Phys 3344: Thursday 12 November

Office Hours: Wed 5:00-6:00

Grades: scaled: roughly A>90, A- >85,B+ >80, B>75, B- >70

make up homework promptly

else it does not benefit you

*Hw 10 and earlier: no re-do after **Sunday***

Final Hw12: Due Wed 02 December

ON EXAM: **degenerate eigenvalues**

Ch 10: Rotational Motion

Ch 11 Coupled Systems

Ch 13 Hamiltonian:

Ch 14 Collisions:

2020 FALL PHYS 3344

#	DAY	LECTURE:	NOTES:	Chpt	TOPIC
1	TUE	08/25/20	First Class	1	Newtons Laws
2	THUR	08/27/20		2	Projectiles
3	TUE	09/01/20		3	Momentum & Angular Momentum
4	THUR	09/03/20		4	Energy
5	TUE	09/08/20		5	Oscillations
6	THUR	09/10/20			
7	TUE	09/15/20			
8	THUR	09/17/20			**EXAM 1**
9	TUE	09/22/20		6	Calculus of Variations
10	THUR	09/24/20		7	Lagrange's Equation
11	TUE	09/29/20			
12	THUR	10/01/20		8	Two Body Problems
13	TUE	10/06/20			
14	THUR	10/08/20		9	Non-Inertial Frames
	TUE	10/13/20	~~**Fall Break**~~	10	Rotational Motion
15	THUR	10/15/20			**EXAM 2**
16	TUE	10/20/20		10	Rotational Motion
17	THUR	10/22/20		11	Coupled Oscillations
18	TUE	10/27/20			
19	THUR	10/29/20		13	Hamiltonian Mechanics
20	TUE	11/03/20			
21	THUR	11/05/20	Drop Date	14	Collision Theory
22	TUE	11/10/20			
23	THUR	11/12/20			**EXAM 3**
24	TUE	11/17/20		15	Special relativity
25	THUR	11/19/20			
26	TUE	11/24/20			
27	THUR	11/26/20	**Thanksgiving**		No Class
28	TUE	12/01/20			No Class
29	THUR	12/03/20	**Last Class**		Review
	WED	**Dec 16**	**FINAL EXAM**		**Wednesday Dec. 16,2020, 11:30am - 2:30**

Exam 3 rules:

Open notes, open book, closed neighbor, closed internet. You may use Mathematica.
Exam is due before Thanksgiving.
Total points: 100. Each problem is 20 points.

PROBLEM 1:

1a) A spider is hanging by a silk thread from a tree in Dallas. Find the orientation and the value of the equilibrium angle that the thread makes with the vertical (i.e. with the direction of gravity), taking into account the rotation of the Earth. Assume that the latitude of Dallas is $\theta \approx 33$ and the radius of the Earth is $R \approx 6,400$km. [Important; note the spider is stationary. This should simplify the problem. Think.]

1b) In Dallas ($\theta \approx 33$), I shoot an arrow east with velocity v=50m/s. What is the magnitude of the Coriolis force compared to the gravitational force? What is the direction?

PROBLEM 2:

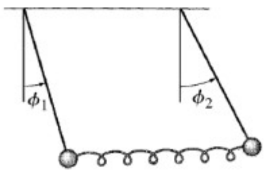
A rigid body consists of 4 point masses:

m at (a,0,0)
2m at (0,a,0)
3m at (0,a,a)
4m at (0,a,-a)

Find a) the moment of inertia tensor, b) the principle moments, and c) the orthogonal principle axes. Note: I suggest you use Mathematica for part of this; but, make sure I can follow your notation!!!

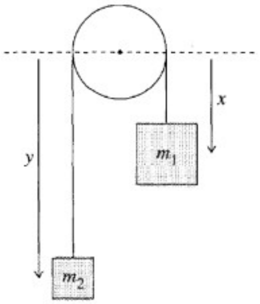
PROBLEM 3:

Consider two identical plane pendulums (each of length L and mass m) that are joined by a massless spring (force constant k) as shown in Figure 11.17. The pendulums' positions are specified by the angles ϕ_1 and ϕ_2 shown. The natural length of the spring is equal to the distance between the two supports, so the equilibrium position is at $\phi_1 = \phi_2 = 0$ with the two pendulums vertical. **(a)** Write down the total kinetic energy and the gravitational and spring potential energies. [Assume that both angles remain small at all times. This means that the extension of the spring is well approximated by $L(\phi_2 - \phi_1)$.] Write down the Lagrange equations of motion. **(b)** Find and describe the normal modes for these two coupled pendulums.



