Quantum Mechanics Core Proficiency Exam
January 2022
Problems provided by the Physics Faculty.
Exam assembled by the CPE Committee

Instructions

The exam consists of two longer questions and two shorter questions.
All four problems will be graded. You have two hours to work on the solutions.
You are allowed one textbook of your choice, one math reference, and a calculator
(no cell phone calculator allowed).

Write your personal identifier (e.g. name, number, etc.) on the cover sheet of this exam.

We will scan the exams to PDF, so please write only on one side of the page.
Sort your copy of the exam together with your scratch paper (exam on top) with the problems in numerical order. You may wish to number the pages so we can check the scanner did not miss a page.

Problem	Points	Score			Final Score
1	20				
2	20				
3	30				
4	30				
Total	100				

This page is intentionally blank

Problem #1 (20 points)

(20 points) What are the wavelengths of

(a) (6 points) a 100 eV photon?

(b) (6 points) an electron with kinetic energy 100 eV?

(c) (6 points) a neutron with kinetic energy 100 eV?

(d) (2 points) Which of these would be most useful to study crystals by using diffraction.
Why?

Problem #2 (20 points)

At time t = 0, a free particle of mass m in three dimensions has the wavefunction:

$$\psi(r, 0) = A \sin(2x/L) \exp[i(y + 3z)/L],$$

where L is a length.

(a) (6 points) If the energy were measured at time t = 0, what values could be obtained and with what probabilities?

(b) (8 points) If, instead of energy, the momentum vector were measured at time t = 0, what values could be obtained and with what probabilities?

(c) (6 points) Neither the energy nor the momentum are measured. At time t = 0, what is the probability of finding the particle at (x, y, z) = (L, 0, 0)?

Problem #3 (30 points)

Consider the one-dimensional infinite square well potential

$$V(x) = \begin{cases} 0 & \text{for } -a < x < a \\ \infty & \text{for } |x| \geq a \end{cases}$$

for which the exact ground state wave function is given by

$$\psi_0(x) = \begin{cases} \frac{1}{\sqrt{a}} \cos\left(\frac{\pi x}{2a}\right) & \text{for } -a < x < a \\ 0 & \text{for } |x| \geq a \end{cases} .$$

23 Points

(a) Derive an approximation to the ground state wave function using the variational principle, starting with the trial wave function

$$\phi(x) = \begin{cases} a^\lambda - |x|^\lambda & \text{for } -a < x < a \\ 0 & \text{for } |x| \geq a \end{cases} .$$

7 Points

(b) What is the ratio of the expectation value of the energy in the approximate ground state if you take $\lambda = 7/4$ (instead of the value you found in the previous part) to that in the exact ground state $\langle E \rangle_{\phi(\lambda=7/4)} / \langle E \rangle_{\psi_0}$?

Problem #4 (30 points)

The following entangled state is prepared using two spin-half electrons

$$\psi = \frac{1}{\sqrt{2}} \left(|\uparrow_{1z}\rangle \otimes |\uparrow_{2z}\rangle + |\downarrow_{1z}\rangle \otimes |\downarrow_{2z}\rangle \right)$$

where $|\uparrow_{1z}\rangle$ is an eigenstate of the spin angular momentum operator $\hat{S}_{1z}$ with eigenvalue $+\frac{1}{2}\hbar$, and similarly for the spin down state with eigenvalue $-\frac{1}{2}\hbar$. The symbol $\otimes$ represents a tensor product of the two Hilbert spaces, one for each particle. A large number of identical entangled particle pairs are prepared. Particle 1 of the pair is always sent to Alice and particle 2 of the pair is always sent to Bob who are each one lightminute from the source of particles. Entangled pairs are sent out every minute. After many trials, Alice and Bob meet to compare notes.

(a) (3 points) If Alice measures the spin of particle 1 along the z-axis, what results can she obtain and with what probabilities?

(b) (6 points) If Alice measures the spin of particle 1 along the x-axis, what results can she obtain and with what probabilities? Show your work.

(c) (6 points) For the first entangled pair sent to Alice and Bob, Alice measures the spin of particle 1 along the z-axis and the result is spin up. Bob measures the spin of particle 2 along the z-axis. What results can Bob obtain and with what probabilities?

(d) (6 points) For the second entangled pair sent to Alice and Bob, Alice measures the spin of particle 1 along the z-axis and the result is spin up. Bob measures the spin of particle 2 along the x-axis. What results can Bob obtain and with what probabilities? Show your work.

(e) (9 points) For the third entangled pair sent to Alice and Bob, Alice measures the spin of particle 1 along the x-axis and the result is spin up. Bob measures the spin of particle 2 along the x-axis. What results can Bob obtain and with what probabilities? Show your work.