Physics 1304 Professor Olness Sept 1995

Gauss' Law

Simplified: $\Phi = EA = \dfrac{Q_{enc}}{\varepsilon_0}$ if ① Flux $\perp$ A ② E is constant

① IF Flux $\perp$ (perpendicular) to A: $EA \Rightarrow \vec{E} \cdot \vec{A} = EA\cos\theta$

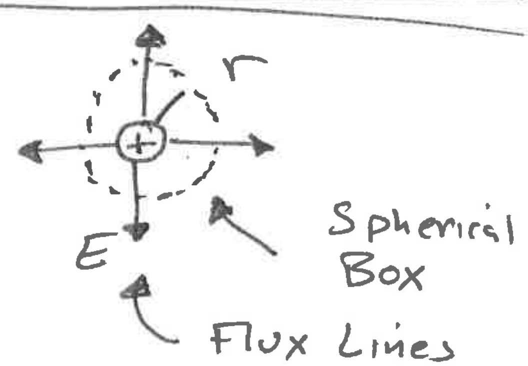
② IF E is not constant $EA = \oint E\, dA$

Zero Dimensional: Point Charge

$E = \dfrac{Q_{enc}}{A\,\varepsilon_0}$ $\quad Q_{enc} = q$

$A = 4\pi r^2$ Area of Sphere

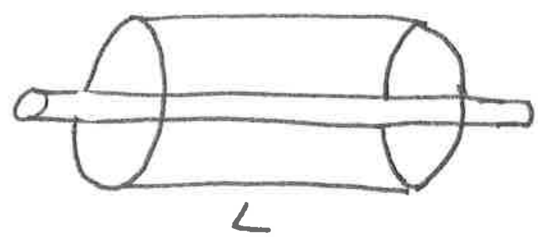
so $E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{r^2}$



One Dimensional: Line Charge

$E = \dfrac{Q_{enc}}{A\,\varepsilon_0}$

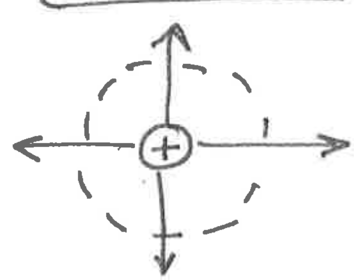
$\lambda = \dfrac{q}{L}$



$Q_{enc} = q = \lambda L$

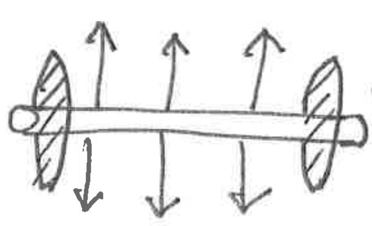
$A = 2\pi r L$ ⇒ Because flux only flows out cylinder. Not ends!!!

End View



$E = \dfrac{\lambda L}{2\pi r L \varepsilon_0} = \dfrac{\lambda}{2\pi\varepsilon_0 r}$

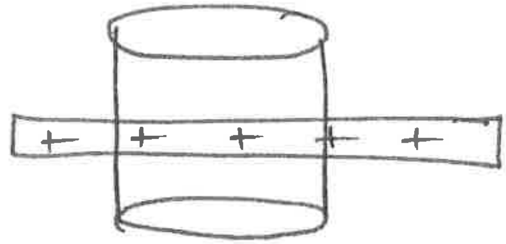
Side view



No Flux flows through here

Two Dimensional | Sheet of Charge

$$E = \frac{Q_{enc}}{A\,\varepsilon_0} \qquad \sigma = \frac{q}{\text{Area}}$$



Area $= \alpha$

$$Q_{enc} = \sigma A = \sigma\,\alpha$$

$$A = 2\alpha$$

There is a top and bottom!!!

$$\therefore \quad E = \frac{\sigma\,\alpha}{2\alpha\,\varepsilon_0} = \frac{\sigma}{2\varepsilon_0}$$

Solid Sphere of Charge

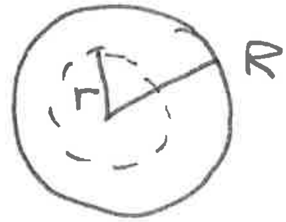
Charge $= Q$

$$\rho = \frac{Q}{V}$$

Do in 2 steps!

