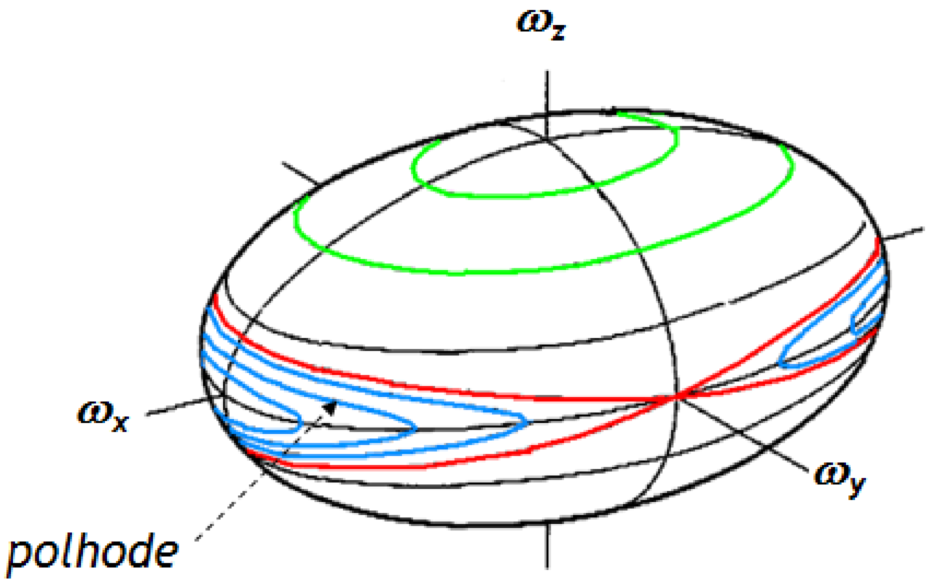
$$L^2 = I_x^2\omega_x^2 + I_y^2\omega_y^2 + I_z^2\omega_z^2$$

$$\frac{\omega_x^2}{(L/I_x)^2} + \frac{\omega_y^2}{(L/I_y)^2} + \frac{\omega_z^2}{(L/I_z)^2} = 1$$

rotational kinetic energy (T_{rot})

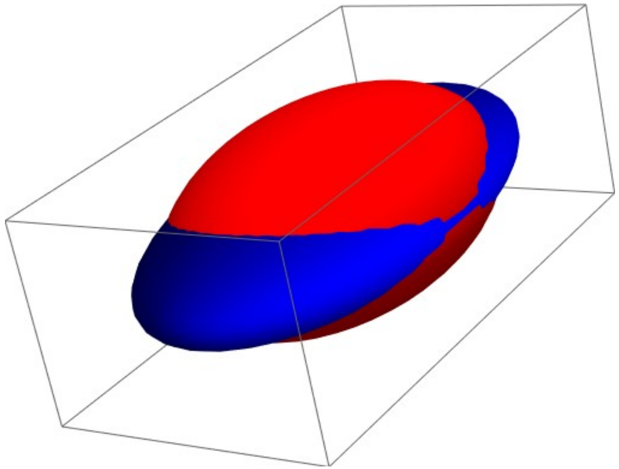
$$T_{rot} = \frac{1}{2}(I_x\,\omega_x^2 + I_y\,\omega_y^2 + I_z\,\omega_z^2)$$

$$\frac{\omega_x^2}{(\sqrt{2T/I_x})^2} + \frac{\omega_y^2}{(\sqrt{2T/I_y})^2} + \frac{\omega_z^2}{(\sqrt{2T/I_z})^2} = 1$$



