Invariants

Space vectors $v = \begin{pmatrix} x \\ y \end{pmatrix}$ $g = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$$\|v\|^2 = v \cdot g \cdot v = \begin{pmatrix} x \\ y \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = x^2 + y^2 = \text{Length}^2$$

Space Time Vectors

Invariant 4-length^2

$v = \begin{pmatrix} t \\ x \end{pmatrix}$ $g = \begin{pmatrix} 1 & \\ & -1 \end{pmatrix}$

$$\|v\|^2 = v \cdot g \cdot v = \begin{pmatrix} t \\ x \end{pmatrix}\begin{pmatrix} +1 & \\ & -1 \end{pmatrix}\begin{pmatrix} t \\ x \end{pmatrix} = t^2 - x^2$$

This does not change under Lorentz TX

Energy Momentum

Invariant

$v = \begin{pmatrix} E \\ p \end{pmatrix}$ $g = \begin{pmatrix} +1 & \\ & -1 \end{pmatrix}$

$$\|v\|^2 = v \cdot g \cdot v = \begin{pmatrix} E \\ p \end{pmatrix}\begin{pmatrix} +1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} E \\ p \end{pmatrix} = E^2 - p^2 \equiv m^2$$

From notes: Note: I use natural units: $c \equiv 1$

$$\beta = \frac{pc}{E} \equiv \frac{mvc}{mc^2} = \frac{vc}{c^2} = \frac{v}{c}$$

$$E = \gamma m = m + (\gamma - 1)m$$

rest mass ↑ Kinetic Energy

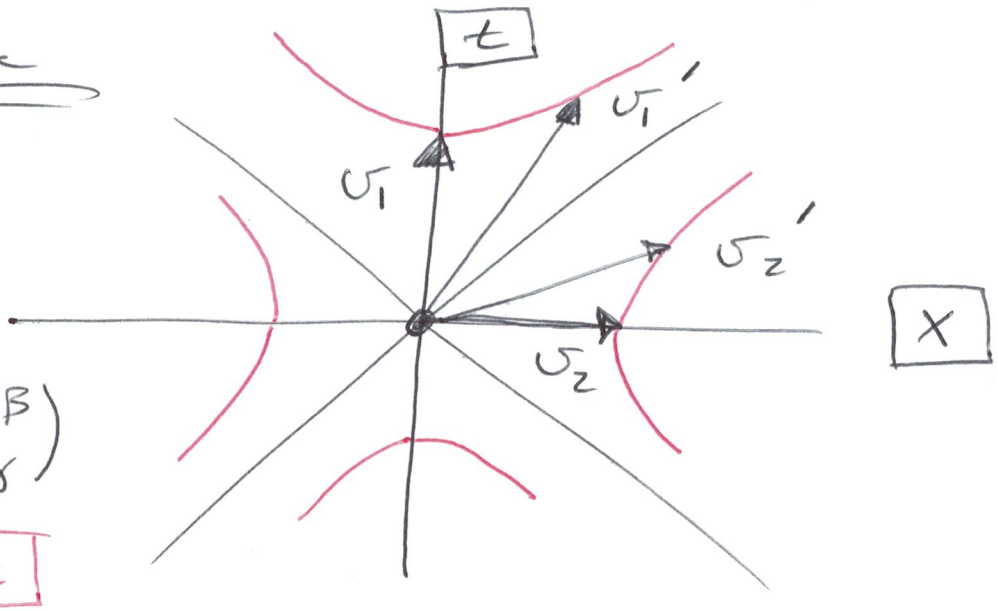
$$E = \sqrt{m^2 + p^2}$$

$$= m\sqrt{1 + \frac{p^2}{m^2}}$$

$$\approx m + \frac{p^2}{2m}$$

$\hookrightarrow \frac{1}{2}mv^2$

Light Cone



$$B = \begin{pmatrix} \gamma & \gamma\beta \\ \gamma\beta & \gamma \end{pmatrix}$$

Time-Like

$$v_1 = (1, 0) \qquad v_1' = B v_1 = \begin{matrix} t \\ (\gamma, \gamma\beta) \end{matrix} \quad \begin{matrix} \\ x \end{matrix}$$

$$\text{velocity} = \frac{\Delta x}{\Delta t} = \frac{\gamma\beta}{\gamma} = \beta$$

Space-Like

$$v_2 = (0, 1) \qquad v_2' = B v_2 = \begin{matrix} t \\ x \end{matrix} \begin{pmatrix} \gamma\beta \\ \gamma \end{pmatrix}$$

The LHC at CERN:

$B = 8$ Tesla Circumference $= 27$ Km

$$F = ma$$

$$qvB = \frac{mv^2}{r} \Rightarrow r = \frac{mv}{qB}$$

$m_{proton} = 1.67 \times 10^{-27}$ kg

$q_{proton} = 1.6 \times 10^{-19}$ C

$\Rightarrow r \approx 0.4$ meters

$$\gamma = \frac{E}{m} = \frac{7{,}000 \text{ GeV}}{\sim 1 \text{ GeV}}$$

$r = 0.4$ m $\rightarrow \sim 2{,}800$ m $= 2.8$ Km

$C = 2\pi r =$ 18 Km