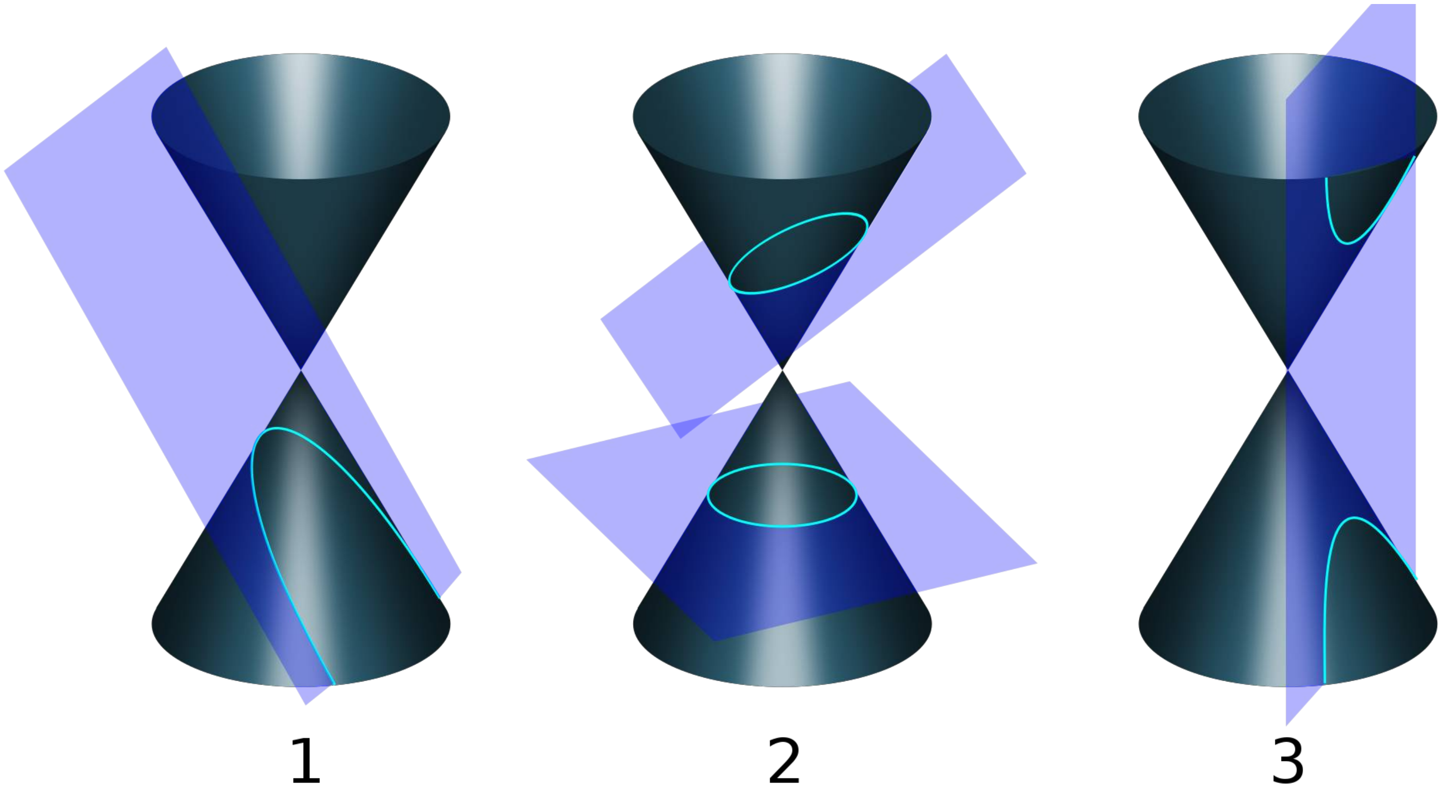


1) The orbit of a planet is an ellipse with the Sun at one of the two foci.

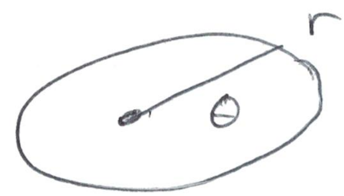
2) A line segment joining a planet and the Sun sweeps out equal areas during equal intervals of time.

