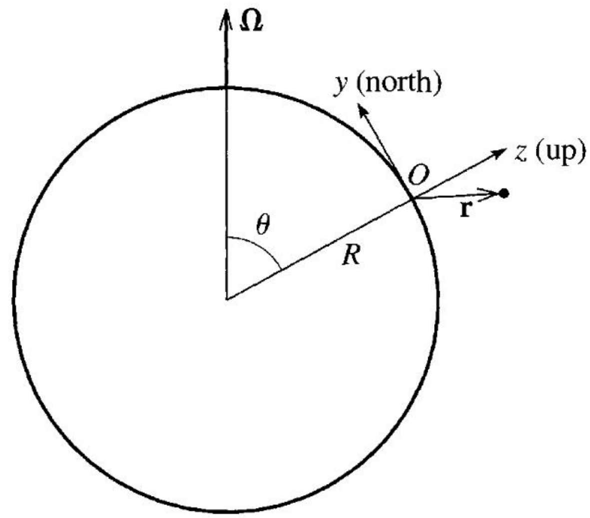
Problem 8: (30 Pts) SpaceX drops a payload from space down to Dallas (θ =57 degrees co-latititude). Assume it's initial motion is in a direct line to the center of the earth, and the velocity is constant v.

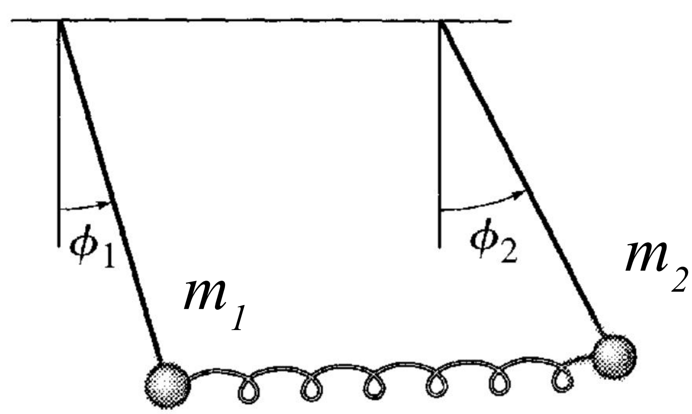


Part a) Compute the magnitude and direction of the Coriolis and centrifugal forces.

Part b) If the object fall at a constant velocity v=100m/s from a height of 10km, find the approximate displacement each due to the Coriolis and centrifugal forces.

Part c) For an object moving in some direction $v_2$near Dallas, is there any v_2 where the direction of the Coriolis and centrifugal forces could be aligned. If so, what is the direction, OR if not, why not?

Problem 9: (30 Points)
The figure shows a system of two coupled pendulums, each of length L and masses m_1 and m_2. Assume the motion is confined to a plane, and the angles ϕ_1 and ϕ_2 are small. The equilibrium position of the spring (with constant k) is for $\phi_1 = \phi_2 = 0$.
(Note, you can ignore gravity.)

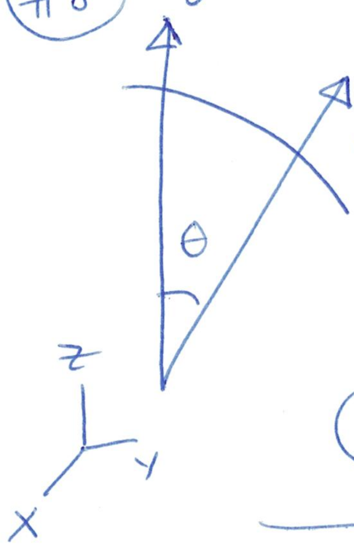


Part a: Write down the total kinetic energy T, the potential energy V, and the Lagrangian L.

Part b: Now, let's set m_1=m_2=m and compute the Lagrange equations of motion.

Part c: Find and describe the normal modes.

