Stat Mech Core Proficiency Exam
January 2022
Problems provided by the Physics Faculty.
Exam assembled by the CPE Committee

Instructions

The exam consists of two longer questions and two shorter questions.
All four problems will be graded. You have two hours to work on the solutions.
You are allowed one textbook of your choice, one math reference, and a calculator
(no cell phone calculator allowed).

Write your personal identifier (e.g. name, number, etc.) on the cover sheet of this exam.

We will scan the exams to PDF, so please write only on one side of the page.
Sort your copy of the exam together with your scratch paper (exam on top) with the problems in numerical order. You may wish to number the pages so we can check the scanner did not miss a page.

Problem	Points	Score			Final Score
1	20				
2	20				
3	30				
4	30				
Total	100				

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Problem #1 (20 points)

a) (10 points) Let the number of states accessible to a room temperature (293 K), macroscopic system with energy E be given by $\Omega(E)$. The thermodynamic temperature can be defined by:

$$kT=\frac{1}{\beta} \qquad \text{where} \quad \beta=\frac{\partial \ln \Omega}{\partial E} \text{ or } \frac{\Delta \ln \Omega}{\Delta E}$$

Calculate the percent increase in the number of states accessible to the system if it absorbs a visible light photon with a wavelength of 500 nanometers.

b) (10 points) Calculate the change in entropy of the same system.

Problem #2 (20 points)

Consider a perfectly insulated box with a partition in the center. One part of the container is filled with an ideal gas, and the other is empty; this is the *initial configuration*. Then, the partition is removed, and the gas is allowed to diffuse throughout the entire container. Let us call the point at which this gas reaches a uniform density the *final configuration*.

a) (5 points) How much work is done by the gas in this process?

b) (5 points) What is the change in the total energy of the gas?

c) (5 points) What is the ratio of the pressure exerted by the gas in the initial configuration to that in the final configuration, in terms of the initial and final volumes (V_i and V_f)?

d) (5 points) What is the ratio of the average speed of a gas particle in the initial and final configurations?

Problem #3 (30 points)

Suppose you have just climbed Mt. Kilimanjaro, and want to celebrate your accomplishment with a cup of tea at the summit (5900 m above sea level). In order to make your tea, you need to boil water.

(Hint: You may assume that the latent heat of vaporization, Δh*, is independent of temperature for the relevant temperatures in this problem, and that the molar volume of water vapor is much greater than the molar volume of liquid water, such that the change in volume from liquid to vapor,* ΔV *, is approximately the molar volume of the gas. The heat of vaporization of water is 42 kJ/mol.)*

a) (15 points) Assume that water boils at pressure P_0 at temperature T_0. The Clapeyron Equation relates the slope of a transition line on a phase diagram (pressure vs. temperature diagram), and is given by:

$$\frac{dP}{dT} = \frac{\Delta h}{T\,\Delta V}$$

By including the relevant assumptions and integrating the differential equation, show that the boiling pressure P is related to the temperature as:

$$P(T) = P_0 e^{\frac{-\Delta h}{R}\left(\frac{1}{T} - \frac{1}{T_0}\right)}$$

b) (15 points) The dependence of the height above sea level (z) of atmospheric pressure is given by the *barometric formula:*

$$P = P_0 e^{\frac{-mgz}{RT_{\text{air}}}}$$

Where m is the molar mass of air, 29 g/mol. Using your result from part (a) and bearing in mind that water boils at sea level at 373 K, what temperature will you need to heat your water to, in order to boil it for tea on Kilimanjaro's summit, where the air temperature is a very cold 250 K?

Problem #4 (30 points)

A system in contact with a reservoir at temperature T posesses three energy levels: E1=ϵ, E2=2ϵ and E3=3ϵ, with degeneracies g(E1)=1, g(E2)=2 and g(E3)=1.

(a) (10 points) Find the partition function Z of the system.

(b) (10 points) Find the total energy E of the system from the partition function.

(c) (10 points) Find the heat capacity C of the system