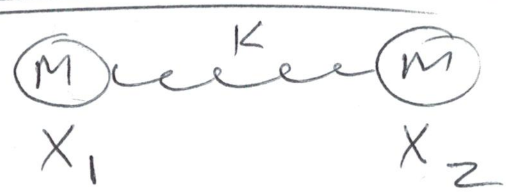
2 Masses on a Spring



$$T = \frac{1}{2} m \left(\dot{X}_1^2 + \dot{X}_2^2 \right) \qquad V = \frac{1}{2} K \left(X_1 - X_2 \right)^2$$

$$L = T - V = \frac{m}{2} \left(\dot{X}_1^2 + \dot{X}_2^2 \right) - \frac{K}{2} \left(X_1 - X_2 \right)^2$$

$$\left[X_1^2 + X_2^2 - 2 X_1 X_2 \right]$$

$$\frac{\partial L}{\partial X_1} = -\frac{K}{2} \left[2X_1 - 2X_2 \right] \qquad \frac{\partial L}{\partial X_2} = -\frac{K}{2} \left[2X_2 - 2X_1 \right]$$

$$\frac{\partial L}{\partial \dot{X}_1} = m \dot{X}_1 \qquad \frac{\partial L}{\partial \dot{X}_2} = m \dot{X}_2$$

$$(1) \Rightarrow \quad m \ddot{X}_1 + K \left[X_1 - X_2 \right] = 0$$

$$(2) \Rightarrow \quad m \ddot{X}_2 + K \left[-X_1 + X_2 \right] = 0$$

$$\overleftrightarrow{M} \ddot{X} + \overleftrightarrow{K} X = 0 \qquad \ddot{X} = \begin{pmatrix} \ddot{X}_1 \\ \ddot{X}_2 \end{pmatrix} \quad X = \begin{pmatrix} X_1 \\ X_2 \end{pmatrix}$$

$$\overleftrightarrow{M} = \begin{pmatrix} m & 0 \\ 0 & m \end{pmatrix} \qquad \overleftrightarrow{K} = K \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$$

Solve $|K - \omega^2 M| = 0$

$$K\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} - \omega^2 \begin{pmatrix} m & 0 \\ 0 & m \end{pmatrix} = \begin{pmatrix} K - \omega^2 m & -K \\ -K & K - \omega^2 m \end{pmatrix}$$

$$(K - \omega^2 m)^2 - K^2 = 0$$

$$\omega^2 m \left[\omega^2 m - 2K\right] = 0 \Rightarrow \boxed{\omega^2 = 0, \ \frac{2K}{m}}$$

Find Eigen Modes

$\omega_1 = 0$

$$(K - \omega_1^2 M)\, v_1 = 0 \qquad v_1 = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$

$$K\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = 0 \Rightarrow \begin{aligned} a_1 - a_2 &= 0 \\ -a_1 + a_2 &= 0 \end{aligned}$$

$$a_1 = a_2 \qquad \boxed{v = \begin{pmatrix} 1 \\ 1 \end{pmatrix}}$$

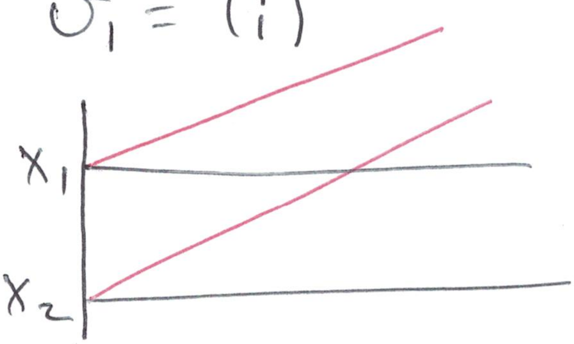
$$\omega_2^2 = \frac{2K}{m}$$

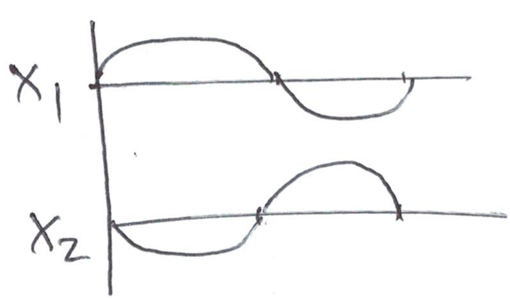
$$(K - \omega_2^2 M) = K\begin{pmatrix} 1-2 & -1 \\ -1 & 1-2 \end{pmatrix} = -K\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = 0 \Rightarrow a_1 + a_2 = 0$$