1) For r inside
2) For r outside

$$E = \frac{Q_{enc}}{A\,\varepsilon_0} \qquad \rho = \frac{Q_{TOT}}{V_{TOT}} = \frac{Q_{enc}}{V_{enc}}$$



$$A = 4\pi r^2 \qquad \therefore \quad Q_{enc} = Q_{TOT}\,\frac{V_{enc}}{V_{TOT}} =$$

$$Q_{enc} = Q_{TOT}\,\frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi R^3} = Q_{TOT}\,\frac{r^3}{R^3}$$

$$E = \frac{1}{4\pi r^2}\,\frac{1}{\varepsilon_0}\,Q_{TOT}\,\frac{r^3}{R^3} = \frac{1}{4\pi\varepsilon_0}\,\frac{r}{R^3}\,Q_{TOT}$$

Note: For $r = R$ $\quad E = \frac{1}{4\pi\varepsilon_0}\,\frac{q}{r^2}$

Part 2: r outside.

$$E = \frac{Q_{enc}}{A\,\varepsilon_0} \qquad Q_{enc} = Q_{TOT} \quad \text{(why???)}$$

$$A = 4\pi r^2$$

$$E = \frac{1}{4\pi\varepsilon_0}\,\frac{Q_{TOT}}{r^2}$$

Hollow Shell - Sphericil

Do in 3 steps:

1) $r < a$: $E = 0$

2) $a < r < b$:

3) $b < r$: $E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$

Part 2: $a < r < b$

$$E = \frac{Q_{enc}}{A\,\varepsilon_0} \qquad \rho = \frac{Q_{TOT}}{V_{TOT}} = \frac{Q_{enc}}{V_{enc}}$$

$$A = 4\pi r^2$$

$$Q_{enc} = Q_{TOT}\,\frac{V_{enc}}{V_{TOT}}$$

$$Q_{enc} = Q_{TOT}\left[\frac{\frac{4}{3}\pi r^3 - \frac{4}{3}\pi a^3}{\frac{4}{3}\pi b^3 - \frac{4}{3}\pi a^3}\right]$$

$$Q_{enc} = Q_{TOT}\left[\frac{r^3 - a^3}{b^3 - a^3}\right]$$

$$E = \frac{1}{4\pi\varepsilon_0}\,\frac{Q_{TOT}}{r^2}\left[\frac{r^3 - a^3}{b^3 - a^3}\right]$$

what happens when $r = b$?

How to use Lenz's Law:

The induced EMF's are always of such a polarity as to oppose the **change** that generated them.

1) Find the direction of the magnetic field B_1.

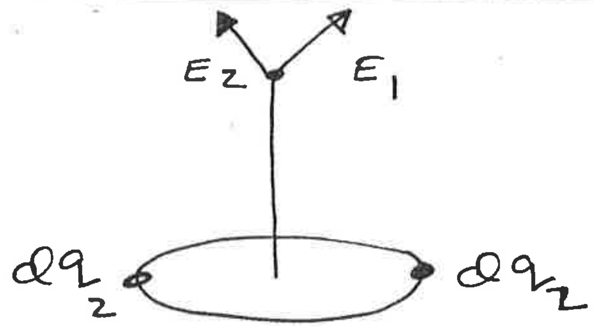
2) Find the direction of the CHANGE magnetic field B_1.

3) Find the direction of the induced field B_2 such that B_2 minimizes the change.
(Careful here. This direction can be the same or different than B_1.)

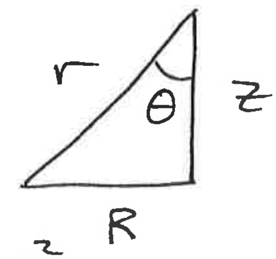
4) Find the direction of the induced current I_2 which generated B_2.

