Orbital Motion

$$E = T + V = \tfrac{1}{2} m \dot{r}^2 + \underbrace{\tfrac{1}{2} m r^2 \dot{\theta}^2}_{\frac{l^2}{2mr^2}} + V(r)$$

$$E = \tfrac{1}{2} m \dot{r}^2 + \left[\frac{l^2}{2mr^2} + V(r)\right] \qquad l = m r^2 \dot{\theta}$$

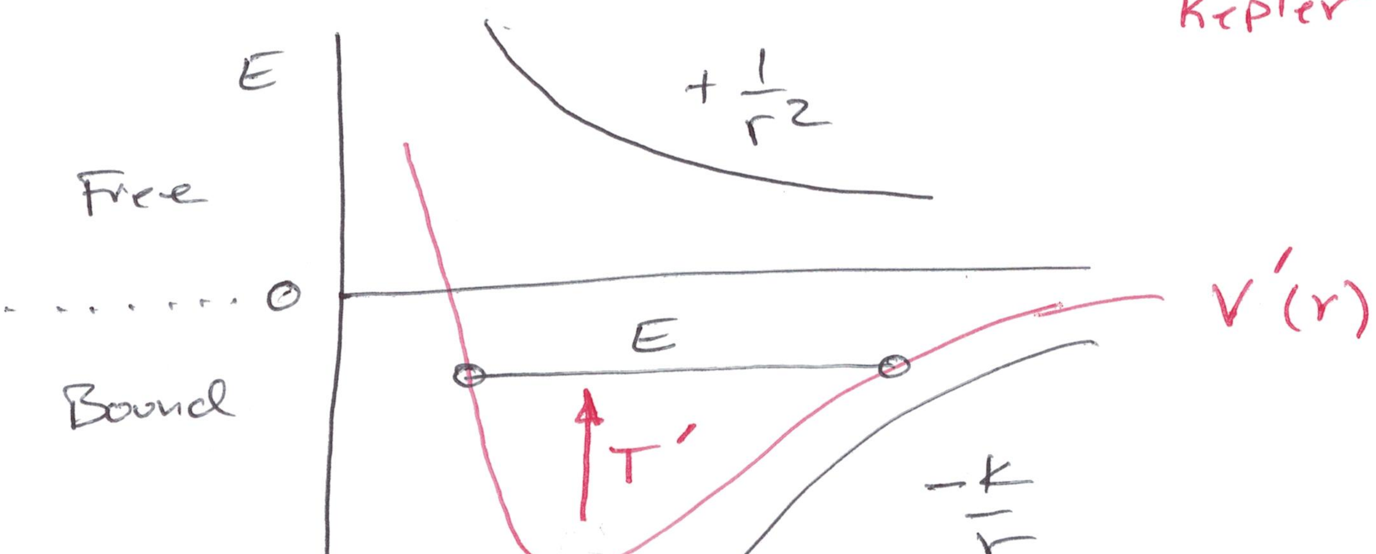
$$E = \tfrac{1}{2} m \dot{r}^2 + V'(r) \equiv T'(r) + V'(r)$$

Effective 1-D problem

$$\dot{r}^2 = \left(\frac{dr}{dt}\right)^2 = \frac{2}{m}\left[E - V(r) - \frac{l^2}{2mr^2}\right]$$

$$V'(r) = +\frac{l^2}{2mr^2} + V(r) \Rightarrow +\frac{l^2}{2mr^2} - \frac{k}{r}$$

Kepler



$$\boxed{E = T' + V'}$$

$$\dot{r} = \frac{dr}{dt} = \sqrt{\dot{r}^2} \Leftarrow \text{should be } \underline{\underline{\text{real}}}$$

$$\dot{r}^2 \geq 0$$

Taylor Expand

$$r_1 = r_0 + \underbrace{\left(\frac{dr}{dt}\right)}_{\dot{r}} \Delta t$$

$$\theta_1 = \theta_0 + \underbrace{\left(\frac{d\theta}{dt}\right)}_{\dot{\theta}} \Delta t$$

