Boas : Ch3 Sec 11 MOD

In[∘]:=
```
Clear["Global`*"]
```

Follow example

In[∘]:=
```
mat = {{3, -1}, {-1, 3}};
Eigensystem[mat]
```

Out[∘]= {{4, 2}, {{-1, 1}, {1, 1}}}

In[∘]:=
```
mat // MatrixForm
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} 3 & -1 \\ -1 & 3 \end{pmatrix}$$

In[∘]:=
```
lam = λ DiagonalMatrix[{1, 1}];
lam // MatrixForm
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix}$$

In[∘]:=
```
mat - lam // MatrixForm
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} 3-\lambda & -1 \\ -1 & 3-\lambda \end{pmatrix}$$

In[∘]:=
```
eq0 = mat - lam // Det
```

Out[∘]= $8 - 6\lambda + \lambda^2$

In[∘]:=
```
Solve[eq0 == 0, λ]
```

Out[∘]= {{λ → 2}, {λ → 4}}

Find Eigenvectors: λ ={2,4}

In[∘]:=
```
vec = {x, y};
eq1 = mat.vec == λ vec
```

Out[∘]= {3 x - y, -x + 3 y} == {x λ, y λ}

In[∘]:=
```
eq2 = eq1 // Thread
```

Out[∘]= {3 x - y == x λ, -x + 3 y == y λ}

In[∘]:=
```
Solve[eq2 /. {λ → 2}, {x, y}]
```

Solve: Equations may not give solutions for all "solve" variables.

Out[∘]= {{y → x}}

In[∘]:=
```
eq3 = Join[eq2, {x^2 + y^2 == 1}]
```
Out[∘]= $\{3x - y == x\lambda, -x + 3y == y\lambda, x^2 + y^2 == 1\}$

In[∘]:=
```
Solve[eq3 /. {λ → 2}, {x, y}]
```
Out[∘]= $\left\{\left\{x \to -\frac{1}{\sqrt{2}}, y \to -\frac{1}{\sqrt{2}}\right\}, \left\{x \to \frac{1}{\sqrt{2}}, y \to \frac{1}{\sqrt{2}}\right\}\right\}$

In[∘]:=
```
Solve[eq3 /. {λ → 4}, {x, y}]
```
Out[∘]= $\left\{\left\{x \to -\frac{1}{\sqrt{2}}, y \to \frac{1}{\sqrt{2}}\right\}, \left\{x \to \frac{1}{\sqrt{2}}, y \to -\frac{1}{\sqrt{2}}\right\}\right\}$

Examine

In[∘]:=
```
eval = Eigenvalues[mat]
```
Out[∘]= {4, 2}

In[∘]:=
```
evec = Eigenvectors[mat]
```
Out[∘]= {{-1, 1}, {1, 1}}

In[∘]:=
```
mat.evec[[1]] // Simplify
```
Out[∘]= {-4, 4}

In[∘]:=
```
mat.evec[[1]] == 4 evec[[1]] // Simplify
```
Out[∘]= True

In[∘]:=
```
mat.evec[[2]] // Simplify
```
Out[∘]= {2, 2}

In[∘]:=
```
mat.evec[[2]] == 2 evec[[2]] // Simplify
```
Out[∘]= True

In[∘]:=
```
evec = Normalize /@ evec
```
Out[∘]= $\left\{\left\{-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right\}, \left\{\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right\}\right\}$

In[∘]:=
```
Transpose[evec].mat.evec // Simplify // MatrixForm
```
Out[∘]//MatrixForm=

$$\begin{pmatrix} 4 & 0 \\ 0 & 2 \end{pmatrix}$$

Boas : Ch3 Sec 11

In[∘]:=
```
Clear["Global`*"]
```

Follow example

In[∘]:=
```
mat = {{5, -2}, {-2, 2}};
mat // MatrixForm
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} 5 & -2 \\ -2 & 2 \end{pmatrix}$$

