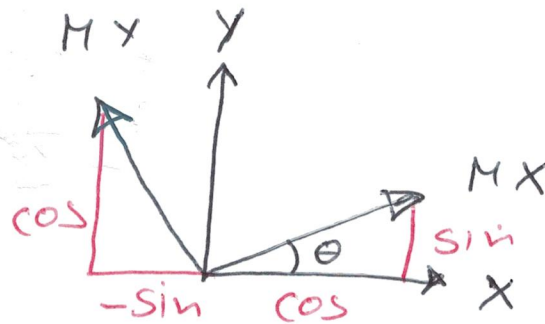


# Rotations 2D

$$M = \begin{pmatrix} \cos\theta & -\sin\theta \\ +\sin\theta & \cos\theta \end{pmatrix} \equiv \begin{pmatrix} c & -s \\ +s & c \end{pmatrix} \quad \begin{matrix} x = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ y = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \end{matrix}$$

$$Mx = \begin{pmatrix} c \\ +s \end{pmatrix}$$

$$My = \begin{pmatrix} -s \\ c \end{pmatrix}$$



Eigenvalues =  $e^{\pm i\theta}$  Not real!

$$\text{Tr}(M) = 2 \cos\theta$$

$$\text{Det } M = |M| = 1$$

$$A = \begin{pmatrix} 1/2 & -\sqrt{3}/2 \\ \sqrt{3}/2 & 1/2 \end{pmatrix}$$

$$\text{Tr } A = 1 \equiv 2 \cos\theta$$

$$\cos\theta = 1/2 \quad \theta = \frac{\pi}{3} = 60^\circ$$

$$\lambda = e^{\pm i\theta} \quad \theta = \pi/3$$

$$B = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix} \quad Bx = x \quad By = 2y$$

$$\text{Det } B = |B| = 2 \neq 1$$

## Rotations 3D

$$M = \begin{pmatrix} c & -s & 0 \\ s & c & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \text{Eigenvalues} = e^{\pm i\theta}, 1$$

$$\lambda = 1 \Rightarrow M \mathbf{v} = \lambda \mathbf{v} = \mathbf{v}$$

Find rotation Angle:

$$\text{Tr } M = 1 + 2\cos\theta \quad \text{Det } M = 1$$

$$A = \begin{pmatrix} 1/2 & -\sqrt{3}/2 & 0 \\ \sqrt{3}/2 & 1/2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\text{Tr } A = 2 = 1 + 2\cos\theta$$
$$\cos\theta = 1/2 \quad \theta = 60^\circ$$