Divergence theorem.
In two dimensions, it is
equivalent to Green's theorem

$$\boxed{\int_V \partial F = \int_{\partial V} F}$$

volume integral

$$\iiint_V (\nabla \cdot \mathbf{F})\, dV = \oiint_S (\mathbf{F} \cdot \mathbf{n})\, dS.$$

surface integral

Name	Integral equations	Differential equations
Gauss's law	surface integral $\oiint_{\partial\Omega} \mathbf{E} \cdot \mathrm{d}\mathbf{S} = 4\pi \iiint_\Omega \rho\, \mathrm{d}V$ volume integral	$\nabla \cdot \mathbf{E} = 4\pi\rho$
Gauss's law for magnetism	$\oiint_{\partial\Omega} \mathbf{B} \cdot \mathrm{d}\mathbf{S} = 0$	$\nabla \cdot \mathbf{B} = 0$
Maxwell–Faraday equation (Faraday's law of induction)	$\oint_{\partial\Sigma} \mathbf{E} \cdot \mathrm{d}\boldsymbol{\ell} = -\frac{1}{c}\frac{\mathrm{d}}{\mathrm{d}t}\iint_\Sigma \mathbf{B} \cdot \mathrm{d}\mathbf{S}$	$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}$
Ampère's circuital law (with Maxwell's addition)	$\oint_{\partial\Sigma} \mathbf{B} \cdot \mathrm{d}\boldsymbol{\ell} = \frac{1}{c}\left(4\pi \iint_\Sigma \mathbf{J} \cdot \mathrm{d}\mathbf{S} + \frac{\mathrm{d}}{\mathrm{d}t}\iint_\Sigma \mathbf{E} \cdot \mathrm{d}\mathbf{S}\right)$	$\nabla \times \mathbf{B} = \frac{1}{c}\left(4\pi \mathbf{J} + \frac{\partial \mathbf{E}}{\partial t}\right)$

Conservative Forces: if F=- ∇U

$$F = -\nabla U$$

$$\int_a^b F = \int_a^b -\nabla U = -U_b + U_a = \Delta U_{ab}$$

Independent of path

Ampere's Law

$$\oint_C B \cdot dl = \mu_0 I$$

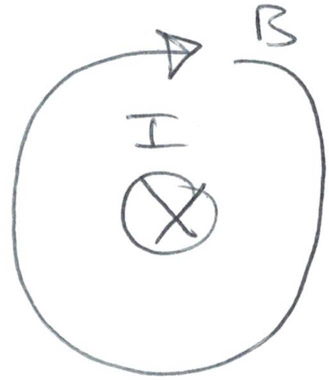
$$\nabla \times B = \mu_0 J$$

$\boxed{\text{Long Wire}}$

$$B L = \mu_0 I$$

$$B\, 2\pi r = \mu_0 I$$

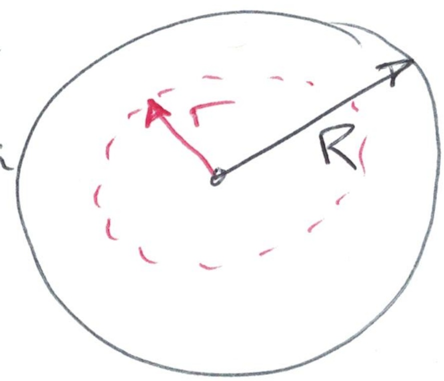
$$B = \frac{\mu_0}{2\pi} \frac{I}{r}$$



$\boxed{\text{Uniform } J}$

large wire
radius R
with uniform

$$J = \frac{I}{A}$$



$$B L = \mu_0 I_{enc} \qquad I_{enc} = J A_{enc}$$

$$B(2\pi r) = \mu_0 J \pi r^2 \qquad = J \pi r^2$$

$$B = \frac{\mu_0}{2} r J$$

$$I_{TOT} = J A_{TOT}$$

$$= J \pi R^2$$

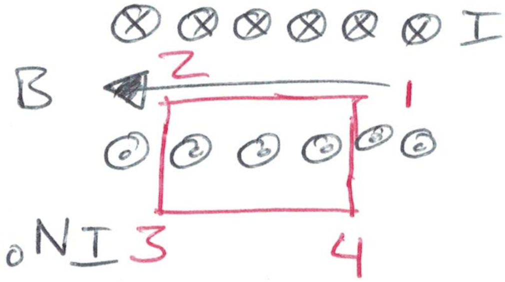
$$B = \frac{\mu_0}{2\pi} I_{TOT} \frac{r}{R^2} \qquad J = \frac{I_{TOT}}{\pi R^2}$$

$\boxed{\text{Sanity Check}}$

$r \to R \quad B \to \frac{\mu_0}{2\pi} \frac{I}{R}$ checks w/ above!!!