PROBLEM 4:

Solve the Atwood machine shown at right using Hamilton's method. Use two variables {x,y}, and obtain Hamilton's equations for both x and y. Then use the constraint x+y=L to find the acceleration of the system and show this equals the expected result.



PROBLEM 5:

a) (This is modeled from Example 14.2 in the text.) 10^6 neutrons are fired through a gold foil 0.1mm thick. Take the gold nucleus cross section to be 98.7 barns. How many neutrons will be scattered. The specific gravity of gold is 19.32.

b) (This is modeled from Example 14.5 in the text.) Using what you learned in Example 14.5, find the fraction of the scattered neutrons that are scattered within 10 degrees of the source; that is between angles 170 and 190 degrees. [Think]

Chapter 13 Hamiltonian Mechanics

Principal Definitions and Equations of Chapter 13

The Hamiltonian

If a system has generalized coordinates $\mathbf{q} = (q_1, \cdots, q_n)$, Lagrangian $\mathcal{L}$, and generalized momenta $p_i = \partial\mathcal{L}/\partial\dot{q}_i$, its **Hamiltonian** is defined as

$$\mathcal{H} = \sum_{i=1}^{n} p_i\dot{q}_i - \mathcal{L}, \qquad \text{[Eq. (13.22)]}$$

always considered as a function of the variables **q** and **p** (and possibly t).

Hamilton's Equations

The time evolution of a system is given by Hamilton's equations

$$\dot{q}_i = \frac{\partial\mathcal{H}}{\partial p_i} \qquad \text{and} \qquad \dot{p}_i = -\frac{\partial\mathcal{H}}{\partial q_i} \qquad [i = 1, \cdots, n]. \quad \text{[Eq. (13.25)]}$$

13.3★ Consider the Atwood machine of Figure 13.2, but suppose that the pulley is a uniform disc of mass M and radius R. Using x as your generalized coordinate, write down the Lagrangian, the generalized momentum p, and the Hamiltonian $\mathcal{H} = p\dot{x} - \mathcal{L}$. Find Hamilton's equations and use them to find the acceleration $\ddot{x}$.

13.4★ The Hamiltonian $\mathcal{H}$ is always given by $\mathcal{H} = p\dot{q} - \mathcal{L}$ (in one dimension), and this is the form you should use if in doubt. However, if your generalized coordinate q is "natural" (relation between q and the underlying Cartesian coordinates is independent of time) then $\mathcal{H} = T + U$, and this form is almost always easier to write down. Therefore, in solving any problem you should quickly check to see if the generalized coordinate is "natural," and if it is you can use the simpler form $\mathcal{H} = T + U$. For the Atwood machine of Example 13.2 (page 527), check that the generalized coordinate was "natural." [*Hint:* There are one generalized coordinate x and two underlying Cartesian coordinates x and y. You have only to write equations for the two Cartesians in terms of the one generalized coordinate and check that they don't involve the time, so it's safe to use $\mathcal{H} = T + U$. This is ridiculously easy!]

Chapter 14
Collision Theory

EXAMPLE 14.1 Shooting Crows in an Oak Tree

A hunter observes 50 crows settling randomly in an oak tree, where he can no longer see them. Each crow has a cross-sectional area $\sigma \approx \frac{1}{2}\ \text{ft}^2$, and the oak has a total area (as seen from the hunter's position) of 150 square feet. If the hunter fires 60 bullets at random into the tree, about how many crows would he expect to hit?

This situation closely parallels our simple scattering experiment. The target density is n_{tar} = (number of crows)/(area of tree) = 50/150 = 1/3 ft^{-2}. The number of incident projectiles is $N_{\text{inc}} = 60$, so, by the analog of (14.2), the expected number of hits is

$$N_{\text{hit}} = N_{\text{inc}}\, n_{\text{tar}}\, \sigma = 60 \times \left(\tfrac{1}{3}\ \text{ft}^{-2}\right) \times \left(\tfrac{1}{2}\ \text{ft}^{2}\right) = 10.$$

EXAMPLE 14.2 Scattering of Neutrons in an Aluminum Foil

If 10,000 neutrons are fired through an aluminum foil 0.1 mm thick and the cross section of the aluminum nucleus is about 1.5 barns,[4] how many neutrons will be scattered? (Specific gravity of aluminum = 2.7.)