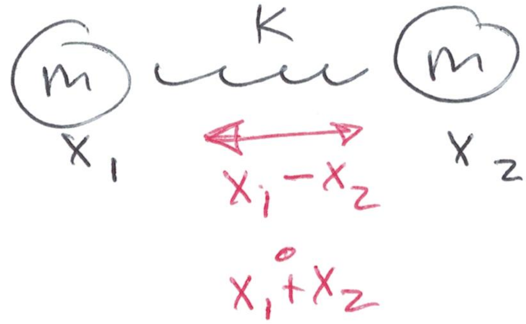
$$v_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$v_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ $\qquad$ $v_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$





2 Masses on Spring $\bar{c}$ Alternate



Let
$$\boxed{Y_1 = \frac{x_1 + x_2}{2} \qquad Y_2 = \frac{x_1 - x_2}{2}}$$

Then $\quad x_1 = (Y_1 + Y_2) \qquad x_2 = (Y_1 - Y_2)$

$$L = T - V \qquad T = \frac{m}{2}\left(\dot{x}_1^2 + \dot{x}_2^2\right) = m\left(\dot{Y}_1^2 + \dot{Y}_2^2\right)$$

cross terms vanish [check this]

$$V = \frac{K}{2}(x_1 - x_2)^2 = 2KY_2^2$$

$$L = m\left(\dot{Y}_1^2 + \dot{Y}_2^2\right) - 2KY_2^2$$

$$\textcircled{1} \Rightarrow \quad 2m\ddot{Y}_1 + 0 = 0$$

$$\textcircled{2} \Rightarrow \quad 2m\ddot{Y}_2 + 4KY_2 = 0$$

$$\textcircled{1} \quad \ddot{Y}_1 = 0 \Rightarrow Y(t) = Y_0 + v_{0Y}\, t$$

$$\textcircled{2} \quad m\ddot{Y}_2 + 2K Y_2 = 0 \Rightarrow Y_2 = A e^{\pm i\omega t}$$

$$\omega^2 = \frac{2K}{m}$$

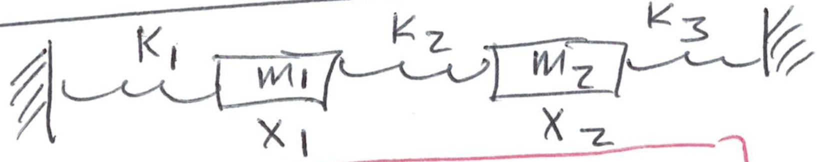
$v_1 = \begin{pmatrix}1\\0\end{pmatrix}$ in Y basis

$v_2 = \begin{pmatrix}0\\1\end{pmatrix}$

In X-basis

$v_1 = \begin{pmatrix}1\\1\end{pmatrix} \qquad v_2 = \begin{pmatrix}1\\-1\end{pmatrix}$

Two Masses + 3 Springs



Simplify $K_1 = K_2 = K_3 = K$

$m_1 = m_2 = m$

$$(11.2) \; ① \quad m\ddot{x}_1 = -2Kx_1 + Kx_2$$
$$② \quad m\ddot{x}_2 = Kx_1 - 2Kx_2$$

Let: $y_1 = \frac{1}{2}(x_1 + x_2)$ $\quad y_2 = \frac{1}{2}(x_1 - x_2)$

$x_1 = (y_1 + y_2)$ $\quad x_2 = (y_1 - y_2)$

$$① \quad m(\ddot{y}_1 + \ddot{y}_2) = -K(y_1 + 3y_2)$$
$$② \quad m(\ddot{y}_1 - \ddot{y}_2) = -K(y_1 - 3y_2)$$

$① + ② \Rightarrow \quad 2m\ddot{y}_1 = -2Ky_1$

$① - ② \Rightarrow \quad 2m\ddot{y}_2 = -2K(3y_2)$

$$\boxed{\begin{aligned} m\ddot{y}_1 + Ky_1 &= 0 \\ m\ddot{y}_2 + 3Ky_2 &= 0 \end{aligned}}$$