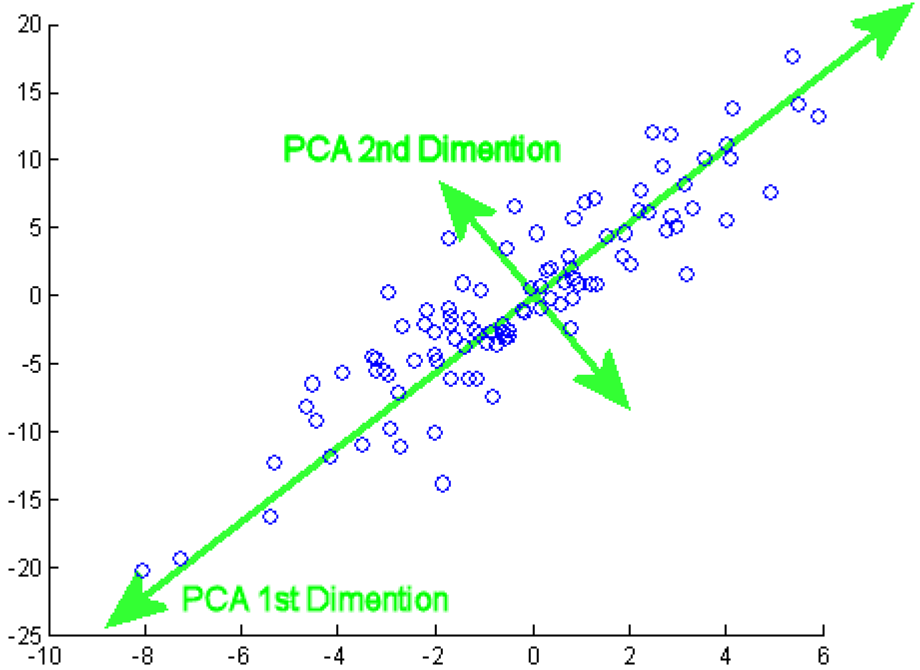
ω_x, ω_y, ω_z : constant

disturbance :