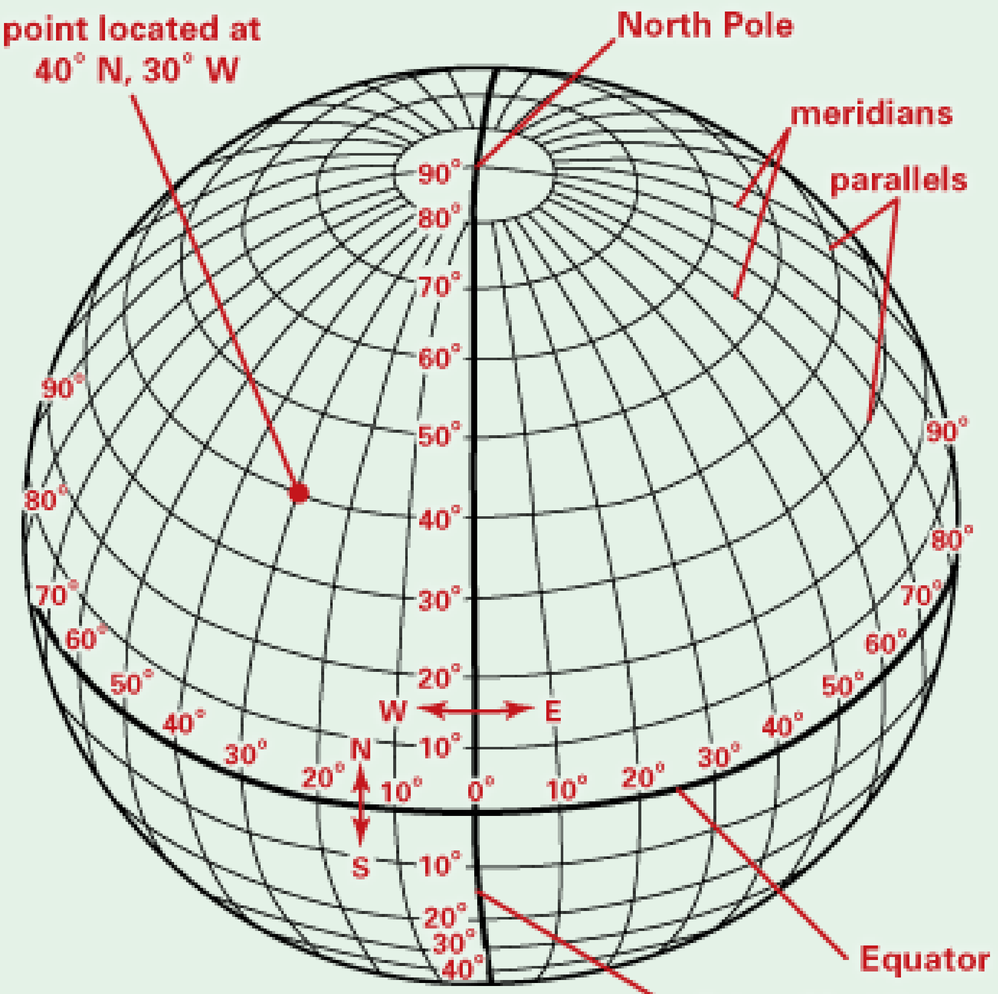
The number of scatterings is given by (14.2), and we already know that $N_{\text{inc}} = 10^4$ and $\sigma = 1.5 \times 10^{-28}$ m^2. Thus all we need to find is the target density n_{tar}, the number of aluminum nuclei per area of the foil. (Of course, the foil contains lots of atomic electrons as well, but these do not contribute appreciably to the scattering of neutrons.) The density of aluminum (mass/volume) is $\varrho = 2.7 \times 10^3\,\text{kg/m}^3$. If we multiply this by the thickness of the foil ($t = 10^{-4}$ m), this will give the mass per area of the foil, and dividing this by the mass of an aluminum nucleus ($m = 27$ atomic mass units), we will have n_{tar}:

$$n_{\text{tar}} = \frac{\varrho t}{m} = \frac{(2.7 \times 10^3\,\text{kg/m}^3) \times (10^{-4}\,\text{m})}{27 \times 1.66 \times 10^{-27}\,\text{kg}} = 6.0 \times 10^{24}\,\text{m}^{-2}. \quad (14.3)$$