Prof. Olness Phys 1304 Sept 94

Find $\vec{E}$ from a ring of charge



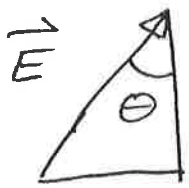
① Horizontal components cancell. Vertical components Remain.



② To Find the angle θ, use →

Note: $\cos\theta = \dfrac{z}{r}$ and $r^2 = R^2 + z^2$

③ The vertical component of E is



$$E_z = E\cos\theta = E\left(\frac{z}{r}\right)$$

④ From Coulomb's Law:

$$dE_z = dE\cos\theta = \cos\theta\left(\frac{1}{4\pi\varepsilon_0}\right)\frac{dq}{r^2}$$

⑤ If $\lambda = \dfrac{q}{s} = \dfrac{dq}{ds} = \dfrac{\text{Charge}}{\text{Length}}$ Then $dq = \lambda\, ds$

$r^2 = (R^2 + z^2)$ Note, r, R, and z are Constant

⑥ ∴

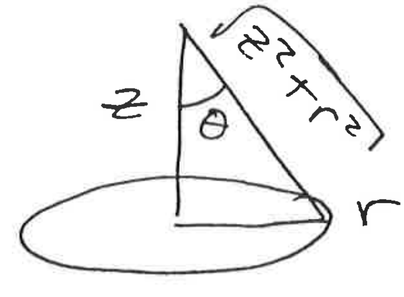
$$E_z = \int dE_z = \int \frac{\cos\theta}{(4\pi\varepsilon_0)} \frac{\lambda\, ds}{(z^2+R^2)} =$$

$$= \frac{\cos\theta}{(4\pi\varepsilon_0)} \frac{\lambda}{(z^2+R^2)} \underbrace{\int ds}_{=2\pi R} = \frac{z\lambda(2\pi R)}{(4\pi\varepsilon_0)(z^2+R^2)^{3/2}}$$

Q: Find E From a Disk

Recall: For a ring:

$$E_z = \left(\frac{1}{4\pi\varepsilon_0}\right) \frac{z\,\lambda(2\pi r)}{(z^2+r^2)^{3/2}}$$



Note: Total Charge on ring

$= dq = \lambda\,(2\pi r)$

Define $\sigma = \dfrac{q}{A} = \dfrac{dq}{dA} = \dfrac{dq}{(2\pi r)\,dr}$

Thus $dq = \sigma\,(2\pi r)\,dr$

For a single ring

$$dE_z = \left(\frac{1}{4\pi\varepsilon_0}\right) \frac{z\,\sigma\,(2\pi r)\,dr}{(z^2+r^2)^{3/2}}$$

Integrate This:

$$E_z = \int dE_z = \left(\frac{1}{4\pi\varepsilon_0}\right) 2\pi z\sigma \underbrace{\int_0^R \frac{r\,dr}{(z^2+r^2)^{3/2}}}_{\left[\frac{-1}{\sqrt{z^2+r^2}}\right]_0^R}$$

$$E_z = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{z}{\sqrt{z^2+R^2}}\right]$$

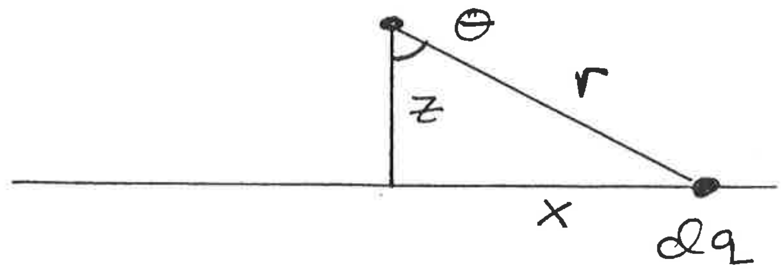
Note, as $R \to \infty$

$$E_z \to \frac{\sigma}{2\varepsilon_0}$$

Very Simple !!!