$y_1 = A_1 e^{\pm i\omega_1 t}$

$y_2 = A_2 e^{\pm i\omega_2 t}$

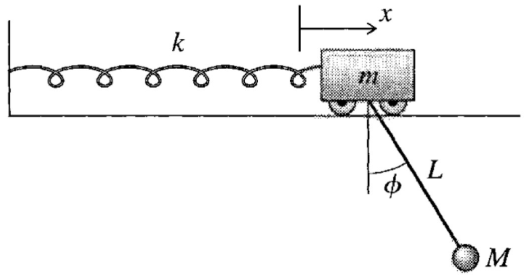
$\omega_1^2 = \frac{K}{m} \qquad \omega_2^2 = \frac{3K}{m}$

$$\boxed{\begin{aligned} x_1(t) &= y_1 + y_2 \\ x_2(t) &= y_1 - y_2 \end{aligned}}$$

See Sec 11.2

Problem 4 [30 Points]

A simple pendulum (mass M and lengh L) is suspended from a cart (mass m) that can oscillate on the end of a spring of force constant k, as shown in the figure below.

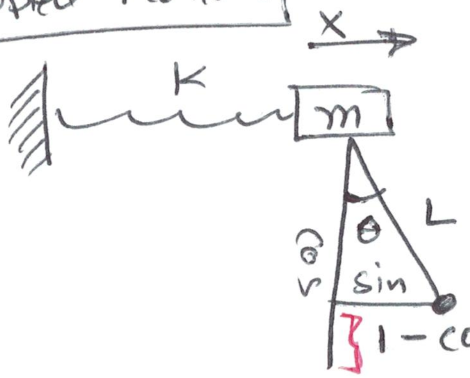


(a) Assuming that the angle ϕ remains small, write down the system's Lagrangian (10 points) and the equations of motion for x and ϕ (5 points).

(b) Assuming that $m = M = L = g = 1$ and $k = 2$ (all in appropriate units) find the normal frequencies (5 points), the normal mode eigenvectors describing the velocities of the two masses undergoing normal oscillations (5 points), and describe the motion of the corresponding normal modes (5 points).

Coupled Motion

2)



$\sin\theta \approx \theta$

$1-\cos\theta \approx \frac{\theta^2}{2}$ ∝ height of M

$$T = \frac{1}{2} m \dot{x}^2 + \frac{1}{2} M (\dot{x} + L\dot{\theta})^2$$

horizontal velocity of M

For small angles, we ignore vertical velocity $\sim \dot{\theta}^4$

$$V = \frac{1}{2} k x^2 + \underbrace{M g L \frac{\theta^2}{2}}_{\text{height of M}}$$

$$\mathcal{L} = T - V$$

$\boxed{10\text{ pts}}$

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{x}}\right) - \frac{\partial \mathcal{L}}{\partial x} = 0 \qquad (m+M)\ddot{x} + LM\ddot{\theta} + kx = 0$$

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{\theta}}\right) - \frac{\partial \mathcal{L}}{\partial \theta} = 0 \qquad LM\ddot{x} + L^2 M\ddot{\theta} + MgL\theta = 0$$

$$T_{matrix} = \begin{pmatrix} m+M & LM \\ LM & L^2 M \end{pmatrix} \qquad V_{matrix} = \begin{pmatrix} k & 0 \\ 0 & gLM \end{pmatrix}$$

$\boxed{5\text{ pts}}$

Eq of Motion $\Rightarrow$ $V_{mat} - T_{mat}\,\omega^2 = 0$

$$\begin{pmatrix} K-(m+M)\omega^2 & -LM\omega^2 \\ -LM\omega^2 & gLM - L^2M\omega^2 \end{pmatrix} = 0$$

Set $m = M = L = g = 1$ and $K = 2$

$$\Rightarrow \begin{pmatrix} 2-2\omega^2 & -\omega^2 \\ -\omega^2 & 1-\omega^2 \end{pmatrix} = 0$$

$$\text{Det} = \omega^4 - 4\omega^2 + 2 = 0$$

$\boxed{\omega^2 = 2 \pm \sqrt{2}}$ Eigen frequencies

5pts

Find Normal Modes

$\omega^2 = 2 + \sqrt{2}$

$$V_{mat} - T_{mat}\,\omega^2 \longrightarrow \begin{pmatrix} \cdots & \cdots \\ -2-\sqrt{2} & -1-\sqrt{2} \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = 0$$

$$\Rightarrow (-2-\sqrt{2})\,a + (-1-\sqrt{2})\,b = 0$$

$$\Rightarrow b = -a\,\frac{(-2-\sqrt{2})}{(-1-\sqrt{2})}\,\frac{(-1+\sqrt{2})}{(-1+\sqrt{2})} = -\sqrt{2}\,a$$