Substituting into (14.2) we find for the number of scatterings

$$N_{\text{sc}} = N_{\text{inc}}\, n_{\text{tar}}\, \sigma = (10^4) \times (6.0 \times 10^{24}\,\text{m}^{-2}) \times (1.5 \times 10^{-28}\,\text{m}^2) = 9.$$

Here, we used the given cross section σ to predict the number N_{sc} of scatterings we should observe. Alternatively, we could have used the observed value of N_{sc} to *find* the cross section σ.


point located at
40° N, 30° W
North Pole
meridians
parallels
90°
80°
70°
60°
50°
40°
30°
20°
10°
0°
W
E
N
S
Equator
Prime Meridian

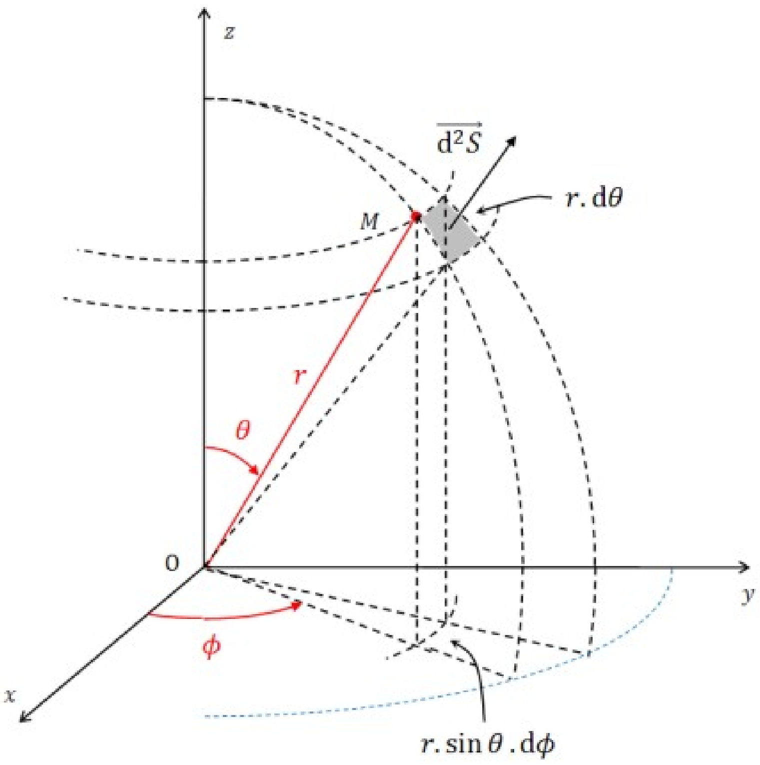


Figure 2: Area element

electron has in the hydrogen atom has the spherical symmetry, because it only depends on r, but not on θ or ϕ.

Given this, let's apply spherical coordinate to obtain the surface area of a sphere with radius r. See Fig. 2. The shaded region is an infinitesimal part of the surface of the sphere. It is a rectangle and the area is given by $(r\sin\theta d\phi)\cdot(rd\theta)$. If we add all these up for different θ and ϕ, we will get the surface area of sphere. Let's do this:

$$A = \int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi}(r\sin\theta d\phi)\cdot(rd\theta) = \int_{\theta=0}^{\pi} r^2\sin\theta d\theta 2\pi = 2\pi r^2(-\cos\pi + \cos 0) = 4\pi r^2 \tag{4}$$

