Phys 3344: Thursday 05 November

Office Hours: Wed 5:00-6:00

Grades: scaled

make up homework promptly

else it does not benefit you

Homework #10: re-do by Friday

Next Hw11: Due next Wed

Ch 13 phase plots

Ch 14

a tour of cern video

	2020 FALL PHYS 3344				
#	DAY	LECTURE:	NOTES:	Chpt	TOPIC
1	TUE	08/25/20	First Class	1	Newtons Laws
2	THUR	08/27/20		2	Projectiles
3	TUE	09/01/20		3	Momentum & Angular Momentum
4	THUR	09/03/20		4	Energy
5	TUE	09/08/20		5	Oscillations
6	THUR	09/10/20			
7	TUE	09/15/20			
8	THUR	09/17/20			**EXAM 1**
9	TUE	09/22/20		6	Calculus of Variations
10	THUR	09/24/20		7	Lagrange's Equation
11	TUE	09/29/20			
12	THUR	10/01/20		8	Two Body Problems
13	TUE	10/06/20			
14	THUR	10/08/20		9	Non-Inertial Frames
	TUE	10/13/20	~~Fall Break~~	10	Rotational Motion
15	THUR	10/15/20			**EXAM 2**
16	TUE	10/20/20		10	Rotational Motion
17	THUR	10/22/20			
18	TUE	10/27/20		11	Coupled Oscillations
19	THUR	10/29/20			
20	TUE	11/03/20		13	Hamiltonian Mechanics
21	THUR	11/05/20	Drop Date		
22	TUE	11/10/20			
23	THUR	11/12/20			**EXAM 3**
24	TUE	11/17/20		14	Collision Theory
25	THUR	11/19/20			
26	TUE	11/24/20		15	Special relativity
27	THUR	11/26/20	**Thanksgiving**		No Class
28	TUE	12/01/20			No Class
29	THUR	12/03/20	**Last Class**		Review
	WED	**Dec 16**	**FINAL EXAM**	**Wednesday Dec. 16,2020, 11:30am – 2:30**	
	Adjustments may be made depending on student interests/needs and unplanned events				

Chapter 13 Hamiltonian Mechanics

Principal Definitions and Equations of Chapter 13

The Hamiltonian

If a system has generalized coordinates $\mathbf{q} = (q_1, \cdots, q_n)$, Lagrangian $\mathcal{L}$, and generalized momenta $p_i = \partial\mathcal{L}/\partial q_i$, its **Hamiltonian** is defined as

$$\mathcal{H} = \sum_{i=1}^{n} p_i \dot{q}_i - \mathcal{L}, \qquad \text{[Eq. (13.22)]}$$

always considered as a function of the variables $\mathbf{q}$ and $\mathbf{p}$ (and possibly t).

Hamilton's Equations

The time evolution of a system is given by Hamilton's equations

$$\dot{q}_i = \frac{\partial\mathcal{H}}{\partial p_i} \qquad \text{and} \qquad \dot{p}_i = -\frac{\partial\mathcal{H}}{\partial q_i} \qquad [i = 1, \cdots, n]. \quad \text{[Eq. (13.25)]}$$

13.3 ★ Consider the Atwood machine of Figure 13.2, but suppose that the pulley is a uniform disc of mass M and radius R. Using x as your generalized coordinate, write down the Lagrangian, the generalized momentum p, and the Hamiltonian $\mathcal{H} = p\dot{x} - \mathcal{L}$. Find Hamilton's equations and use them to find the acceleration $\ddot{x}$.

13.4 ★ The Hamiltonian $\mathcal{H}$ is always given by $\mathcal{H} = pq - \mathcal{L}$ (in one dimension), and this is the form you should use if in doubt. However, if your generalized coordinate q is "natural" (relation between q and the underlying Cartesian coordinates is independent of time) then $\mathcal{H} = T + U$, and this form is almost always easier to write down. Therefore, in solving any problem you should quickly check to see if the generalized coordinate is "natural," and if it is you can use the simpler form $\mathcal{H} = T + U$. For the Atwood machine of Example 13.2 (page 527), check that the generalized coordinate was "natural." [*Hint:* There are one generalized coordinate x and two underlying Cartesian coordinates x and y. You have only to write equations for the two Cartesians in terms of the one generalized coordinate and check that they don't involve the time, so it's safe to use $\mathcal{H} = T + U$. This is ridiculously easy!]

Chapter 14 Collision Theory

EXAMPLE 14.1 Shooting Crows in an Oak Tree

A hunter observes 50 crows settling randomly in an oak tree, where he can no longer see them. Each crow has a cross-sectional area $\sigma \approx \frac{1}{2}$ ft^2, and the oak has a total area (as seen from the hunter's position) of 150 square feet. If the hunter fires 60 bullets at random into the tree, about how many crows would he expect to hit?