$\therefore$ Eigenvector #1 $= v_1 = (1, -\sqrt{2})$

$\omega_1 = 2 + \sqrt{2} \cong 3.4$

$\omega_2 = 2 - \sqrt{2}$

$$V - T\omega^2 \longrightarrow \begin{pmatrix} \circ\circ\circ & \circ\circ\circ \\ -2+\sqrt{2} & -1+\sqrt{2} \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = 0$$

$$\Rightarrow b = -a\,\frac{(-2+\sqrt{2})}{(-1+\sqrt{2})}\,\frac{(-1-\sqrt{2})}{(-1-\sqrt{2})} = +\sqrt{2}\,a$$

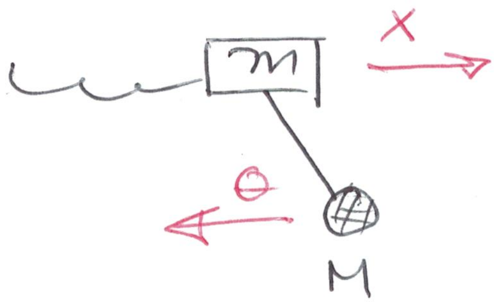
Eigenvector #2 $= v_2 = (1, +\sqrt{2})$

$\omega_2 = 2 - \sqrt{2} \cong 0.6$

$\boxed{5\text{pts}}$

#4

Mode #1: $\omega_1 \sim 3.4 > \omega_2 \sim 0.6$

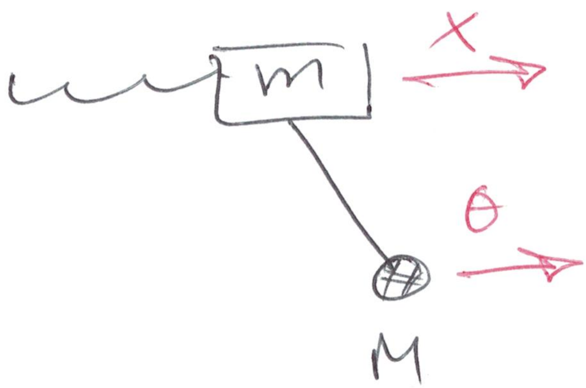


$v_1 = (1, -\sqrt{2})$

Anti-symmetric

- Motion is in opposite directions
- Frequency is LARGE

Mode #2 $\omega_2 \sim 0.6 < \omega_1 \sim 3.4$



$v_2 = (1, +\sqrt{2})$

Symmetric

- Motion is in same direction
- Frequency is small

$\boxed{5\text{pts}}$

Problem 2 [20 Points]

A particle of mass M at rest decays into a particle of mass m and a massless particle. Find

(a) (10 points) the total energy of the massive decay product

(b) (10 points) the momentum of the massless decay product

#2

2

Before:
$$P_0 = (M\ 0\ 0\ 0)$$
$$s = P_0^2 = M^2$$

After
$$P_1 = (E\ 0\ 0\ p)$$
$$P_2 = (p\ 0\ 0\ -p)$$
$$P_{12} = (E+P,\ 0\ 0\ 0)$$
$$s = P_{12}^2 = (E+P)^2$$

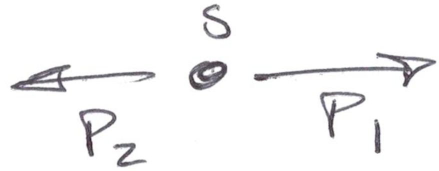
$$E^2 - p^2 = m^2$$

$s = s \Rightarrow$
$$M^2 = (E+P)^2$$
$$M = (E+P)$$
$$P = M - E \qquad E^2 - p^2 = m^2$$

$$E^2 - (M-E)^2 = m^2$$
$$\cancel{E^2} - M^2 + 2ME + \cancel{E^2} = m^2$$
$$E = \left(\frac{M^2 + m^2}{2M}\right) c^2$$