In[∘]:=
```
lam = λ DiagonalMatrix[{1, 1}];
lam // MatrixForm
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix}$$

In[∘]:=
```
mat - lam // MatrixForm
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} 5-\lambda & -2 \\ -2 & 2-\lambda \end{pmatrix}$$

In[∘]:=
```
eq0 = mat - lam // Det
```

Out[∘]= $6 - 7\lambda + \lambda^2$

In[∘]:=
```
Solve[eq0 == 0, λ]
```

Out[∘]= {{λ → 1}, {λ → 6}}

Find Eigenvectors: λ ={6,1}

In[∘]:=
```
vec = {x, y};
eq1 = mat.vec == λ vec
```

Out[∘]= $\{5x - 2y, -2x + 2y\} == \{x\lambda, y\lambda\}$

In[∘]:=
```
eq2 = eq1 // Thread
```

Out[∘]= $\{5x - 2y == x\lambda, -2x + 2y == y\lambda\}$

In[∘]:=
```
Solve[eq2 /. {λ → 6}, {x, y}]
```

Solve: Equations may not give solutions for all "solve" variables.

Out[∘]= $\{\{y \to -\frac{x}{2}\}\}$

In[∘]:=
```
eq3 = Join[eq2, {x^2 + y^2 == 1}]
```

Out[∘]= $\{5x - 2y == x\lambda, -2x + 2y == y\lambda, x^2 + y^2 == 1\}$

In[∘]:= `Solve[eq3 /. {λ → 6}, {x, y}]`

Out[∘]= $\left\{\left\{x \to -\frac{2}{\sqrt{5}},\ y \to \frac{1}{\sqrt{5}}\right\},\ \left\{x \to \frac{2}{\sqrt{5}},\ y \to -\frac{1}{\sqrt{5}}\right\}\right\}$

In[∘]:= `Solve[eq3 /. {λ → 1}, {x, y}]`

Out[∘]= $\left\{\left\{x \to -\frac{1}{\sqrt{5}},\ y \to -\frac{2}{\sqrt{5}}\right\},\ \left\{x \to \frac{1}{\sqrt{5}},\ y \to \frac{2}{\sqrt{5}}\right\}\right\}$

Examine

In[∘]:= `eval = Eigenvalues[mat]`

Out[∘]= {6, 1}

In[∘]:= `evec = Eigenvectors[mat]`

Out[∘]= {{-2, 1}, {1, 2}}

In[∘]:= `mat.evec[[1]] // Simplify`

Out[∘]= {-12, 6}

In[∘]:= `mat.evec[[2]] // Simplify`

Out[∘]= {1, 2}

In[∘]:= `evec = Normalize /@ evec`

Out[∘]= $\left\{\left\{-\frac{2}{\sqrt{5}},\ \frac{1}{\sqrt{5}}\right\},\ \left\{\frac{1}{\sqrt{5}},\ \frac{2}{\sqrt{5}}\right\}\right\}$

In[∘]:= `Transpose[evec].mat.evec // Simplify // MatrixForm`

Out[∘]//MatrixForm=

$$\begin{pmatrix} 6 & 0 \\ 0 & 1 \end{pmatrix}$$

Boas : Ch3 Example 3 p.165

In[∘]:= `Clear["Global`*"]`

Follow example

In[∘]:= pot = $\frac{1}{2} k\, x^2 + \frac{1}{2} k\, (x - y)^2 + \frac{1}{2} k\, y^2$ // Simplify

Out[∘]= $k\left(x^2 - x\, y + y^2\right)$

In[∘]:=
```
D[pot, x]
D[pot, y]
```

Out[∘]= $k\left(2x - y\right)$

Out[∘]= $k\left(-x + 2y\right)$

In[∘]:=
```
mat = {{2, -1}, {-1, 2}};
mat // MatrixForm
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix}$$

In[∘]:=
```
lam = λ DiagonalMatrix[{1, 1}];
lam // MatrixForm
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix}$$