Q: Find $\vec{E}$ from a finite wire of charge

$$E = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2}$$



$$dE_z = \frac{1}{4\pi\varepsilon_0} \frac{dq}{r^2} \cos\theta$$

$$r^2 = x^2 + z^2 \qquad \cos\theta = \frac{z}{r}$$

$$\tan\theta = \frac{x}{z} \qquad r = \frac{z}{\cos\theta} = z\sec\theta$$

$$x = z\tan\theta$$

Goal: Express all vars. in term θ

Define $\lambda = \dfrac{q}{x} = \dfrac{dq}{dx}$

$\sec\theta = \dfrac{1}{\cos\theta}$

or $\quad q = x\lambda = z\lambda\tan\theta$

$$dq = z\lambda \sec^2\theta \, d\theta$$

Express all vars. in terms of θ

$$dE_z = \frac{1}{4\pi\varepsilon_0} \frac{z\lambda\sec^2\theta \, d\theta}{z^2\sec^2\theta} \cos\theta =$$

$$\int dE_z = \frac{1}{4\pi\varepsilon_0}\frac{\lambda}{z}\int_{-\theta_0}^{\theta_0}\cos\theta \, d\theta = \frac{1}{4\pi\varepsilon_0}\frac{\lambda}{z}\left[\sin\theta\right]_{-\theta_0}^{+\theta_0}$$

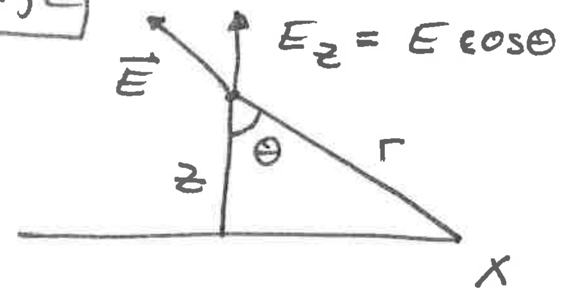
For Infinite wire: $\theta_0 = \pi/2$, $\sin(\pi/2) = 1$

$$E_z = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{z}$$

Simple

Physics 1304 Prof. Olness Sept. 11 1995

Compute E from a line of charge



Vector $\vec{E}$: $\vec{E} = \left(\frac{1}{4\pi\varepsilon_0}\right)\frac{q}{r^2}$

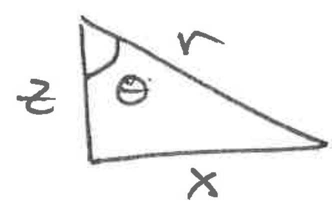
Differential: $d\vec{E} = \left(\frac{1}{4\pi\varepsilon_0}\right)\frac{dq}{r^2}$

Z component: $dE_z = dE\cos\theta = \left(\frac{1}{4\pi\varepsilon_0}\right)\frac{dq}{r^2}\cos\theta$

Integrate: $E_z = \int dE_z = \left(\frac{1}{4\pi\varepsilon_0}\right)\int \frac{dq}{r^2}\cos\theta$

To do integral, must convert dq, r, $\cos\theta$ into same variable

We will convert to θ:



① $\cos\theta = \frac{z}{r}$

$$\boxed{r = \frac{z}{\cos\theta}}$$

② $q = \lambda x$

$dq = \lambda\, dx$

$\tan\theta = \frac{x}{z} \Rightarrow x = z\tan\theta$

$$\boxed{dx = z\frac{d\theta}{\cos^2\theta}}$$

Now sub into integral

$$E_z = \left(\frac{1}{4\pi\varepsilon_0}\right)\int \underbrace{\left(\frac{\lambda z\, d\theta}{\cos^2\theta}\right)}_{\text{"}dq\text{"}}\cdot\underbrace{\left(\frac{\cos^2\theta}{z^2}\right)}_{\text{"}\frac{1}{r^2}\text{"}}\cos\theta = \left(\frac{1}{4\pi\varepsilon_0}\right)\frac{\lambda}{z}\int\cos\theta\, d\theta$$