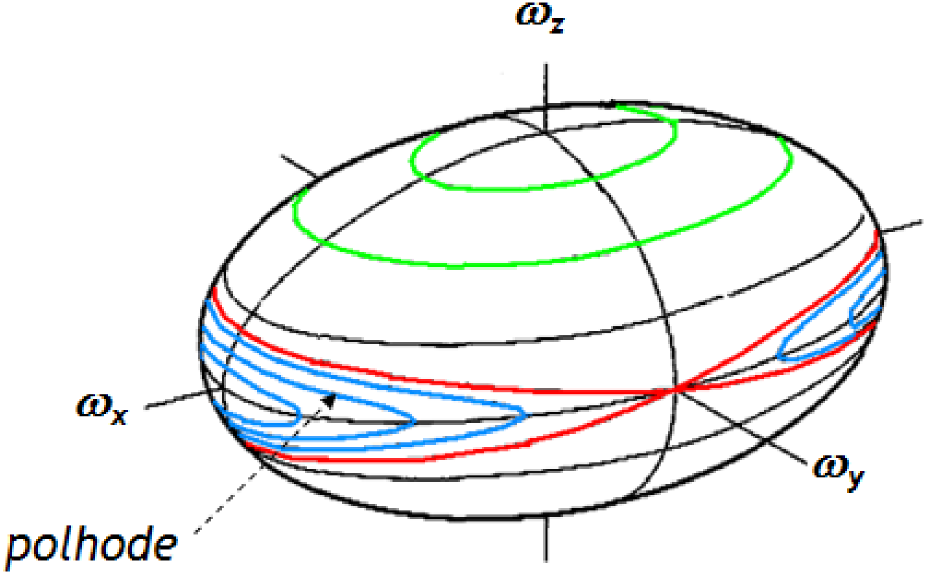
$I_z > I_y > I_x$

intermediate moment of inertia

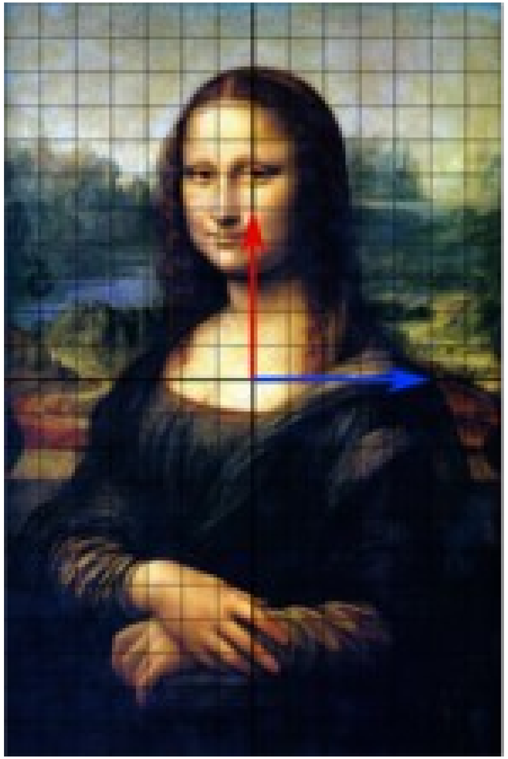


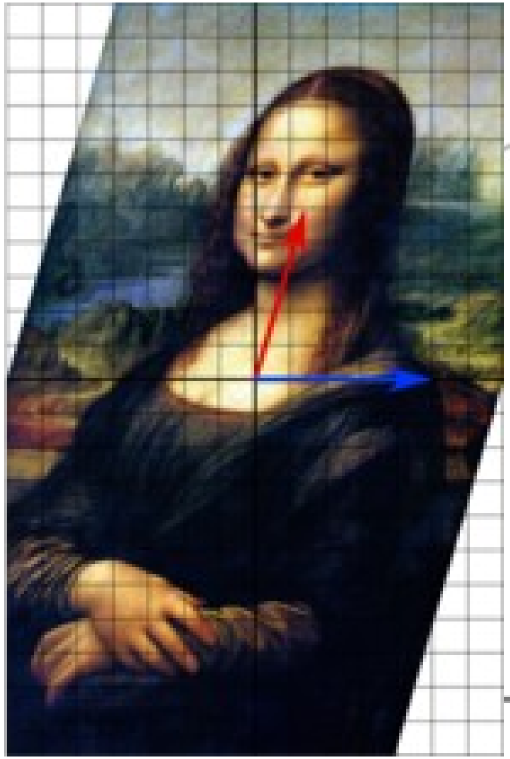
$$I_{xy} = -\sum_{\alpha} m_{\alpha} x_{\alpha} y_{\alpha},$$

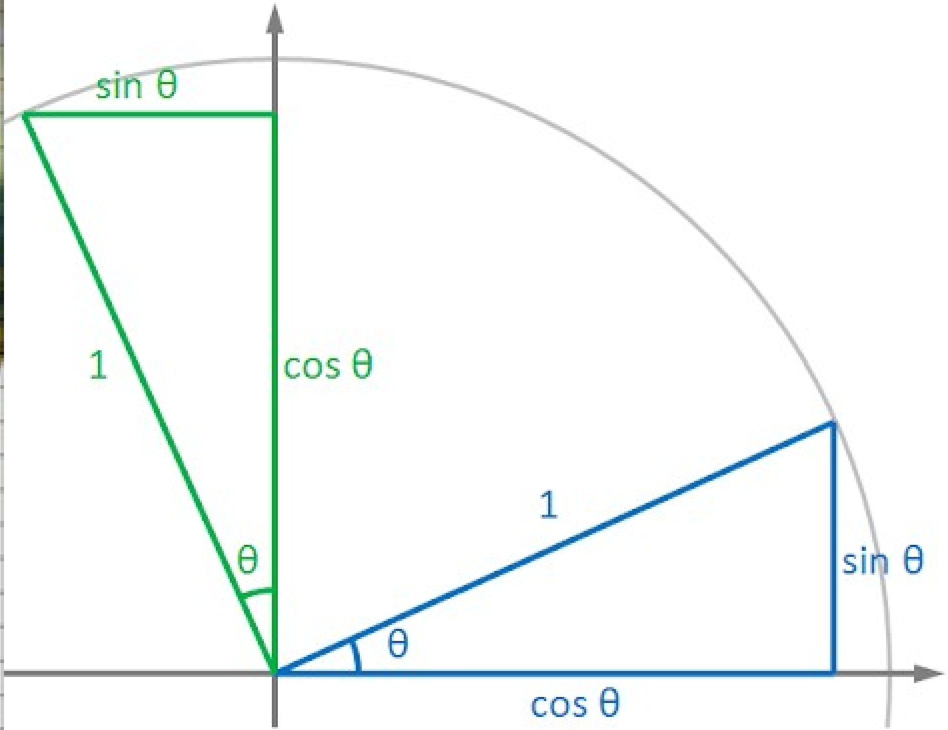
$$I_{xx} = \sum_{\alpha} m_{\alpha}(y_{\alpha}^{2} + z_{\alpha}^{2}), \text{ etc.}$$