In[∘]:=
```
mat - lam // MatrixForm
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} 2-\lambda & -1 \\ -1 & 2-\lambda \end{pmatrix}$$

In[∘]:=
```
eq0 = mat - lam // Det
```

Out[∘]= $3 - 4\lambda + \lambda^2$

In[∘]:=
```
Solve[eq0 == 0, λ]
```

Out[∘]= $\{\{\lambda \to 1\}, \{\lambda \to 3\}\}$

Find Eigenvectors: λ ={1,3}

In[∘]:=
```
vec = {x, y};
eq1 = mat.vec == λ vec
```

Out[∘]= $\{2x - y, -x + 2y\} == \{x\lambda, y\lambda\}$

In[∘]:=
```
eq2 = eq1 // Thread
```

Out[∘]= $\{2x - y == x\lambda, -x + 2y == y\lambda\}$

In[∘]:=
```
Solve[eq2 /. {λ → 1}, {x, y}]
```

Out[∘]= $\{\{y \to x\}\}$

In[∘]:=
```
eq3 = Join[eq2, {x^2 + y^2 == 1}]
```

Out[∘]= $\{2x - y == x\lambda, -x + 2y == y\lambda, x^2 + y^2 == 1\}$

In[∘]:= `Solve[eq3 /. {λ → 1}, {x, y}]`

Out[∘]= $\left\{\left\{x \to -\frac{1}{\sqrt{2}}, y \to -\frac{1}{\sqrt{2}}\right\}, \left\{x \to \frac{1}{\sqrt{2}}, y \to \frac{1}{\sqrt{2}}\right\}\right\}$

In[∘]:= `Solve[eq3 /. {λ → 3}, {x, y}]`

Out[∘]= $\left\{\left\{x \to -\frac{1}{\sqrt{2}}, y \to \frac{1}{\sqrt{2}}\right\}, \left\{x \to \frac{1}{\sqrt{2}}, y \to -\frac{1}{\sqrt{2}}\right\}\right\}$

Examine

In[∘]:= `eval = Eigenvalues[mat]`

Out[∘]= {3, 1}

In[∘]:= `evec = Eigenvectors[mat]`

Out[∘]= {{-1, 1}, {1, 1}}

In[∘]:= `mat.evec[[1]] // Simplify`

Out[∘]= {-3, 3}

In[∘]:= `mat.evec[[1]] == 3 evec[[1]] // Simplify`

Out[∘]= True

In[∘]:= `mat.evec[[2]] // Simplify`

Out[∘]= {1, 1}

In[∘]:= `mat.evec[[2]] == 1 evec[[2]] // Simplify`

Out[∘]= True

In[∘]:= `evec = Normalize /@ evec`

Out[∘]= $\left\{\left\{-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right\}, \left\{\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right\}\right\}$

In[∘]:= `Transpose[evec].mat.evec // Simplify // MatrixForm`

Out[∘]//MatrixForm=

$$\begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix}$$

Change coordinates

In[∘]:= `eq1 =` $\left\{ z1 == \frac{x+y}{\sqrt{2}}, \; z2 == \frac{x-y}{\sqrt{2}} \right\}$

Out[∘]= $\left\{ z1 == \frac{x+y}{\sqrt{2}}, z2 == \frac{x-y}{\sqrt{2}} \right\}$

In[∘]:=
```
sol = Solve[eq1, {x, y}][[1]] // Simplify
```

Out[∘]= $\left\{x \to \frac{z1 + z2}{\sqrt{2}},\ y \to \frac{z1 - z2}{\sqrt{2}}\right\}$

In[∘]:=
```
pot
```

Out[∘]= $k\left(x^2 - x\,y + y^2\right)$

In[∘]:=
```
pot /. sol // Simplify
```

Out[∘]= $\frac{1}{2}\,k\left(z1^2 + 3\,z2^2\right)$