Fourier Trans form

$$F(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} d\omega \; g(\omega) \; e^{-i\omega x}$$

$$g(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dx \; F(x) \; e^{+i\omega x}$$

Note: $\int d\omega \; e^{i\omega(x-x')} = 2\pi \, \delta(x-x')$

Check sub $g(\omega)$ in $F(x)$

$$F(x) = \frac{1}{\sqrt{2\pi}} \int d\omega \left[\frac{1}{\sqrt{2\pi}} \int dx' \, F(x') \, e^{+i\omega x'} \right] e^{-i\omega x}$$

$$= \frac{1}{2\pi} \int dx' \int d\omega \; e^{+i\omega(x'-x)} \qquad F(x')$$

$$= \frac{1}{2\pi} \int dx' \; F(x') \; 2\pi \, \delta(x-x')$$

$$= F(x)$$

$$\ddot{x} + F\dot{x} + \omega_i^2 x(t) = Q(t)$$

Fourier Transform:

$$Q(t) = \frac{1}{\sqrt{2\pi}} \int G(\omega)\, e^{-i\omega t}\, d\omega$$

$$x(t) = \frac{1}{\sqrt{2\pi}} \int \tilde{x}(\omega)\, e^{-i\omega t}\, d\omega$$

$$(D^2 + F D + \omega_i^2)\, x(t) = Q(t)$$

$$\frac{1}{\sqrt{2\pi}} \int d\omega\, e^{-i\omega t} (-\omega^2 - i\omega F + \omega_i^2)\, \tilde{x}(\omega)$$

$$\equiv \frac{1}{\sqrt{2\pi}} \int G(\omega)\, e^{-i\omega t}\, d\omega$$

Therefore

$$(-\omega^2 - i\omega F + \omega_i^2)\, \tilde{x}(\omega) = G(\omega)$$

$$\tilde{x}(\omega) = \frac{G(\omega)}{(\omega_i^2 - \omega^2) - i\omega F}$$

$$x(t) = \frac{1}{\sqrt{2\pi}} \int \tilde{x}(\omega)\, e^{-i\omega t}\, d\omega$$

$$= \frac{1}{\sqrt{2\pi}} \int d\omega\, e^{-i\omega t} \frac{G(\omega)}{(\omega_i^2 - \omega^2) - i\omega F}$$