3) The square of a planet's orbital period is proportional to the cube of the length of the semi-major axis of its orbit.

$$\mu = \frac{m_1 m_2}{m_1 + m_2},$$

$$\mathbf{R} = \frac{m_1 \mathbf{r}_1 + m_2 \mathbf{r}_2}{m_1 + m_2} = \frac{m_1 \mathbf{r}_1 + m_2 \mathbf{r}_2}{M},$$

Orbits



$$T = \frac{1}{2} m (\dot{r}^2 + r^2\dot{\theta}^2)$$

$V = V(r)$ independent of θ

$$L = T - V = \frac{m}{2}\dot{r}^2 + \frac{m}{2} r^2 \dot{\theta}^2 - V(r)$$

$$\frac{\partial L}{\partial \theta} = 0 \Rightarrow \frac{\partial L}{\partial \dot{\theta}} = \text{const} = \boxed{m r^2 \dot{\theta} = l}$$

angular momentum conserved !!!

R eq

$$\frac{\partial L}{\partial r} = m r \dot{\theta}^2 - \frac{\partial V(r)}{\partial r} \qquad \frac{\partial L}{\partial \dot{r}} = m\dot{r}$$

$$\text{Eq} \Rightarrow m r \dot{\theta}^2 - \frac{\partial V}{\partial r} - m\ddot{r} = 0$$

$$\frac{l^2}{mr^3} + F - m\ddot{r} = 0$$

$$\dot{\theta} = \frac{l}{mr^2} \qquad F = -\frac{\partial V(r)}{\partial r}$$

$$\boxed{\text{Eq:}} \quad \underbrace{\frac{l^2}{mr^3} + F}_{F_{\text{effective}}} = \underbrace{m\ddot{r}}_{ma}$$

Note $F = -\frac{\partial V}{\partial r}$

Example: Coulomb: $F = +\frac{K}{r^2}$ $V = \frac{K}{r}$ $F = \frac{-\partial V}{\partial r}$

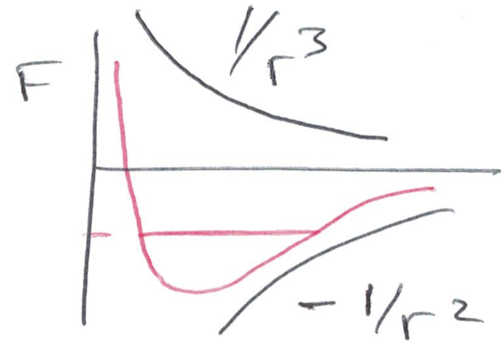
Gravity $F = -\frac{K}{r^2}$ $V = -\frac{K}{r}$ $K = GMm$

$\boxed{\text{Eq:}}$ $ma = m\ddot{r} = F + \frac{\ell^2}{mr^3} \equiv F_{\text{effective}}$

$F_{\text{eff}} = \frac{-K}{r^2} + \frac{\ell^2}{mr^3}$

Kepler force

Centrifugal force



	Force	Potential
	$F = -\frac{\partial V}{\partial r}$	V
Kepler	$-\frac{K}{r^2}$	$-\frac{K}{r}$
Centrifugal	$+\frac{\ell^2}{mr^3}$	$+\frac{\ell^2}{2mr^2}$

Energy $E = T + V$

$$E = \tfrac{1}{2} m (\dot{r}^2 + r^2 \dot{\theta}^2) + V(r)$$

$$E = \underbrace{\tfrac{1}{2} m \dot{r}^2 + \frac{\ell^2}{2mr^2}}_{\text{Kinetic}} + \underbrace{V(r)}_{\text{Potential}}$$

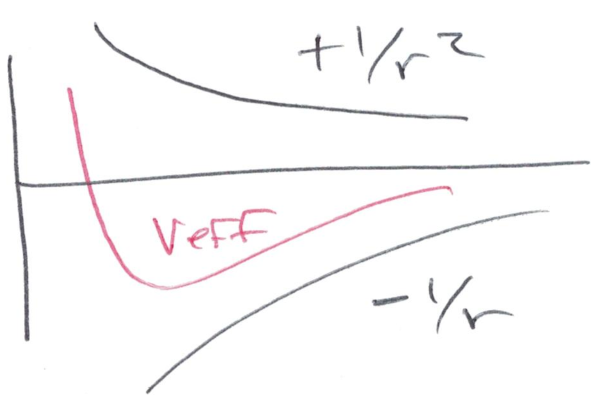
$$E = \underbrace{\tfrac{1}{2} m \dot{r}^2}_{\text{Kinetic}} + \underbrace{\frac{\ell^2}{2mr^2} + V(r)}_{\text{Effective Potential}}$$