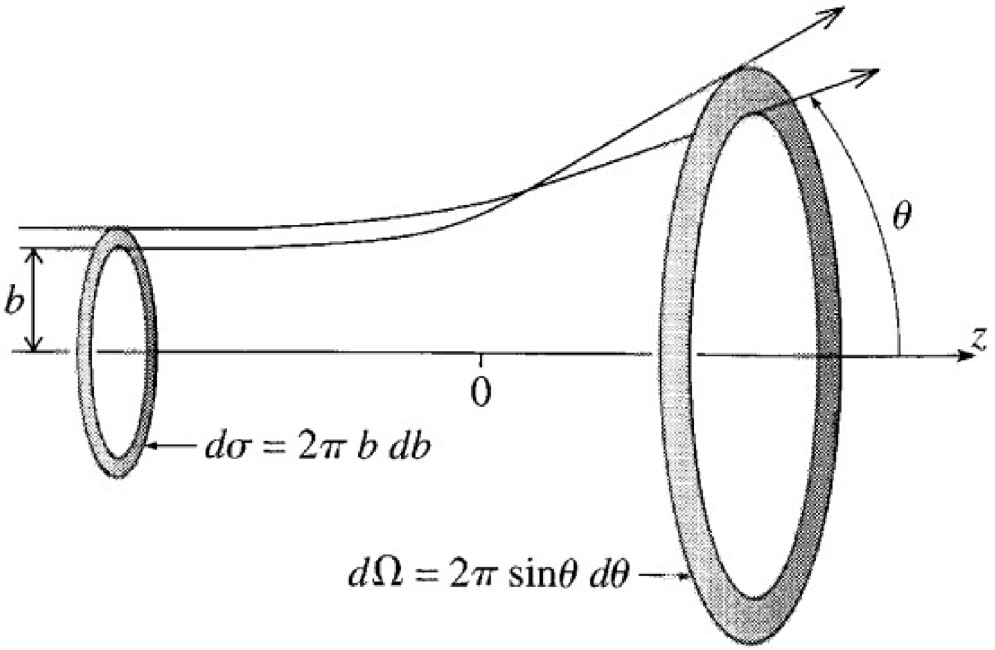
So, this is the surface area of the sphere. Now, what is the enclosed volume of the sphere? The inner region of a sphere is called a "ball." So what is the volume of ball with radius R? See Fig.3. We see that the infinitesimal volume area is given as follows:

$$dV = (r\sin\theta d\phi)\cdot(rd\theta)\cdot dr \tag{5}$$

Integrating, we get:

$$V = \int_{r=0}^{r=R}\int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} dV = \int_{r=0}^{r=R}\int_{\theta=0}^{\pi} dr(r^2\sin\theta d\theta 2\pi) \tag{6}$$

$$= \int_{r=0}^{r=R} 4\pi r^2 dr = \frac{4}{3}\pi R^3 \tag{7}$$

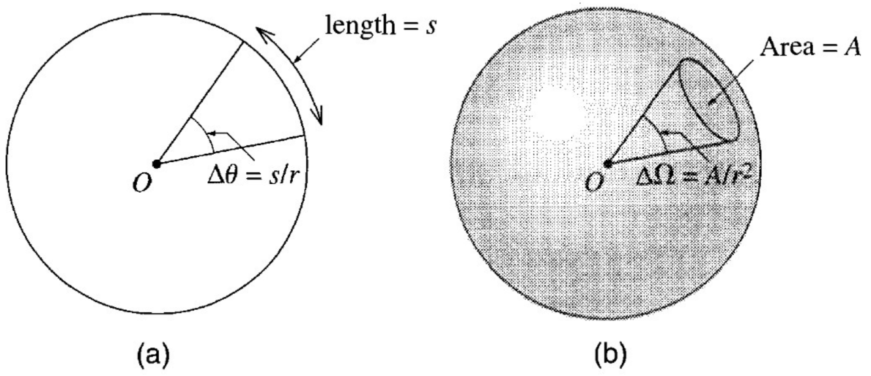


$$\frac{d\sigma}{d\Omega} = \frac{b}{\sin\theta}\left|\frac{db}{d\theta}\right| \qquad (14.23)$$