$E_z = \left(\frac{1}{4\pi\varepsilon_0}\right)\frac{\lambda}{z}\sin\theta$

For infinite line charge

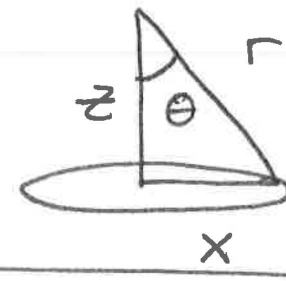
$\sin\theta \to +2$

$E_z = \frac{\lambda}{2\pi\varepsilon_0 z}$

Compute E For a disk of Charge

We already know for ring of charge

$$E_z = \left(\frac{1}{4\pi\varepsilon_0}\right) \frac{q}{r^2} \cos\theta$$



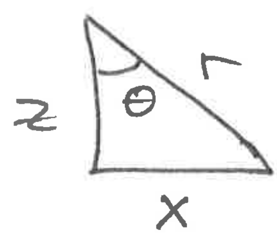
Add up many rings:

Differential: $dE_z = \left(\frac{1}{4\pi\varepsilon_0}\right) \frac{dq}{r^2} \cos\theta$

Integral: $E_z = \int dE_z = \left(\frac{1}{4\pi\varepsilon_0}\right) \int \frac{dq}{r^2} \cos\theta$

To do integral, must convert dq, r, $\cos\theta$ to same variable

We will convert to r:



$\sigma = \frac{q}{A}$ $\quad q = \sigma A = \sigma \pi r^2 = \sigma\pi(z^2+x^2)$

$\boxed{dq = \sigma\, 2\pi r\, dr} = \sigma\, 2\pi x\, dx$

$\boxed{\cos\theta = \frac{z}{r}}$

Error

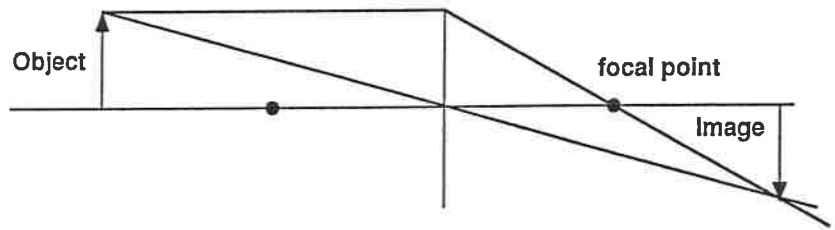
Now Sub in integral

$$E_z = \left(\frac{1}{4\pi\varepsilon_0}\right) \int \underbrace{(\sigma\, 2\pi r\, dr)}_{\text{"}dq\text{"}} \frac{1}{r^2} \underbrace{\frac{z}{r}}_{\text{"}\cos\theta\text{"}} = \frac{\sigma z}{2\varepsilon_0} \int_z^\infty \frac{dr}{r^2}$$

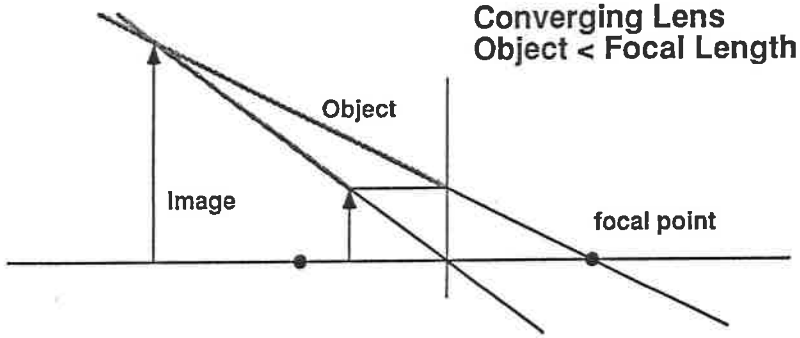
$$E_z = \frac{\sigma z}{2\varepsilon_0}\left[\frac{-1}{r}\right]_z^\infty = \frac{\sigma z}{2\varepsilon_0}\left[0 - \left(-\frac{1}{z}\right)\right] = \frac{\sigma}{2\varepsilon}$$

