4321 Final Exam

Take-Home portion

Due: Monday 12 May at 11:30am (the start of the in-class portion)

Rules: Open book, open notes, closed neighbor.

WEB USAGE: I prefer you work these out on your own without the web, but if you find something similar in your materials, be sure this matches the details of our version, and I need to see all the intermediate steps.

IMPORTANT: SUBMITTING YOUR EXAM

I will need a 1) hand written portion and 2) a Mathematica portion.

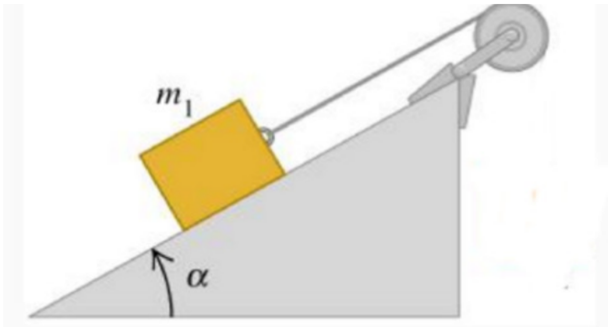
1. Please submit the hand written portion
    1. SINGLE SIDED, ONE PROBLEM Per PAGE.
       (I will scan this)
    2. This must be reasonable size hand-writting.
       (Most homeworks are too small and hard to read)

2. For the Mathematica portion, please upload a notebook file that I can use to reproduce your results.

In-Class portion

This will be taken from homework and exam problems (including the take home portion). These will be relatively simple items (I put the longer problems on the take-home portion), but I'll be spot-checking key concepts.

PROBLEM #1:



Consider an Atwood machine with mass m_1 on a frictionless ramp with a **MASSIVE** pulley of mass m_2 and moment of inertia of $I = (2/5)m_2 r^2$. The string is wound around the pulley and does not slip. For coordinates, measure the distance of m_1 along the ramp to be x, and the rotation of the pulley to be θ.

(a) (5 Points) Compute the Lagrangian L, and obtain the associated equations of motion in terms of $\{x, x', \theta, \theta'\}$ using a Lagrange multiplier λ.

(b) (5 Points) Find the acceleration of the m_1. Also solve the Lagrange multiplier λ and compare this to the tension T in the string.

PROBLEM #2:

a) The Electron Ion Collider (EIC) is designed with an electron beam energy of 18 GeV and a proton energy of 275 GeV. Compute the total mass-energy ($\sqrt{s}$) available to produce final state particles.

b) The Z boson is a neutral particle with a mass of 90 GeV. (For reference, in these units the proton mass is 1 GeV). If we create a Z boson in the process: pp → ppZ , what is the minimum energy required of each initial-state proton if we collide them in the center-of-mass frame? (Don't forget to include the 2 protons in the final state.)

c) Repeat the above, but now do it in the frame where one proton is at rest; what is the minimum energy of the initial-state moving proton required to create the Z boson?

PROBLEM #3:

a) Starting from Maxwell's equations in a vacuum, use the proper vector formulas to show this give rise to electromagnetic waves. b) Compute the speed of the waves in terms of fundamental constants, and check that the units are correct. c) Write an expression for the solution of E and B as a function of time using the angular frequency w and the wave number k, and identify the relations between {c,w,k}.

PROBLEM #4:

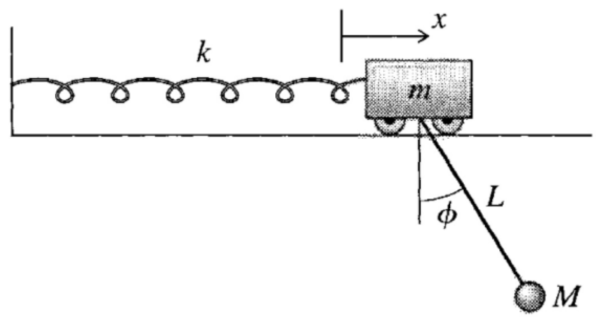
a) Compute the convolution of two Gaussian functions, one of width a, and the other of width b.
Find the result, and compare the width of this to the initial functions.

PROBLEM #5:

a) For the function x^3, compute the Fourier Series on the interval $[-\pi,\pi]$.
b) Plot this on the interval and compare to x^3 to verify it is correct.
c) Plot this on the interval $[-3\pi,3\pi]$ to show it is correctly periodic.

PROBLEM #6:

A simple pendulum (mass M and lengh L) is suspended from a cart (mass m) that can oscillate on the end of a spring of force constant k, as shown in the figure below.



(a) Assuming that the angle φ remains <u>small</u>, write down the system's Lagrangian and the equations of motion for x and φ.

(b) Assuming that m = M = L = g = 1 and k = 2 (all in appropriate units) find the normal frequencies, the normal mode eigenvectors describing the velocities of the two masses undergoing normal oscillations, and describe the motion of the corresponding normal modes.

PROBLEM #7:

Design a RLC band-pass filter to tune to frequency a channel of the 2.4 GHz WiFi spectrum GHz (802.11b/g/n/ax/be).

Please use the following channels: Damondre Channel 1: Luke Channel 6; Josh Channel 11.

a) Choose a reasonable value for L and C. Choose your R so that the signal (current) is equal to 10% of the peak value at the edge of the frequency range of ±11 MHz. (See table.)

b) Plot this including grid-lines so we can verify graphically this satisfies the condition.

Note: There are 14 channels are designated in the 2.4 GHz range, spaced 5 MHz apart from each other except for a 12 MHz space before channel 14.

		D
#	F_0 (MHz)	Frequency range (MHz)
1	2412	2401–2423
2	2417	2406–2428
3	2422	2411–2433
4	2427	2416–2438
5	2432	2421–2443
6	2437	2426–2448
7	2442	2431–2453
8	2447	2436–2458
9	2452	2441–2463
10	2457	2446–2468
11	2462	2451–2473
12	2467	2456–2478
13	2472	2461–2483
14	2484	2473–2495