#8



$\swarrow v$

Coriolis: $2m\,v \times \Omega$ $\otimes$ F_{cor} East

$F_{cor} = 2mv\Omega \sin\theta$

Centrifugal: $m(\Omega \times r) \times \Omega$ $\xrightarrow{F_{cf}}$ Y direction

$= m\Omega^2 r \sin\theta$

(b) $v = 100 \frac{m}{s}$ $X = 10^4 m$ $t = 100$ sec

Displacement East due to Coriolis Force

$F_{cor} = 2mv\Omega \sin\theta$

$F = ma$

$a = \frac{F}{m}$

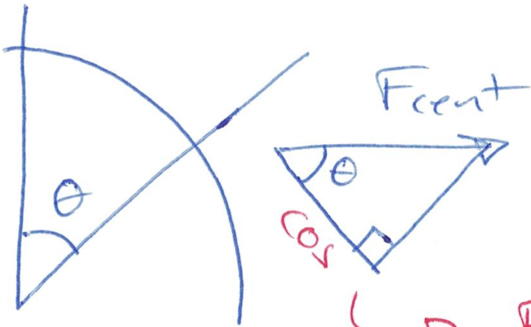
$X = X_0 + v_0 t + \frac{1}{2}at^2$

$\quad \hookrightarrow 0 \;\; \hookrightarrow 0$

$X = \frac{1}{2}(2v\Omega \sin\theta)t^2$

$= 60$ meters

Displacement South due to Centrifugal



↳ $F_{cent} \cos\theta$ deflects to South

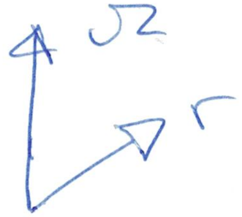
$$F_{south} = [m\Omega^2 r \sin\theta] \cos\theta$$

$$a = \frac{F_{south}}{m} = \Omega^2 r \sin\theta \cos\theta$$

$$X = \frac{1}{2} a t^2 = \frac{1}{2}[\Omega^2 r \sin\theta \cos\theta] t^2$$

$= 77$ meters

(c)



$m(\Omega \times r) \times \Omega \Rightarrow \xrightarrow{F_{cent.}}$

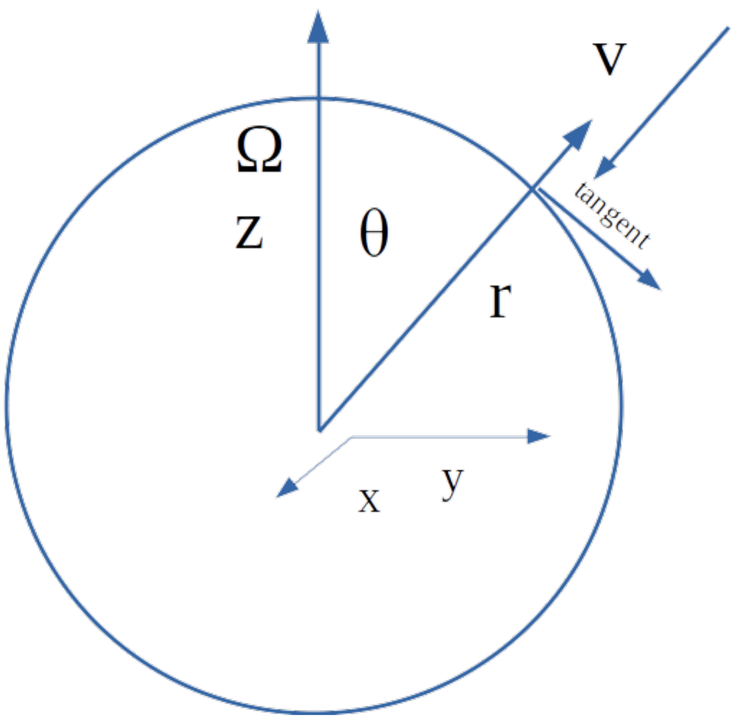
IF $v = \otimes$

$F_{cor} = 2mv \times \Omega = \xrightarrow{F_{cor}}$

Forces Align

Problem #8)Non-Intertial Frames

See handwritten for full solution



In[117]:=
```
Ω = Ω0 {0, 0, 1};
r = r0 {0, Sin[θ], Cos[θ]};
v = v0 {0, -Sin[θ], -Cos[θ]};
```

In[120]:=
```
values = {θ → 57 Degree, v0 -> 100, t → 100, Ω0 → 2π/(24 × 60 × 60), r0 → 6400 × 10^3};
```

In[121]:=
```
(* Direction :east:   -x direction *)
coriolis = 2 m Cross[v, Ω]
```

Out[121]= {-2 m v0 Ω0 Sin[θ], 0, 0}

In[122]:=
```
(* East is negative x axis *)
eastForce = {-1, 0, 0}.coriolis
```

Out[122]= 2 m v0 Ω0 Sin[θ]

In[123]:=
```
(* Direction :   +y direction *)
centrifugal = m Cross[Cross[Ω, r], Ω]
```