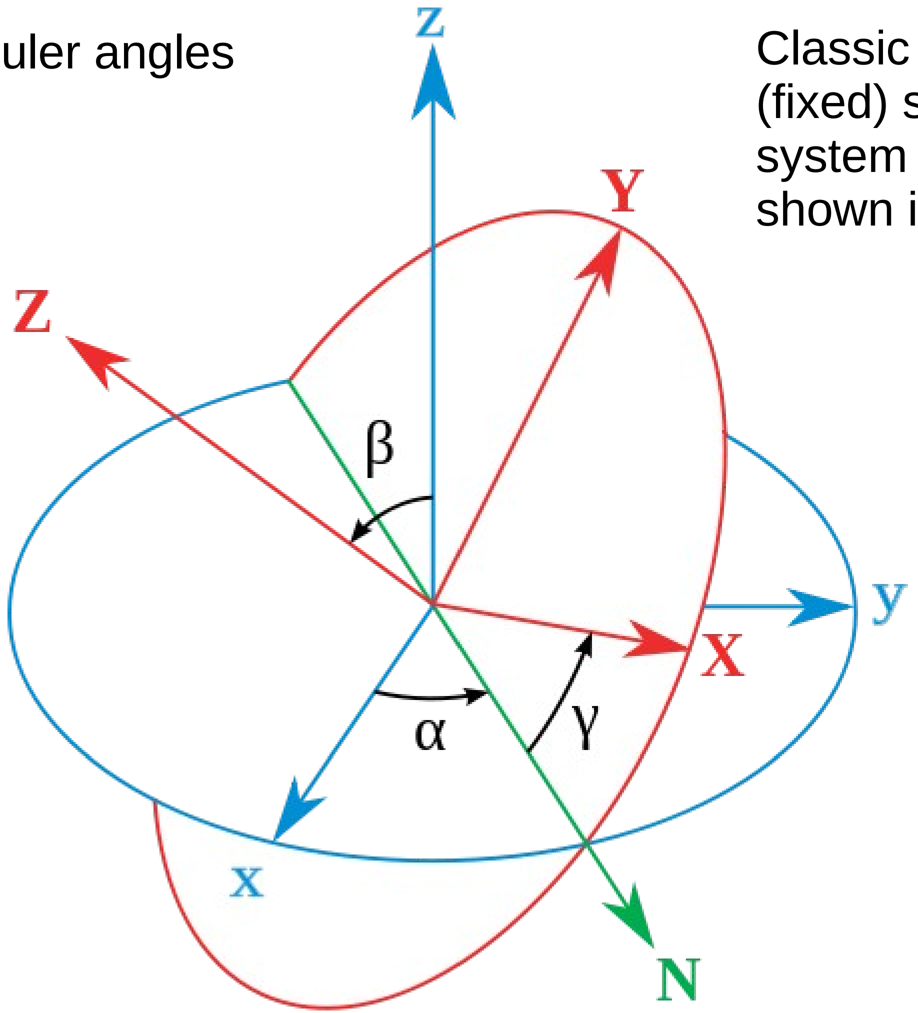
Eigen vectors and rotations







Euler angles



Classic Euler angles geometrical definition. The xyz (fixed) system is shown in blue, the XYZ (rotated) system is shown in red. The line of nodes (N) is shown in green