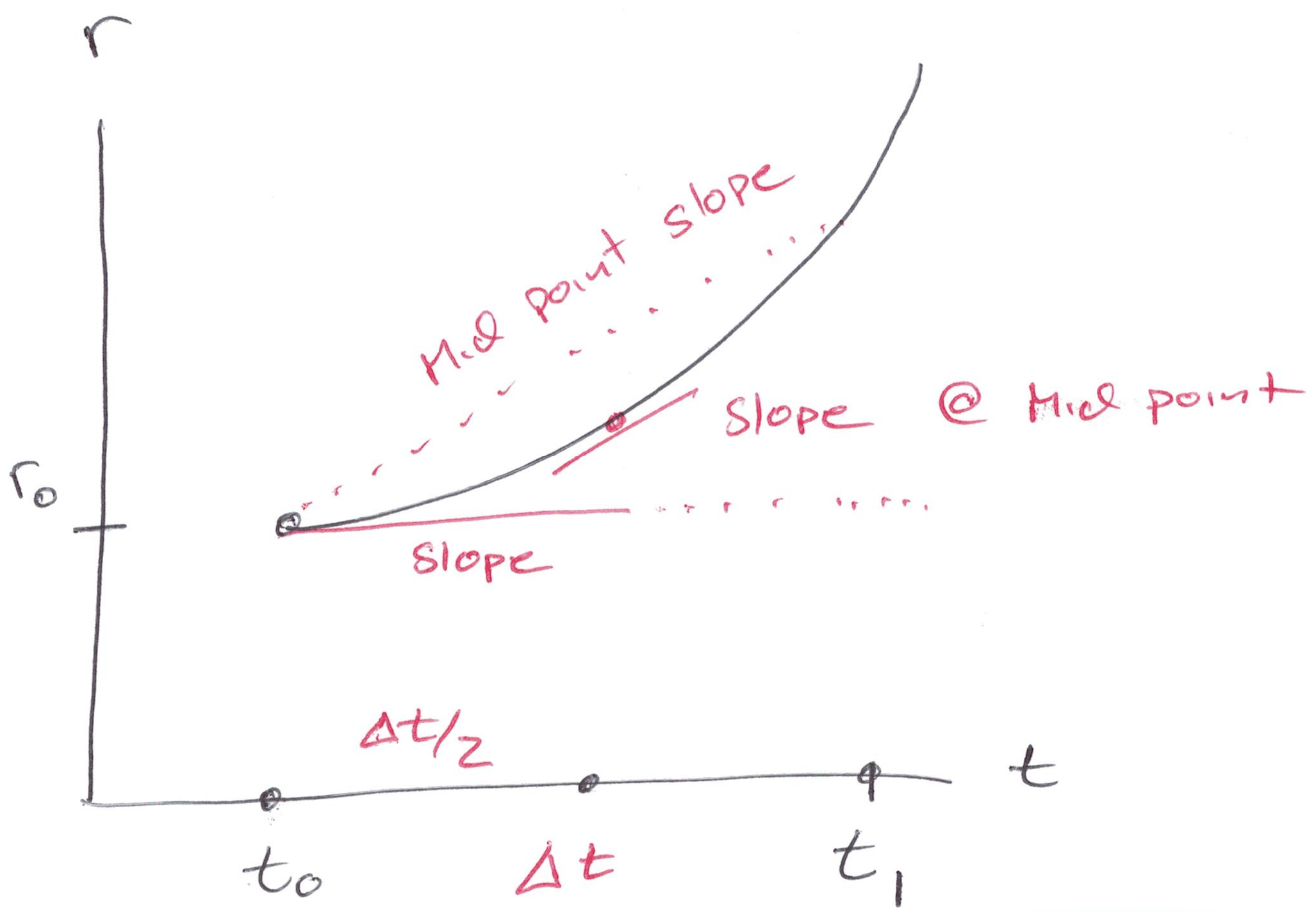
$$l = mr^2\dot{\theta} \Longrightarrow \dot{\theta} = \frac{l}{mr^2}$$

$\boxed{\text{Euler Method}}$ 1-Step Method

Other Methods:

- Mid-Point (2-Step) $\quad \dot{r} = \dot{r}(r_{\text{mid point}})$
- 4-Step Runge Kutta

$$\dot{r} = \frac{1}{6}\left(\dot{r}_1 + 2\dot{r}_2 + 2\dot{r}_3 + \dot{r}_4\right)$$



Mid-Point Method

$$r_m \cong r_0 + \mathring{r}(r_0)\,\frac{\Delta t}{2}$$

$\mathring{r}(r)$

$\mathring{\theta}(r)$

$$\mathring{r}_m = \mathring{r}(r_m)$$

$$r_1 = r_0 + \mathring{r}(r_m)\,\Delta t$$