Out[123]= {0, m r0 Ω0^2 Sin[θ], 0}

In[124]:=
```
(* Project component of centrifugal tangent to the earth's surface *)
```

In[125]:=
```
tangent = {0, Cos[θ], -Sin[θ]};
(* Check it is perpendicular to v (and r) and points south *)
{tangent.v, tangent.r}
```
Out[126]= {0, 0}

In[127]:=
```
southForce = centrifugal.tangent
```
Out[127]= $m\, r0\, \Omega 0^2 \,Cos[\theta]\, Sin[\theta]$

In[128]:=
```
eastAcc = eastForce/m
```
Out[128]= $2\, v0\, \Omega 0\, Sin[\theta]$

In[129]:=
```
southAcc = southForce/m
```
Out[129]= $r0\, \Omega 0^2\, Cos[\theta]\, Sin[\theta]$

In[130]:=
```
(* x=x0+v0 t + 1/2 at^2*)
southDisplacement = 1/2 southAcc t^2
```
Out[130]= $\frac{1}{2} r0\, t^2\, \Omega 0^2\, Cos[\theta]\, Sin[\theta]$

In[131]:=
```
southDisplacement //. values // N
```
Out[131]= 77.3005

In[132]:=
```
(* x=x0+v0 t + 1/2 at^2*)
eastDisplacement = 1/2 eastAcc t^2
```
Out[132]= $t^2\, v0\, \Omega 0\, Sin[\theta]$

In[133]:=
```
eastDisplacement //. values // N
```
Out[133]= 60.9898

Part c: let v be East (negative x direction)

In[134]:=
```
Ω = Ω0 {0, 0, 1};
r = r0 {0, Sin[θ], Cos[θ]};
v = v0 {-1, 0, 0};
```

In[137]:=
```
(* Direction : +y direction *)
coriolis = 2 m Cross[v, Ω]
```
Out[137]= {0, 2 m v0 Ω0, 0}

In[138]:=
```
(* Direction :   +y direction *)
centrifugal = m Cross[Cross[Ω, r], Ω]
```
Out[138]= $\{0,\ m\ r0\ \Omega0^2\ \mathrm{Sin}[\theta],\ 0\}$

Problem #9) Without gravity

In[139]:=
```
Clear["Global`*"]
```

In[140]:= $T = \frac{1}{2} m1\ r^2\ \phi1\ '[t]^2 + \frac{1}{2} m2\ r^2\ \phi2\ '[t]^2;$

In[141]:= $V = \frac{1}{2} k\ (r\ \phi1[t] - r\ \phi2[t])^2;$

In[142]:=
```
lag = T - V
```
Out[142]= $-\frac{1}{2} k\ (r\ \phi1[t] - r\ \phi2[t])^2 + \frac{1}{2} m1\ r^2\ \phi1'[t]^2 + \frac{1}{2} m2\ r^2\ \phi2'[t]^2$

In[143]:=
```
D[D[lag, ϕ1 '[t]], t] - D[lag, ϕ1[t]]  // Expand
```
Out[143]= $k\ r^2\ \phi1[t] - k\ r^2\ \phi2[t] + m1\ r^2\ \phi1''[t]$

In[144]:=
```
D[D[lag, ϕ2 '[t]], t] - D[lag, ϕ2[t]] // Expand
```
Out[144]= $-k\ r^2\ \phi1[t] + k\ r^2\ \phi2[t] + m2\ r^2\ \phi2''[t]$

In[145]:= $Tmat = \{\{m1\ r^2, 0\}, \{0, m2\ r^2\}\};$
```
Tmat // MatrixForm
```
Out[146]//MatrixForm=

$$\begin{pmatrix} m1\ r^2 & 0 \\ 0 & m2\ r^2 \end{pmatrix}$$

In[147]:= $Vmat = k\ r^2\ \{\{1, -1\}, \{-1, 1\}\};$
```
Vmat // MatrixForm
```
Out[148]//MatrixForm=

$$\begin{pmatrix} k\ r^2 & -k\ r^2 \\ -k\ r^2 & k\ r^2 \end{pmatrix}$$

In[149]:=
```
mat = Vmat - Tmat ω2;
mat // MatrixForm
```
Out[150]//MatrixForm=

$$\begin{pmatrix} k\ r^2 - m1\ r^2\ \omega2 & -k\ r^2 \\ -k\ r^2 & k\ r^2 - m2\ r^2\ \omega2 \end{pmatrix}$$