Divergence theorem.
In two dimensions, it is equivalent to Green's theorem

$$\boxed{\int_V \partial F = \int_{\partial V} F}$$

volume integral

$$\iiint_V (\nabla \cdot \mathbf{F})\, dV = \oiint_S (\mathbf{F} \cdot \mathbf{n})\, dS.$$

surface integral

Name	Integral equations	Differential equations
Gauss's law	surface integral $\oiint_{\partial\Omega} \mathbf{E} \cdot \mathrm{d}\mathbf{S} = 4\pi \iiint_\Omega \rho\, \mathrm{d}V$ volume integral	$\nabla \cdot \mathbf{E} = 4\pi\rho$
Gauss's law for magnetism	$\oiint_{\partial\Omega} \mathbf{B} \cdot \mathrm{d}\mathbf{S} = 0$	$\nabla \cdot \mathbf{B} = 0$
Maxwell–Faraday equation (Faraday's law of induction)	$\oint_{\partial\Sigma} \mathbf{E} \cdot \mathrm{d}\boldsymbol{\ell} = -\frac{1}{c}\frac{\mathrm{d}}{\mathrm{d}t} \iint_\Sigma \mathbf{B} \cdot \mathrm{d}\mathbf{S}$	$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}$
Ampère's circuital law (with Maxwell's addition)	$\oint_{\partial\Sigma} \mathbf{B} \cdot \mathrm{d}\boldsymbol{\ell} = \frac{1}{c}\left(4\pi \iint_\Sigma \mathbf{J} \cdot \mathrm{d}\mathbf{S} + \frac{\mathrm{d}}{\mathrm{d}t}\iint_\Sigma \mathbf{E} \cdot \mathrm{d}\mathbf{S}\right)$	$\nabla \times \mathbf{B} = \frac{1}{c}\left(4\pi \mathbf{J} + \frac{\partial \mathbf{E}}{\partial t}\right)$

Conservative Forces: if $F=-\nabla U$

$$F = -\nabla U$$

$$\int_a^b F = \int_a^b -\nabla U = -U_b + U_a = \Delta U_{ab}$$

Independent of path

Maxwell's Eq $\Rightarrow$ Wave Eq

$\nabla \cdot E = \rho$ $\qquad \nabla \times E = \frac{\partial B}{\partial t} + \phi$

$\nabla \cdot B = 0$

$\nabla \times B = \frac{\partial E}{\partial t} + \mu_0 I$

Homework for students:
Repeat with all μ_0 ε_0 to find c
in terms of $\mu_0 \varepsilon_0$

In Vacuum: $\rho \to 0 \quad I \to 0$

$$\nabla \times \left(\nabla \times E = \frac{\partial B}{\partial t} \right) \Rightarrow \nabla^2 E = \frac{\partial}{\partial t} (\nabla \times B)$$

$$= \frac{\partial}{\partial t} \left(\frac{\partial E}{\partial t} \right) = \frac{\partial^2 E}{\partial t^2}$$

$$\Rightarrow \boxed{\nabla^2 E = \partial_t^2 E}$$

Likewise $\nabla^2 B = \partial_t^2 B$

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