For infinite Sheet

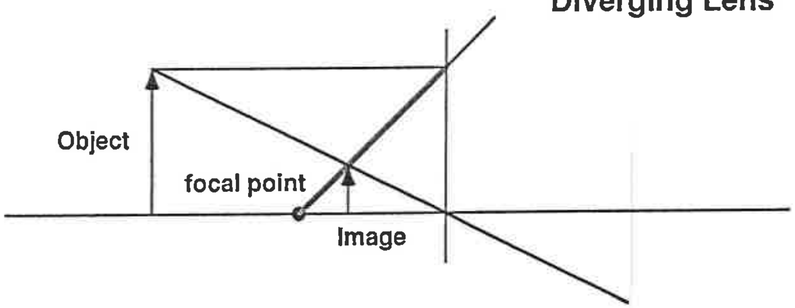
Converging Lens
Object > Focal Length



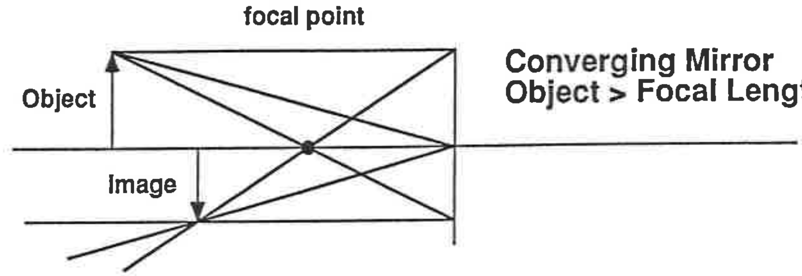
Converging Lens
Object < Focal Length



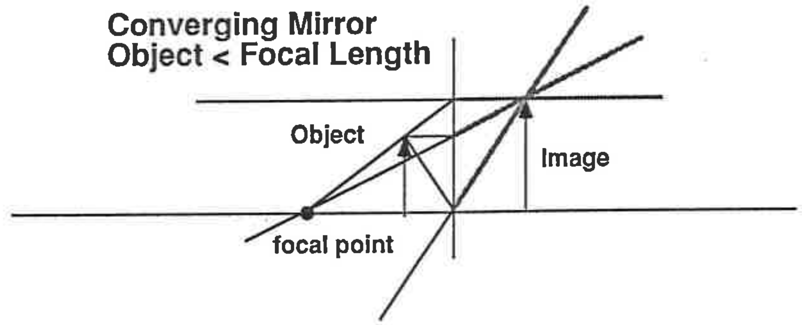
Diverging Lens



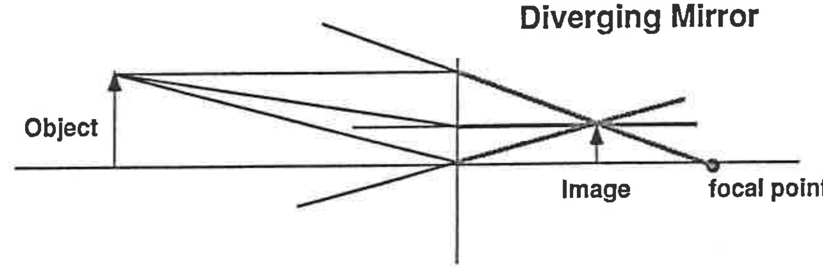
Converging Mirror
Object > Focal Length



Converging Mirror
Object < Focal Length



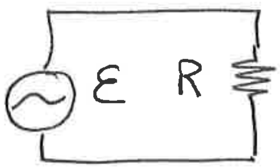
Diverging Mirror



AC Circuits :
Physics 1403

Professor Olness
April 1993

Resistor



$$RI = V = \mathcal{E}$$

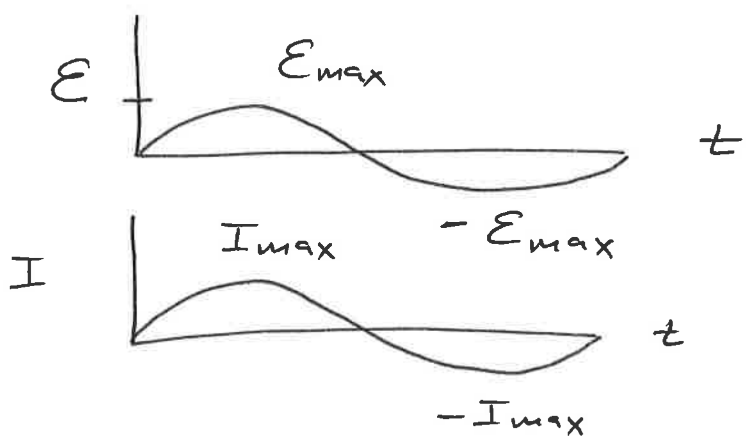
$$I = \frac{\mathcal{E}}{R}$$

Let $\mathcal{E}(t) = \mathcal{E}_{max} \sin(\omega t)$

Then $I(t) = \frac{\mathcal{E}(t)}{R} = \frac{\mathcal{E}_{max}}{R} \sin(\omega t)$

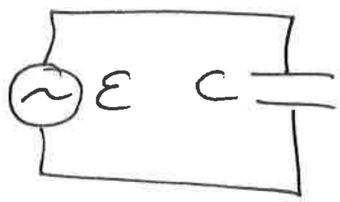
$\therefore I(t) = I_{max} \sin(\omega t)$

