Classical Mechanics Core Proficiency Exam
January 2022
Problems provided by the Physics Faculty.
Exam assembled by the CPE Committee

Instructions

The exam consists of two longer questions and two shorter questions.
All four problems will be graded. You have two hours to work on the solutions.
You are allowed one textbook of your choice, one math reference, and a calculator
(no cell phone calculator allowed).

Write your personal identifier (e.g. name, number, etc.) on the cover sheet of this exam.

We will scan the exams to PDF, so please write only on one side of the page.
Sort your copy of the exam together with your scratch paper (exam on top) with the problems in numerical order. You may wish to number the pages so we can check the scanner did not miss a page.

Problem	Points	Score			Final Score
1	20				
2	20				
3	30				
4	30				
Total	100				

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Problem #1 (20 points)

(a) (6 points) Write and number Kepler's laws.

(b) (4 points) Which law(s), if any, are only approximately true in an exact 1/r potential?

(c) (6 points) Which law(s), if any, are true for an exact 1/r potential, but are false if the potential is proportional to $1/r^{1.1}$?

(d) (4 points) Which law(s), if any, are true for any central force field?

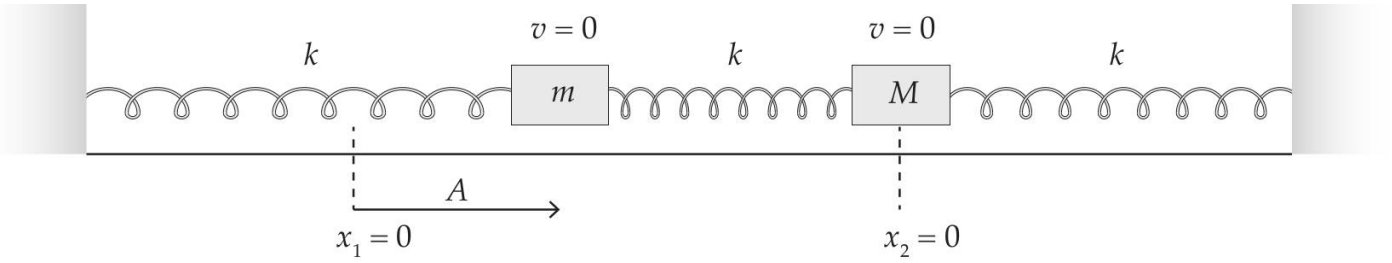
Problem #2 (20 points)

Consider a uniform density Earth.

(a) (10 points) With the convention that the gravitational potential is zero infintely far away from Earth, what is the gravitational potential at the center of the Earth?
Give a formula, then a numerical value in MKS units.

(b) (10 points) If a tunnel is dug through the Earth from north pole to south pole and a small stone is dropped through the hole, what is the speed of the stone when it is at the center of Earth? Neglect air resistance.
Give a formula, then a numerical value in MKS units.

Problem #3 (30 points)



Consider two blocks of mass m and M attached to identical springs with the spring constant k as shown in the figure. Both blocks are initially at rest (t = 0). The block of mass m starts at point $x_1 = +A$ while the block of mass M starts at its equilibrium location ($x_2 = 0$).

a) Write down the Lagrangian of the two-block system. Derive the equations of motion for each mass from the Lagrangian.

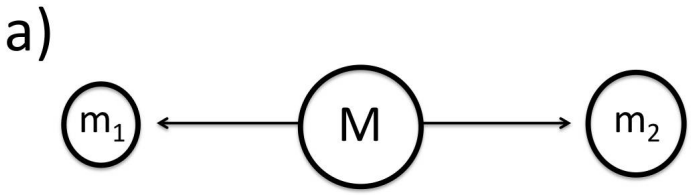
b) Find the normal frequencies of this system.

c) Assume that m = M . Find the normal frequencies of the system and the corresponding normal modes.

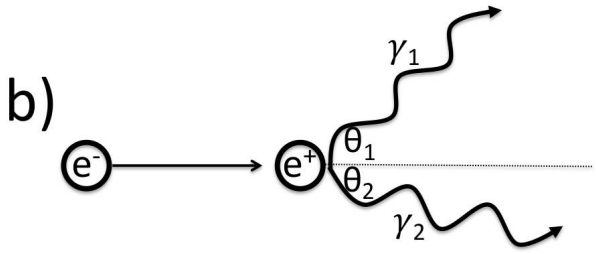
d) Given the normal modes found in part c), propose a set of initial conditions that will only excite a single mode. [There are multiple answers; we just need one.]

Problem #4 (30 points)

The following problems should be solved using the special theory of relativity.



a) A relativistic particle with mass M initially at rest (v=0) decays into two particles with masses m_1 and m_2 (see Figure a). Calculate the total energies (E_1 and E_2) and momentum (p_1 and p_2) of the produced particles in terms of M, m_1, m_2, and the speed of light c.



b) An electron e− with mass m collides with a positron e+ with mass m at rest (v = 0). They annihilate and produce two photons γ_1 and γ_2 (see Figure b). Derive the momentum p(e−) of the electron and the photon energy E_1 of γ_1 in terms of m, θ_1. c,