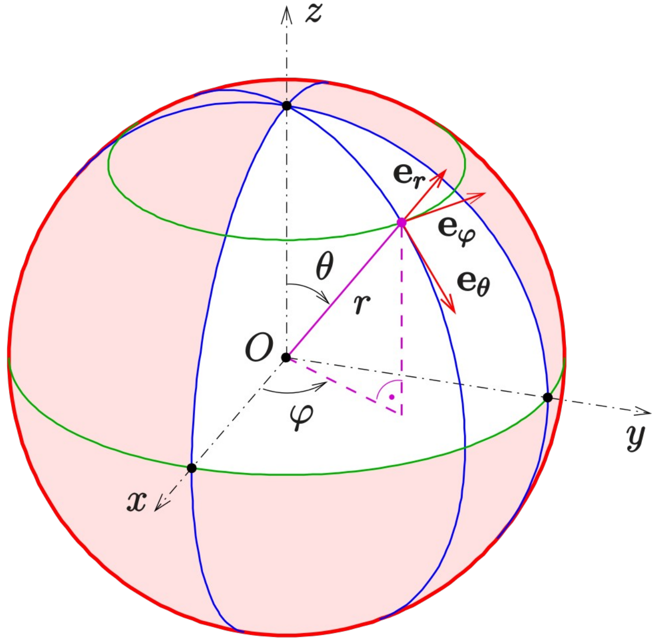
Figure 14.9 All projectiles incident between b and $b + db$ are scattered between angles θ and $\theta + d\theta$. The area on which these particles impinge is $d\sigma = 2\pi b\, db$, and the solid angle into which they scatter is $d\Omega = 2\pi \sin\theta\, d\theta$.

$$\sigma = \int \frac{d\sigma}{d\Omega}(\theta, \phi)\, d\Omega = \int_0^{\pi} \sin\theta\, d\theta \int_0^{2\pi} d\phi\, \frac{d\sigma}{d\Omega}(\theta, \phi) \qquad (14.18)$$

$$\Delta\Omega = A/r^2. \quad (14.14)$$



$$d\Omega = \sin\theta \, d\theta \, d\phi. \quad (14.15)$$



$$\int \Omega = \int_0^{\pi} \sin\theta \, d\theta \int_0^{2\pi} d\phi = 2 \times 2\pi = 4\pi$$

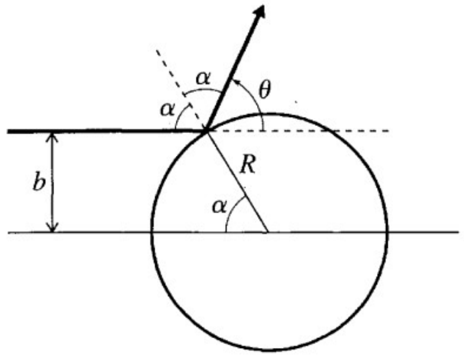


Figure 14.10 A point projectile bouncing off a fixed rigid sphere obeys the law of reflection, that the two adjacent angles labelled α are equal. The impact parameter is $b = R\sin\alpha$, and the scattering angle is $\theta = \pi - 2\alpha$.

EXAMPLE 14.5 Hard Sphere Scattering

As a first example of the use of (14.23), find the differential cross section for scattering of a point projectile off a fixed rigid sphere of radius R. Integrate your result over all solid angles to find the total cross section.

Our first task is to find the trajectory of a scattered projectile, as shown in Figure 14.10. The crucial observation is that when the projectile bounces off the hard sphere, its angles of incidence and reflection (both shown as α in the picture) are equal. (This "law of reflection" follows from conservation of energy and angular momentum — see Problem 14.13.) Inspection of the picture shows that the impact parameter is $b = R\sin\alpha$, and the scattering angle is $\theta = \pi - 2\alpha$. Combining these two equations we find that

$$b = R\sin\frac{\pi - \theta}{2} = R\cos(\theta/2), \tag{14.24}$$

and from (14.23), we find the differential cross section

$$\frac{d\sigma}{d\Omega} = \frac{b}{\sin\theta}\left|\frac{db}{d\theta}\right| = \frac{R\cos(\theta/2)}{\sin\theta}\frac{R\sin(\theta/2)}{2} = \frac{R^2}{4}. \tag{14.25}$$

The most striking thing about this result is that the differential cross section is isotropic; that is, the number of particles scattered into a solid angle $d\Omega$ is the same in all directions. To find the total cross section, we have only to integrate this result over all solid angles:

$$\sigma = \int \frac{d\sigma}{d\Omega}d\Omega = \int \frac{R^2}{4}d\Omega = \pi R^2,$$

which is, of course, the cross-sectional area of the target sphere.