This situation closely parallels our simple scattering experiment. The target density is $n_{\text{tar}} =$ (number of crows)/(area of tree) $= 50/150 = 1/3$ ft^{-2}. The number of incident projectiles is $N_{\text{inc}} = 60$, so, by the analog of (14.2), the expected number of hits is

$$N_{\text{hit}} = N_{\text{inc}}\, n_{\text{tar}}\, \sigma = 60 \times \left(\tfrac{1}{3}\ \text{ft}^{-2}\right) \times \left(\tfrac{1}{2}\ \text{ft}^{2}\right) = 10.$$

EXAMPLE 14.2 Scattering of Neutrons in an Aluminum Foil

If 10,000 neutrons are fired through an aluminum foil 0.1 mm thick and the cross section of the aluminum nucleus is about 1.5 barns,[4] how many neutrons will be scattered? (Specific gravity of aluminum = 2.7.)

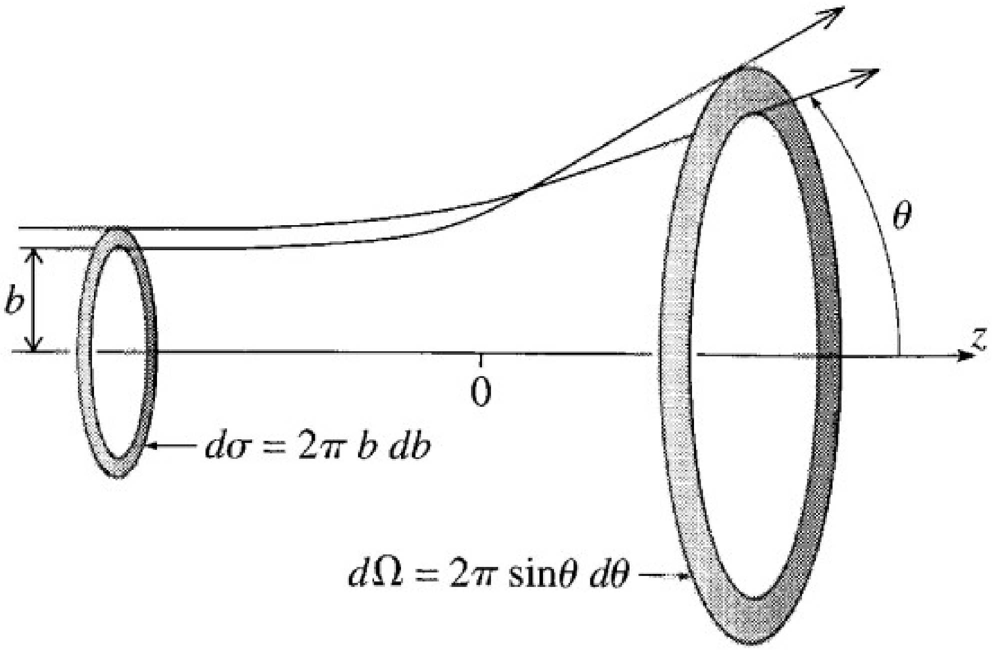
The number of scatterings is given by (14.2), and we already know that $N_{\text{inc}} = 10^4$ and $\sigma = 1.5 \times 10^{-28}$ m^2. Thus all we need to find is the target density n_{tar}, the number of aluminum nuclei per area of the foil. (Of course, the foil contains lots of atomic electrons as well, but these do not contribute appreciably to the scattering of neutrons.) The density of aluminum (mass/volume) is $\varrho = 2.7 \times 10^3$ kg/m^3. If we multiply this by the thickness of the foil ($t = 10^{-4}$ m), this will give the mass per area of the foil, and dividing this by the mass of an aluminum nucleus ($m = 27$ atomic mass units), we will have n_{tar}:

$$n_{\text{tar}} = \frac{\varrho t}{m} = \frac{(2.7 \times 10^3\,\text{kg/m}^3) \times (10^{-4}\,\text{m})}{27 \times 1.66 \times 10^{-27}\,\text{kg}} = 6.0 \times 10^{24}\,\text{m}^{-2}. \quad (14.3)$$

Substituting into (14.2) we find for the number of scatterings

$$N_{\text{sc}} = N_{\text{inc}}\, n_{\text{tar}}\, \sigma = (10^4) \times (6.0 \times 10^{24}\,\text{m}^{-2}) \times (1.5 \times 10^{-28}\,\text{m}^2) = 9.$$

Here, we used the given cross section σ to predict the number N_{sc} of scatterings we should observe. Alternatively, we could have used the observed value of N_{sc} to *find* the cross section σ.

