Problem 1:

a) Given $\phi = -(x^2+y^2+z^2)$, Compute: a) $\nabla\phi$, b) $\nabla \bullet \nabla\phi$, c) $\nabla \times \nabla\phi$,

b) Given $\vec{V} = \{x, 0, 0\}$ Compute: a) $\nabla \bullet \vec{V}$, b) $\nabla(\nabla \bullet \vec{V})$,

c) Given $\vec{V} = \{y, -x, 0\}$
Compute: a) $\nabla \times \vec{V}$, b) $\nabla \bullet (\nabla \times \vec{V})$, c) $\nabla(\nabla \times \vec{V})$, d) $\nabla \times (\nabla \times \vec{V})$,

Problem 2:

We found that the complex square root function ($\sqrt{z}$) has a discontinuity along the negative axis (with the conventional definition). This is also the case for the n-th root ($z^{1/n}$).

a) Let's start with the cube root ($z^{1/3}$).
Plot this in Mathematica similar to the displayed plot.

b) Now make a 2D plot showing the discontinuity for the cube root across the negative axis, similar to the plot displayed.

c) In general, compute the discontinuity for the n-th root ($z^{1/n}$).

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Plot3D[Im[Sqrt[x + I y]],{x,-4,4},{y,-4,4}]
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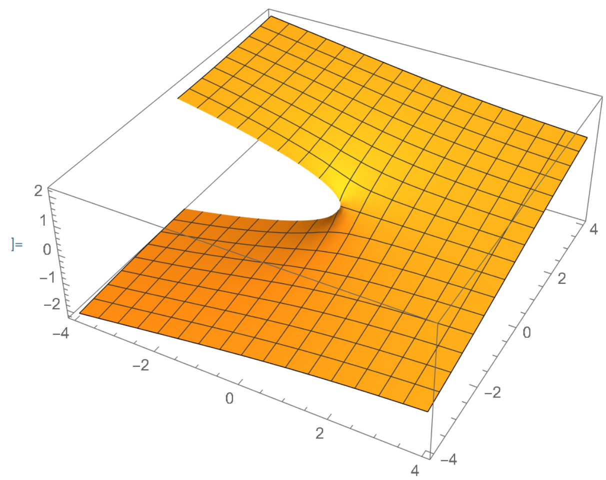

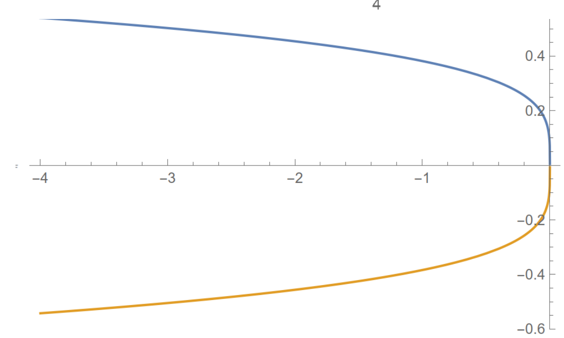

Problem 3:

The below matrix represents a rotation about some axis.
a) Demonstrate this is an orthogonal matrix.
b) Find the axis of rotation.
c) Find the rotation angle.
Hint: Careful, some of the eigenvalues and vectors are complicated.

$$\begin{pmatrix} 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} \\ 1 & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix}$$

Problem 4:
Consider the matrix M.
a) Find the eigenvalues. (Hint: there are degenerate values.)
b) Find the eigenvectors. (Hint: there are degenerate values.)
c) Normalize the eigenvectors and show that $V^T.V=1$.
d) Show that $V^T.M.V$ diagonalizes M.
(Hint: you may need to order the eigenvectors carefully.)

$$\begin{pmatrix} 3 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 1 & 2 \end{pmatrix}$$

Problem 5:
In 3D space there are 3 rotations, and they will rotate a vector v={x,y,z}.
There are 3 Pauli matrices in 2D complex space, and these will rotate a 2D complex vector.

a) For {x,y,z} rotations, show that $R_z^\dagger \vec{v} R_z$ rotates the vector v about the z axis.
Note: the dagger symbol represents the Conjugate Transpose!!!

b,c) Repeat for x and y.

Note: technically, $zrot = e^{i\theta/2\sigma_k} \simeq \mathbb{I} + \frac{i\theta}{2}\sigma_k + ...$

$$\sigma_1 = \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

$$\text{vec1} == \begin{pmatrix} z & x - i\, y \\ x + i\, y & -z \end{pmatrix}$$

$$\text{zrot} = \text{s0} + \frac{I\,\theta}{2}\,\text{s3}$$

$$\begin{pmatrix} 1 + \frac{i\,\theta}{2} & 0 \\ 0 & 1 - \frac{i\,\theta}{2} \end{pmatrix}$$