where $I_{max} = \mathcal{E}_{max}/R$



Capacitor

$$C = \frac{Q}{V} \qquad I = \frac{dQ}{dt}$$



$$\mathcal{E} = \frac{Q}{C}$$

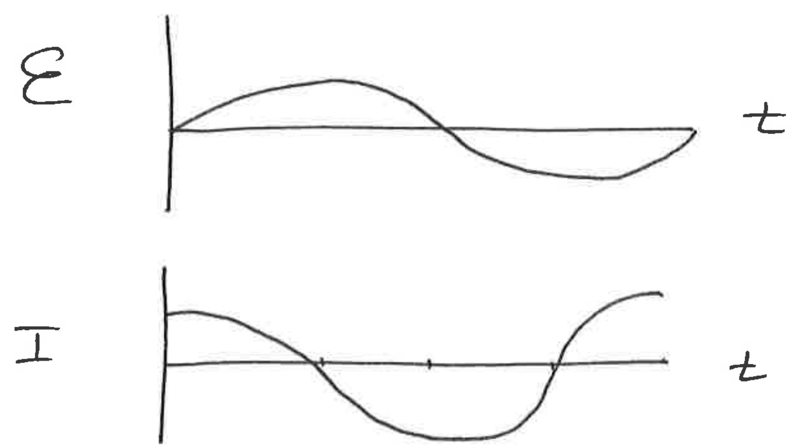
$$\frac{d\mathcal{E}}{dt} = \frac{1}{C}\frac{dQ}{dt} = \frac{1}{C} I$$

Let $\mathcal{E}(t) = \mathcal{E}_{max} \sin(\omega t)$

$$I(t) = C\frac{d\mathcal{E}(t)}{dt} = C\,\mathcal{E}_{max}\,\omega\, \cos(\omega t)$$

$\therefore$ $I(t) = I_{max} \cos(\omega t)$

with $I_{max} = \mathcal{E}_{max}(\omega C)$



$\boxed{I\ C\ E}$

Note: to make above look like resistor case

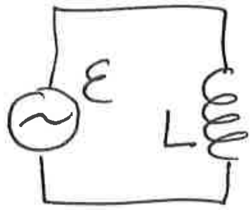
define $X_c = \dfrac{1}{\omega C}$ ~~[illegible]~~

Then for resistor: $I_{max} = \dfrac{\mathcal{E}_{max}}{R}$

For capacitor: $I_{max} = \dfrac{\mathcal{E}_{max}}{X_c} = \mathcal{E}_{max}\,\omega C$