$$\Rightarrow P = M - E = \left(\frac{M^2 - m^2}{2M}\right) c$$

Note General Process



$$P_1 = (E_1 \; 0 \; 0 \; +p) \qquad E_1^2 - p^2 = m_1^2$$

$$P_2 = (E_2 \; 0 \; 0 \; -p) \qquad E_2^2 - p^2 = m_2^2$$

$$E_{\substack{1\\2}} = \frac{s \pm m_1^2 \mp m_2^2}{2\sqrt{s}}$$

$$p = \frac{\Delta(s, m_1^2, m_2^2)}{2\sqrt{s}}$$

$$\Delta(a,b,c) = \sqrt{a^2 + b^2 + c^2 - 2(ab+bc+ca)}$$

Check: $m_2 = 0 \qquad \Delta \to s - m_1^2 \to M^2 - m_1^2$

Let $s = M^2$

$$E_1 = \frac{M^2 + m_1^2}{2M} \qquad p = \frac{M^2 - m_1^2}{2M}$$

$$E_2 = \frac{M^2 - m_1^2}{2M} \equiv p$$

Check!

Greens Functions :

Hard Problem : $\mathbb{D}\ F(x) = \phi(x)$

Easier Problem : $\mathbb{D}\, G(x) = \delta(x)$

or $\mathbb{D}\, G(x-x') = \delta(x-x')$

If we solve easier problem, we can solve hard problem, because :

$$F(x) = \int_{-\infty}^{\infty} G(x-x')\ \phi(x')\, dx'$$

Check :

$$\mathbb{D}\ F(x) = \int_{-\infty}^{\infty} \underbrace{\mathbb{D}\, G(x-x')}_{\delta(x-x')}\ \phi(x')\, dx'$$

$$= \int_{-\infty}^{\infty} dx'\ \delta(x-x')\ \phi(x')$$

$$\mathbb{D}\, F(x) = \phi(x)$$

QED.

Continious Fourier Transform:

$$F(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} d\omega \; g(\omega) \; e^{-i\omega x}$$

$$g(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dx \; F(x) \; e^{+i\omega x}$$

Check:

$$F(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} d\omega \left\{ \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dx' \, F(x') \, e^{i\omega x'} \right\} e^{-i\omega x}$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} dx' \; F(x') \; \underbrace{\int_{-\infty}^{\infty} d\omega \; e^{i\omega(x'-x)}}_{2\pi \; \delta(x'-x)}$$

$$= 1 \cdot \int_{-\infty}^{\infty} dx' \; F(x') \; \delta(x'-x)$$

$$F(x) = F(x)$$

Note:

$$2\pi \; \delta(x-x') = \int d\omega \; e^{i\omega(x-x')}$$

Now, how to solve easy problem:

Use Fourier transforms.

Turns differential eq $\Rightarrow$ algebraic eq.

$\boxed{\text{Ex.}}$ $$(a\, D^2 + bD + c)\, F(x) = \delta(x)$$

use: $$F(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dw\, g(w)\, e^{-iwx}$$

$$\delta(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} dw\, e^{\pm iwx}$$ $\left\{ \begin{array}{l} \text{either} \\ \text{sign} \\ \text{works} \end{array} \right.$

$\therefore$

$$(aD^2 + bD + c)\, \frac{1}{\sqrt{2\pi}} \int dw\, g(w)\, e^{-iwx}$$

$$= \frac{1}{\sqrt{2\pi}} \int dw\, g(w) \left[-w^2 a - iwb + c\right] e^{-iwx}$$

$$\equiv \frac{1}{2\pi} \int dw\, e^{-iwx}$$

$\therefore \Rightarrow$ $$g(w) \left[-w^2 a - iwb + c\right] = \frac{1}{\sqrt{2\pi}}$$

$$g(w) = \frac{1}{\sqrt{2\pi}} \frac{1}{\left[-w^2 a - iwb + c\right]}$$

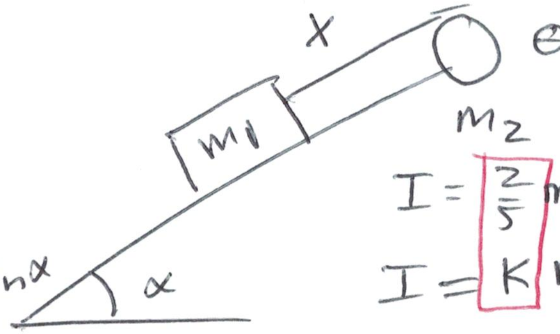
Full solution:

$$F(x) = \frac{1}{\sqrt{2\pi}} \int d\omega \quad g(\omega) \; e^{-i\omega x}$$

$$= \frac{1}{2\pi} \int d\omega \quad \frac{e^{-i\omega x}}{\left[-\omega^2 a - i\omega b + c\right]}$$