Proper Euler angles	Tait–Bryan angles
$X_1 Z_2 X_3 = \begin{bmatrix} c_2 & -c_3 s_2 & s_2 s_3 \\ c_1 s_2 & c_1 c_2 c_3 - s_1 s_3 & -c_3 s_1 - c_1 c_2 s_3 \\ s_1 s_2 & c_1 s_3 + c_2 c_3 s_1 & c_1 c_3 - c_2 s_1 s_3 \end{bmatrix}$	$X_1 Z_2 Y_3 = \begin{bmatrix} c_2 c_3 & -s_2 & c_2 s_3 \\ s_1 s_3 + c_1 c_3 s_2 & c_1 c_2 & c_1 s_2 s_3 - c_3 s_1 \\ c_3 s_1 s_2 - c_1 s_3 & c_2 s_1 & c_1 c_3 + s_1 s_2 s_3 \end{bmatrix}$
$X_1 Y_2 X_3 = \begin{bmatrix} c_2 & s_2 s_3 & c_3 s_2 \\ s_1 s_2 & c_1 c_3 - c_2 s_1 s_3 & -c_1 s_3 - c_2 c_3 s_1 \\ -c_1 s_2 & c_3 s_1 + c_1 c_2 s_3 & c_1 c_2 c_3 - s_1 s_3 \end{bmatrix}$	$X_1 Y_2 Z_3 = \begin{bmatrix} c_2 c_3 & -c_2 s_3 & s_2 \\ c_1 s_3 + c_3 s_1 s_2 & c_1 c_3 - s_1 s_2 s_3 & -c_2 s_1 \\ s_1 s_3 - c_1 c_3 s_2 & c_3 s_1 + c_1 s_2 s_3 & c_1 c_2 \end{bmatrix}$
$Y_1 X_2 Y_3 = \begin{bmatrix} c_1 c_3 - c_2 s_1 s_3 & s_1 s_2 & c_1 s_3 + c_2 c_3 s_1 \\ s_2 s_3 & c_2 & -c_3 s_2 \\ -c_3 s_1 - c_1 c_2 s_3 & c_1 s_2 & c_1 c_2 c_3 - s_1 s_3 \end{bmatrix}$	$Y_1 X_2 Z_3 = \begin{bmatrix} c_1 c_3 + s_1 s_2 s_3 & c_3 s_1 s_2 - c_1 s_3 & c_2 s_1 \\ c_2 s_3 & c_2 c_3 & -s_2 \\ c_1 s_2 s_3 - c_3 s_1 & c_1 c_3 s_2 + s_1 s_3 & c_1 c_2 \end{bmatrix}$
$Y_1 Z_2 Y_3 = \begin{bmatrix} c_1 c_2 c_3 - s_1 s_3 & -c_1 s_2 & c_3 s_1 + c_1 c_2 s_3 \\ c_3 s_2 & c_2 & s_2 s_3 \\ -c_1 s_3 - c_2 c_3 s_1 & s_1 s_2 & c_1 c_3 - c_2 s_1 s_3 \end{bmatrix}$	$Y_1 Z_2 X_3 = \begin{bmatrix} c_1 c_2 & s_1 s_3 - c_1 c_3 s_2 & c_3 s_1 + c_1 s_2 s_3 \\ s_2 & c_2 c_3 & -c_2 s_3 \\ -c_2 s_1 & c_1 s_3 + c_3 s_1 s_2 & c_1 c_3 - s_1 s_2 s_3 \end{bmatrix}$
$Z_1 Y_2 Z_3 = \begin{bmatrix} c_1 c_2 c_3 - s_1 s_3 & -c_3 s_1 - c_1 c_2 s_3 & c_1 s_2 \\ c_1 s_3 + c_2 c_3 s_1 & c_1 c_3 - c_2 s_1 s_3 & s_1 s_2 \\ -c_3 s_2 & s_2 s_3 & c_2 \end{bmatrix}$	$Z_1 Y_2 X_3 = \begin{bmatrix} c_1 c_2 & c_1 s_2 s_3 - c_3 s_1 & s_1 s_3 + c_1 c_3 s_2 \\ c_2 s_1 & c_1 c_3 + s_1 s_2 s_3 & c_3 s_1 s_2 - c_1 s_3 \\ -s_2 & c_2 s_3 & c_2 c_3 \end{bmatrix}$
$Z_1 X_2 Z_3 = \begin{bmatrix} c_1 c_3 - c_2 s_1 s_3 & -c_1 s_3 - c_2 c_3 s_1 & s_1 s_2 \\ c_3 s_1 + c_1 c_2 s_3 & c_1 c_2 c_3 - s_1 s_3 & -c_1 s_2 \\ s_2 s_3 & c_3 s_2 & c_2 \end{bmatrix}$	$Z_1 X_2 Y_3 = \begin{bmatrix} c_1 c_3 - s_1 s_2 s_3 & -c_2 s_1 & c_1 s_3 + c_3 s_1 s_2 \\ c_3 s_1 + c_1 s_2 s_3 & c_1 c_2 & s_1 s_3 - c_1 c_3 s_2 \\ -c_2 s_3 & s_2 & c_2 c_3 \end{bmatrix}$