Physics 3344:
Homework #12: *(the last)*
Prof. Olness
Fall 2020
Due 02 December 2020

PROBLEM #1: Consider the metric:

$$g_{\mu\nu} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

and vector:

$$v = \begin{pmatrix} x \\ y \end{pmatrix}$$

Note: Please do parts a), b), c) BY HAND! Part d) can be done with any plotting program you like.

a) Compute the length-squared of the vector v.
b) Write down a rotation matrix R.
c) Show the length of v is invariant under a rotation.
d) Using **Mathematica**, make a ContourPlot of the length of the vector in the {x,y} space. Comment about the result.

PROBLEM #2: Consider the metric:

$$g_{\mu\nu} = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

and vector:

$$v = \begin{pmatrix} x \\ y \end{pmatrix}$$

Note: Please do part a) BY HAND! Part b) can be done with any plotting program you like.

a) Compute the length-squared of the vector v.
b) Using **Mathematica**, make a ContourPlot of the length of the vector in the {x,y} space. Comment about the result. [Note, for this one, just focus on the first quadrant; I'll explain in class.]
c) Can you think of a physical example where this is the appropriate metric to use to measure distance??? *(There are more than one answer, but the more natural the better; think creatively)*

PROBLEM #3: Consider the metric:

$$g_{\mu\nu} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

and vector:

$$v = \begin{pmatrix} t \\ x \end{pmatrix}$$

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Note: Please do parts a), b), c), d), e) BY HAND! Part f) can be done with any plotting program you like.

a) Compute the length-squared of the vector v.
b) Write down a boost matrix B in terms of β and γ.
c) Show the length of v is invariant under a boost.
d) Write down a boost matrix B in terms of cosh and sinh.
e) Show the length of v is invariant under a boost in terms of cosh and sinh.
f) Using **Mathematica**, make a ContourPlot of the length of the vector in the {t,x} space. Comment about the result.

PROBLEM #4: For the general mass case of $p_1 + p_2 \to p_{12}$ compute the components of all 4-vectors in terms of invariants of the problem: $\{m_1^2, m_2^2, s\}$. Assume the 3-momentum lies along the z-axis. Hint: The z-component of the momentum should be proportional to: $\Delta(s, m_1^2, m_2^2)$, where $\Delta(a, b, c) = \sqrt{a^2 + b^2 + c^2 - 2(ab + bc + ca)}$.

PROBLEM #5: Consider the reaction: $pp \to ppH$ where H is the Higgs boson and has a rest mass of $126 GeV$. Note, a proton has a mass 1 GeV Compute the threshold beam energy for

a) colliding beams, and

b) for a fixed target experiment.

PROBLEM #6

a) Two particles collide in the CMS frame: $p_1^\mu = \{E_1, +p\}$, $p_2^\mu = \{E_2, -p\}$, with masses $m_{1,2}$ respectively. Find the boost matrix B that transforms this to the rest frame of p_2^μ. Express all variable $\{E_{1,2}, p, \gamma, \beta, cosh, sinh\}$ in terms of invariants $\{m_1, m_2, s\}$.

PROBLEM #7

Note: I recommend you do this on Mathematica, but is not required. If so, do the comparison with the book by hand.

Consider a boost B along the x-axis using coordinates $\{t, x, y, z\}$ with metric $g = \{1, -1, -1, -1\}$.

Write down the 4x4 B matrix in terms of $\{\gamma, \beta\}$. Then transform $F^{\alpha\beta}$ to see how the $\{E, B\}$ fields transform: $F' = BFB^T$. Compare with the results in the book, and comment. [Note, the book uses different coordinate order and metric. Hint: I suggest you use mathmatica.]

$$F^{\alpha\beta} = \begin{pmatrix} 0 & E_x & E_y & E_z \\ -E_x & 0 & B_z & -B_y \\ -E_y & -B_z & 0 & B_x \\ -E_z & B_y & -B_x & 0 \end{pmatrix}$$

PROBLEM #8

Starting from Maxwell's equations in a vacuum, use these to obtain an EM wave, and compute the speed of the waves in terms of μ_0 and ϵ_0.