Problem #1

$$T = \frac{1}{2} m_1 \dot{x}^2 + \frac{1}{2} I \dot{\theta}^2$$



$$I = \boxed{\frac{2}{5}} m_2 r^2$$

$$I = \boxed{K} m_2 r^2$$

$$V = mgh = -m_1 g x \sin\alpha$$

$$L = T - V = \frac{m_1}{2} \dot{x}^2 + \frac{I}{2} \dot{\theta}^2 + m_1 g x \sin\alpha$$

$$\frac{\partial L}{\partial x} = mg \sin\alpha \qquad \frac{\partial L}{\partial \dot{x}} = m_1 \dot{x} \qquad \frac{\partial L}{\partial \theta} = 0 \qquad \frac{\partial L}{\partial \dot{\theta}} = I \dot{\theta}$$

Constraint: $x = r\theta \Rightarrow f = x - r\theta = 0$

$$\frac{\partial f}{\partial x} = +1 \qquad \frac{\partial f}{\partial \theta} = -r$$

$$\boxed{\frac{d}{dt} \frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial q} = \lambda \frac{\partial f}{\partial q}}$$

$\boxed{x}$ $\quad m_1 \ddot{x} - m_1 g \sin\alpha = \lambda$

$\boxed{\theta}$ $\quad I \ddot{\theta} - 0 = -r\lambda \quad \Rightarrow \quad K m_2 r^2 \frac{\ddot{x}}{r} = -r\lambda$

$\boxed{f}$ $\quad f \Rightarrow \ddot{x} = r\ddot{\theta} \qquad K m_2 \ddot{x} = -\lambda$

$x \Rightarrow$

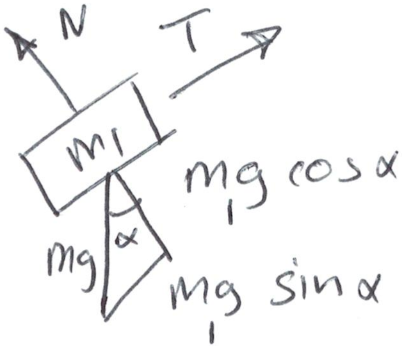
$$m_1 \ddot{x} - m_1 g \sin\alpha = \lambda = -K m_2 \ddot{x}$$

$$\ddot{x}(m_1 + K m_2) = m_1 g \sin\alpha$$

$$\ddot{x} = \frac{m_1 g \sin\alpha}{(m_1 + K m_2)} \qquad K = \frac{2}{5}$$

$$\lambda = -K m_2 \ddot{x} = -\frac{K m_1 m_2 g \sin\alpha}{(m_1 + K m_2)}$$

Force Method



$$F = ma$$

① $m_1 g\sin\alpha - T = m_1 a$

$$\tau = I\alpha$$

$$rT = I\alpha \qquad \alpha = \frac{a}{r}$$

$$rT = K m_2 r^2 \frac{a}{r}$$

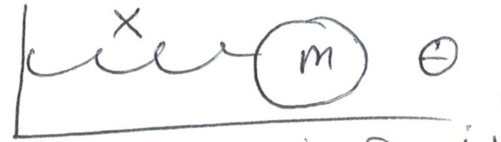
② $T = K m_2 a$

① ② $\rightarrow$ $m_1 g\sin\alpha - K m_2 a = m_1 a$

$$\boxed{\frac{m_1 g\sin\alpha}{m_1 + K m_2} = a}$$

$$T = K m_2 a = K\frac{m_1 m_2 g\sin\alpha}{m_1 + K m_2}$$

Problem #2



$X=0$ is Equilibrium

$$E_{before} = E_{After}$$

$$U = T$$ Kinetic Energy

$\gamma = \frac{1}{2}$ ← my notation

$$\frac{1}{2} K X^2 = \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2$$

$$= \frac{1}{2} m v^2 + \frac{1}{2} \gamma m r^2 \frac{v^2}{r^2}$$

$$\frac{1}{2} K X^2 = \frac{1}{2} m v^2 (1 + \gamma)$$

Translation ← Rotational

(b) Rearrange $L = T - U$

$$T = \frac{1}{2} m \dot{x}^2 + \frac{1}{2} \gamma m \dot{x}^2 \Rightarrow \frac{1}{2} m (1+\gamma) \dot{x}^2$$