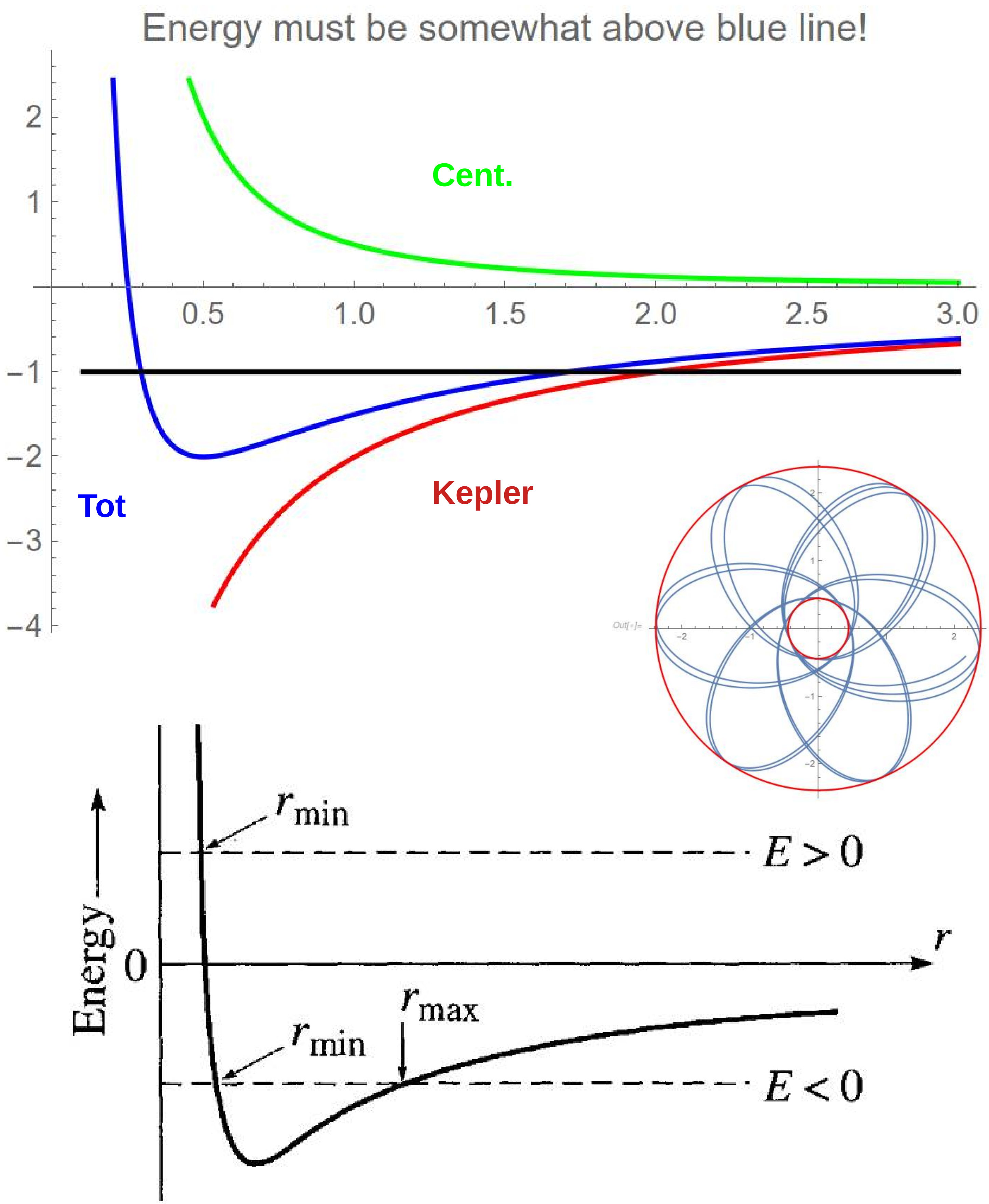
$$E = T_{eff} + V_{eff}$$

$$V_{eff} \equiv \frac{\ell^2}{2mr^2} + V(r)$$

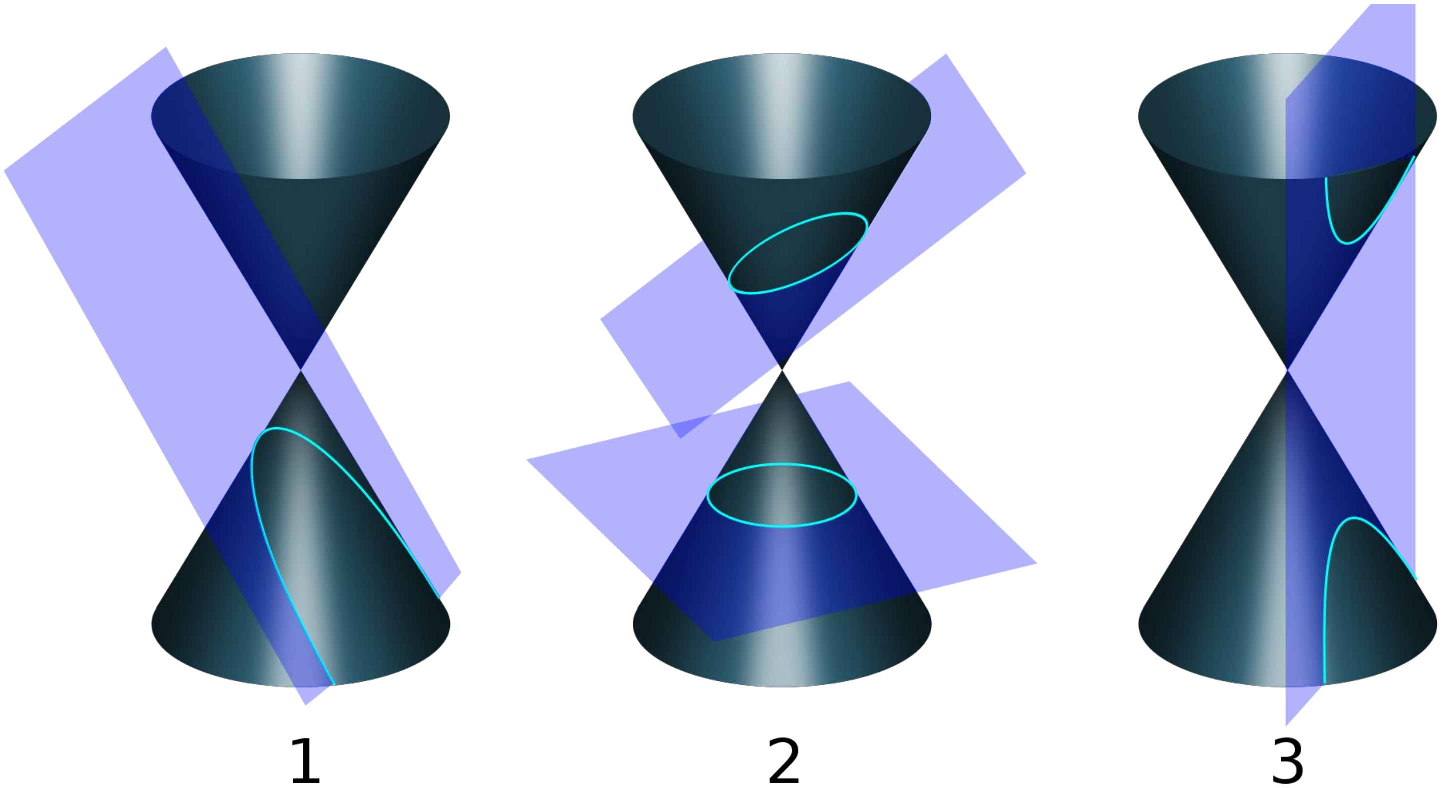
Kepler

$$= \frac{\ell^2}{2mr^2} - \frac{k}{r}$$





Energy must be somewhat above blue line!
Cent.
Tot
Kepler
r_{min}
$E > 0$
Energy
0
r
r_{max}
r_{min}
$E < 0$



$$r(\phi) = \frac{c}{1 + \epsilon \cos\phi},$$

eccentricity	energy	orbit
$\epsilon = 0$	$E < 0$	circle
$0 < \epsilon < 1$	$E < 0$	ellipse
$\epsilon = 1$	$E = 0$	parabola
$\epsilon > 1$	$E > 0$	hyperbola