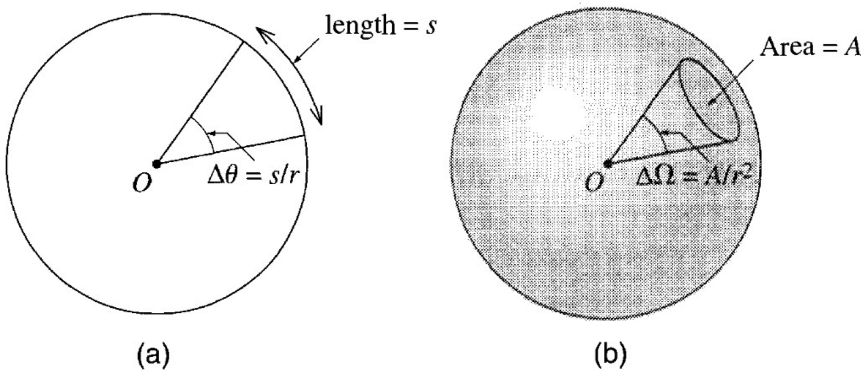


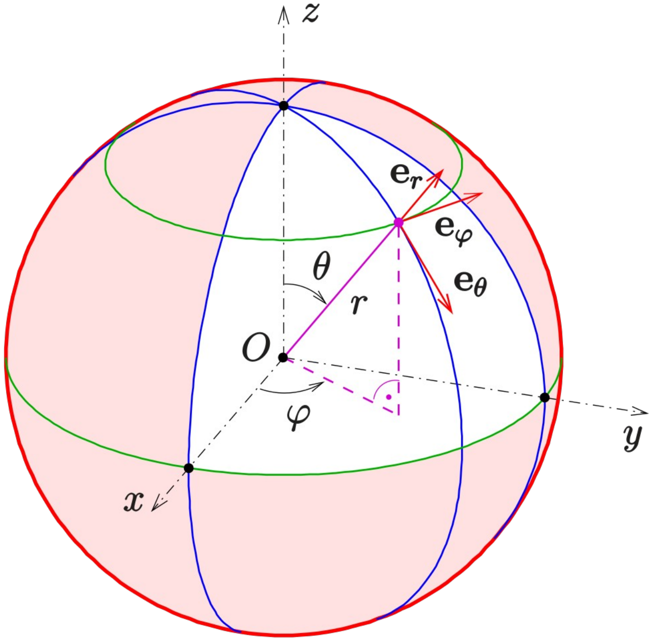
$$\frac{d\sigma}{d\Omega} = \frac{b}{\sin\theta}\left|\frac{db}{d\theta}\right| \qquad (14.23)$$

Figure 14.9 All projectiles incident between b and $b + db$ are scattered between angles θ and $\theta + d\theta$. The area on which these particles impinge is $d\sigma = 2\pi b\, db$, and the solid angle into which they scatter is $d\Omega = 2\pi \sin\theta\, d\theta$.

$$\sigma = \int \frac{d\sigma}{d\Omega}(\theta, \phi)\, d\Omega = \int_0^{\pi} \sin\theta\, d\theta \int_0^{2\pi} d\phi\, \frac{d\sigma}{d\Omega}(\theta, \phi) \qquad (14.18)$$

$$\Delta\Omega = A/r^2. \quad (14.14)$$





$$d\Omega = \sin\theta \, d\theta \, d\phi. \quad (14.15)$$

$$\int \Omega = \int_0^{\pi} \sin\theta \, d\theta \int_0^{2\pi} d\phi = 2 \times 2\pi = 4\pi$$

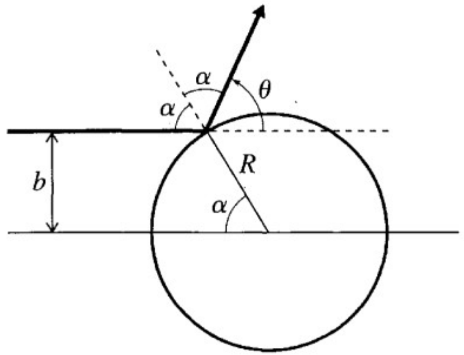


Figure 14.10 A point projectile bouncing off a fixed rigid sphere obeys the law of reflection, that the two adjacent angles labelled α are equal. The impact parameter is $b = R\sin\alpha$, and the scattering angle is $\theta = \pi - 2\alpha$.

EXAMPLE 14.5 Hard Sphere Scattering

As a first example of the use of (14.23), find the differential cross section for scattering of a point projectile off a fixed rigid sphere of radius R. Integrate your result over all solid angles to find the total cross section.

Our first task is to find the trajectory of a scattered projectile, as shown in Figure 14.10. The crucial observation is that when the projectile bounces off the hard sphere, its angles of incidence and reflection (both shown as α in the picture) are equal. (This "law of reflection" follows from conservation of energy and angular momentum — see Problem 14.13.) Inspection of the picture shows that the impact parameter is $b = R\sin\alpha$, and the scattering angle is $\theta = \pi - 2\alpha$. Combining these two equations we find that

$$b = R\sin\frac{\pi - \theta}{2} = R\cos(\theta/2), \tag{14.24}$$

and from (14.23), we find the differential cross section

$$\frac{d\sigma}{d\Omega} = \frac{b}{\sin\theta}\left|\frac{db}{d\theta}\right| = \frac{R\cos(\theta/2)}{\sin\theta}\,\frac{R\sin(\theta/2)}{2} = \frac{R^2}{4}. \tag{14.25}$$

The most striking thing about this result is that the differential cross section is isotropic; that is, the number of particles scattered into a solid angle $d\Omega$ is the same in all directions. To find the total cross section, we have only to integrate this result over all solid angles:

$$\sigma = \int \frac{d\sigma}{d\Omega}d\Omega = \int \frac{R^2}{4}d\Omega = \pi R^2,$$

which is, of course, the cross-sectional area of the target sphere.