$$U = \frac{1}{2} K X^2 \qquad L = T - U$$

$$\frac{\partial L}{\partial X} = -K X \qquad \frac{\partial L}{\partial \dot{X}} = m(1+\gamma)\dot{X}$$

$$\Rightarrow \quad m(1+\gamma)\ddot{X} + K X = 0$$

$$\left[-\omega^2 m (1+\gamma) + K\right] X = 0$$

$$\left[\begin{array}{l} X = A e^{i\omega t} \\ \dot{X} = i\omega X \\ \ddot{X} = -\omega^2 X \end{array}\right.$$

$$\omega^2 = \frac{K}{m(1+\gamma)} \equiv \frac{2K}{3m}$$

$$\gamma = \frac{1}{2} \qquad 1 + \gamma = \frac{3}{2}$$

#3)

(a) $X = vt = (3\times10^{8}\,\frac{m}{s})(2.2\times10^{-6}\,s)$

$X = 660\text{ m}$

(b) $X_1 = 660\text{ m}$ $\quad X_2 = 10{,}000\text{ m}$

$$\gamma = \frac{X_2}{X_1} = \frac{10{,}000}{660} = 15.15$$

(c) $\gamma = \frac{1}{\sqrt{1-\beta^2}}$ $\quad \beta^2 = 1 - \frac{1}{\gamma^2} = 0.9956$

$\beta = 0.9978$

(d) $$\beta_{12} = \frac{\beta_1 + \beta_2}{1 + \beta_1\beta_2} = 0.94 < 1$$

Before

$\boxed{\text{Center of Mass}}$

(e) $P_1 = (E, 0 0 + P)$

$P_2 = (E, 0 0 - P)$

$P_{12} = (2E, 000)$

$S = P_{12}^2 = 4E^2$ initial invariant mass^2

After: work in CMS

$P_3 = (m\ 000)$

$P_4 = (m\ 000)$

$P_5 = (M, 000)$

$m = 1$

$M = 90$

$P_{345} = (2m + M, 000)$

$s = P_{345}^2 = (2m+M)^2 = (2+90)^2$

$s = s$

$4E^2 = (2m+M)^2$

$2E = 2m + M$

$E = \dfrac{2m+M}{2} = \dfrac{2+90}{2} = 46$ GeV

(F) **Lab System**

__Before__

$$P_1 = (E, 0\,0\,P) \longrightarrow E^2 - P^2 = m^2$$

$$P_2 = (M, 0\,0\,0)$$

$$P_{12} = (E+M,\ 0\,0\,P)$$

$$s = P_{12}^2 = (E+M)^2 - P^2$$

$= m^2$

$$= E^2 + 2EM + m^2 - P^2$$

$$s = 2EM + 2m^2$$

$$S_{After} = (2m + M)^2$$ same as before

$$s = S \Rightarrow 2EM + 2m^2 = (2m + M)^2$$

$$E = \frac{(M+2m)^2 - 2m^2}{2m} = 4231 \text{ GeV}$$

Example $pp \to ppp\bar{p}$

Before: CMS

$$P_1 = (E\ 0\ 0\ p)$$
$$P_2 = (E\ 0\ 0\ -p)$$
$$P_{12} = (2E, 0\,0\,0) \qquad P_{12}^2 = s = 4E^2$$

After: $P_{3456} = (4m, 0\,0\,0) \qquad s = 16m^2$

$$s = s \Rightarrow 4E^2 = 16m^2 \Rightarrow E = 2m \sim 2\,\text{GeV}$$

Lab frame

Before:

$$P_1 = (E\ 0\ 0\ p)$$
$$P_2 = (m\ 0\ 0\ 0)$$
$$P_{12} = (E+m, 0\,0\,p)$$
$$s = 2Em + 2m^2$$

$$s = s \Rightarrow 2Em + 2m^2 = 16m^2$$
$$2Em = 14m^2$$
$$E = 7m \mathrel{\hat{=}} 7\,\text{GeV}$$