Inductor

$$\mathcal{E} = L \frac{dI}{dt}$$



Then $dI = dt \, \frac{\mathcal{E}}{L}$

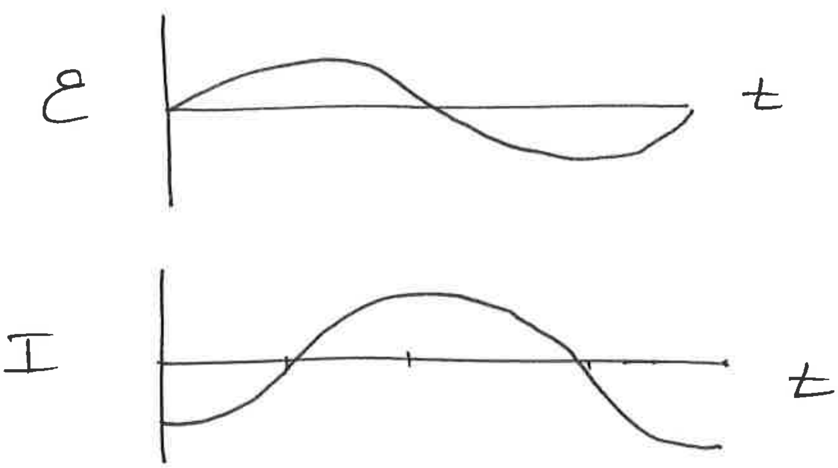
Integrate: $I = \int dt \, \frac{\mathcal{E}}{L}$

Let $\mathcal{E}(t) = \mathcal{E}_{max} \sin(\omega t)$

$$I(t) = -\frac{\mathcal{E}_{max}}{\omega L} \cos(\omega t) = -I_{max} \cos(\omega t$$

with $I_{max} = \frac{\mathcal{E}_{max}}{\omega L} \equiv \frac{\mathcal{E}_{max}}{X_L}$

with $X_L = \omega L$



$\boxed{\text{ELI}}$

Summary: $RI = V$

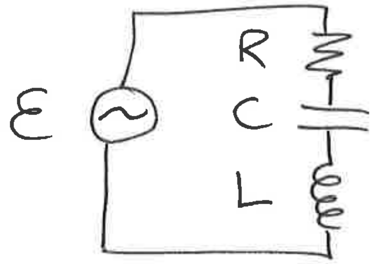
$XI = \mathcal{E} \Rightarrow I = \frac{\mathcal{E}}{X}$

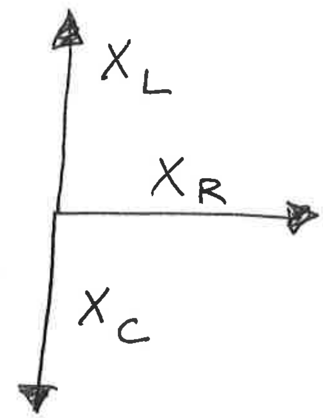
Resistor $X_R = R$ I and $\mathcal{E}$ are in phase

Capacitor $X_C = \frac{1}{\omega C}$ ICE

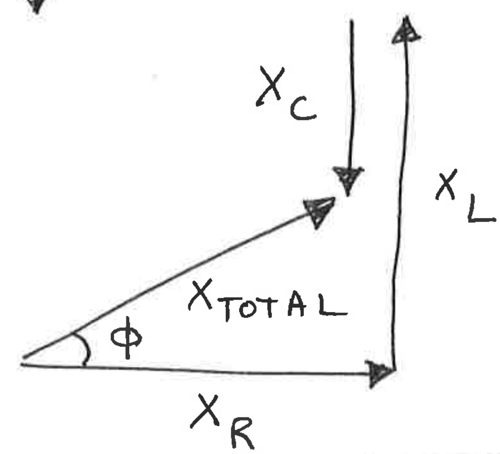
Inductor $X_L = \omega L$ ELI

Put together RLC Circuit





Add Them up



$$Z = X_{TOTAL} = \sqrt{X_R^2 + (X_L - X_C)^2} = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

$$\tan \phi = \frac{X_L - X_C}{X_R}$$

$$RI = V$$

$$ZI = \mathcal{E} \Rightarrow I = \frac{\mathcal{E}}{Z} \Rightarrow I_{Max} = \frac{\mathcal{E}_{max}}{Z}$$