Solenoid

$$\oint_C B \cdot dL = \mu_0 I_{enc}$$



1 → 2 : $BL = \mu_0 N I$

2 → 3 : $B \cdot L = 0$

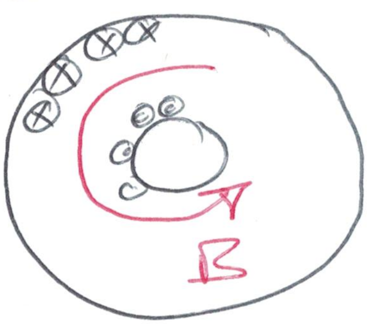
3 → 4 : $B = 0$

4 → 1 : $B \cdot L = 0$

$$B = \mu_0 \frac{NI}{L} = \mu_0 n I$$

$$n = \frac{N}{L} = \frac{\#\text{ Turns}}{\text{length}}$$

Torid



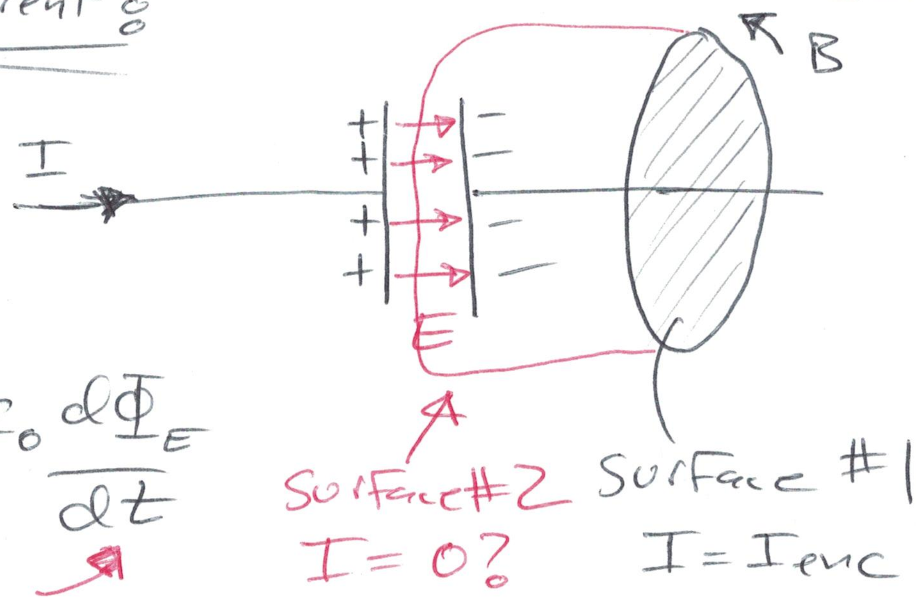
$$\oint B \cdot dL = \mu_0 I_{enc}$$

$$B \, 2\pi r = \mu_0 N I$$

$$B = \frac{\mu_0}{2\pi} \frac{NI}{r}$$

Displacement Current:

$$\boxed{\oint B \cdot dL = \mu_0 I_{enc}}$$



Solution

$$\oint B \cdot dL = \mu_0 I + \varepsilon_0 \frac{d\Phi_E}{dt}$$

resolves problem

Maxwells Eq

$$\oint E \cdot dA = \frac{Q}{\varepsilon_0} \qquad \nabla \cdot E = \frac{\rho}{\varepsilon_0}$$

$$\oint B \cdot dA = 0 \qquad \nabla \cdot B = 0$$

$$\oint E \cdot dL = -\frac{d\Phi_B}{dt} \qquad \nabla \times E = -\frac{\partial B}{\partial t}$$

$$\oint B \cdot dL = \mu_0 I + \varepsilon_0 \frac{d\Phi_E}{dt} \qquad \nabla \times B = \mu_0 J + \mu_0 \varepsilon_0 \frac{\partial E}{\partial t}$$

$\boxed{\text{Question}}$ What if Magnetic Monopoles Exist?

Maxwells Eq in Vacuum:

$$\nabla \cdot E = 0 \qquad \nabla \times E = -\frac{\partial B}{\partial t}$$

$$\nabla \cdot B = 0 \qquad \nabla \times B = \mu_0 \varepsilon_0 \frac{\partial E}{\partial t}$$

$$\nabla \times \left(\nabla \times E = -\frac{\partial B}{\partial t} \right)$$

$$\nabla \times (\nabla \times E) = -\frac{\partial}{\partial t} (\nabla \times B)$$

$$\hookrightarrow \mu_0 \varepsilon_0 \frac{\partial E}{\partial t}$$

$$\nabla \times (\nabla \times E) = \nabla(\nabla \cdot E) - \nabla^2 E$$

$$\hookrightarrow 0$$

$$\therefore \quad \boxed{+\nabla^2 E = +\mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2}}$$

Wave Equation

Guess $E = E_0 e^{i(kx + \omega t)}$ $\qquad \nabla^2 E = -k^2 E$

$v = \frac{\omega}{k}$ $\qquad \omega \sim \frac{1}{sec}$ $\qquad \partial_t^2 E = -\omega^2 E$

$k \sim \frac{1}{meters}$

$$\hookrightarrow \quad k^2 = \mu_0 \varepsilon_0 \omega^2 \Rightarrow \frac{\omega^2}{k^2} = \frac{1}{\mu_0 \varepsilon_0} = v^2$$

$$v = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

$\varepsilon_0 = 8.85 \times 10^{-12} \ \frac{\text{Farads}}{m}$

$\mu_0 = 1.26 \times 10^{-6} \ H/m$

$$v = c = 3 \times 10^{8} \ m/s$$