
Boas : Ch3 Sec 11 MOD

```
In[1]:= Clear["Global`*"]
```

Follow example

```
In[2]:= mat = {{3, -1}, {-1, 3}};  
Eigensystem[mat]
```

```
Out[2]= {{4, 2}, {-1, 1}, {1, 1}}
```

```
In[3]:= mat // MatrixForm
```

```
Out[3]//MatrixForm=
```

$$\begin{pmatrix} 3 & -1 \\ -1 & 3 \end{pmatrix}$$

```
In[4]:= lam = \lambda DiagonalMatrix[{1, 1}];  
lam // MatrixForm
```

```
Out[4]//MatrixForm=
```

$$\begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix}$$

```
In[5]:= mat - lam // MatrixForm
```

```
Out[5]//MatrixForm=
```

$$\begin{pmatrix} 3 - \lambda & -1 \\ -1 & 3 - \lambda \end{pmatrix}$$

```
In[6]:= eq0 = mat - lam // Det
```

```
Out[6]= 8 - 6 \lambda + \lambda^2
```

```
In[7]:= Solve[eq0 == 0, \lambda]
```

```
Out[7]= \{\{\lambda \rightarrow 2\}, \{\lambda \rightarrow 4\}\}
```

Find Eigenvectors: $\lambda = \{2, 4\}$

```
In[8]:= vec = {x, y};  
eq1 = mat.vec == \lambda vec
```

```
Out[8]= \{3 x - y, -x + 3 y\} == \{x \lambda, y \lambda\}
```

```
In[9]:= eq2 = eq1 // Thread
```

```
Out[9]= \{3 x - y == x \lambda, -x + 3 y == y \lambda\}
```

```
In[10]:= Solve[eq2 /. {\lambda \rightarrow 2}, {x, y}]
```

... **Solve**: Equations may not give solutions for all "solve" variables. [i](#)

```
Out[10]= \{y \rightarrow x\}
```

```
In[]:= eq3 = Join[eq2, {x^2 + y^2 == 1}]
Out[=] {3 x - y == x λ, -x + 3 y == y λ, x^2 + y^2 == 1}

In[]:= Solve[eq3 /. {λ → 2}, {x, y}]
Out[=] {{x → -1/Sqrt[2], y → -1/Sqrt[2]}, {x → 1/Sqrt[2], y → 1/Sqrt[2]}}

In[]:= Solve[eq3 /. {λ → 4}, {x, y}]
Out[=] {{x → -1/Sqrt[2], y → 1/Sqrt[2]}, {x → 1/Sqrt[2], y → -1/Sqrt[2]}}
```

Examine

```
In[]:= eval = Eigenvalues[mat]
Out[=] {4, 2}

In[]:= evec = Eigenvectors[mat]
Out[=] {{-1, 1}, {1, 1}}

In[]:= mat.evec[[1]] // Simplify
Out[=] {-4, 4}

In[]:= mat.evec[[1]] == 4 evec[[1]] // Simplify
Out[=] True

In[]:= mat.evec[[2]] // Simplify
Out[=] {2, 2}

In[]:= mat.evec[[2]] == 2 evec[[2]] // Simplify
Out[=] True

In[]:= evec = Normalize /@ evec
Out[=] {{-1/Sqrt[2], 1/Sqrt[2]}, {1/Sqrt[2], 1/Sqrt[2]}}

In[]:= Transpose[evec].mat.evec // Simplify // MatrixForm
Out[=] //MatrixForm=

$$\begin{pmatrix} 4 & 0 \\ 0 & 2 \end{pmatrix}$$

```

Boas : Ch3 Sec 11

```
In[]:= Clear["Global`*"]
```

Follow example

```
In[1]:= mat = {{5, -2}, {-2, 2}};
mat // MatrixForm

Out[1]//MatrixForm=

$$\begin{pmatrix} 5 & -2 \\ -2 & 2 \end{pmatrix}$$


In[2]:= lam = \lambda DiagonalMatrix[{1, 1}];
lam // MatrixForm

Out[2]//MatrixForm=

$$\begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix}$$


In[3]:= mat - lam // MatrixForm

Out[3]//MatrixForm=

$$\begin{pmatrix} 5 - \lambda & -2 \\ -2 & 2 - \lambda \end{pmatrix}$$


In[4]:= eq0 = mat - lam // Det
Out[4]= 6 - 7 \lambda + \lambda^2

In[5]:= Solve[eq0 == 0, \lambda]
Out[5]= \{\{\lambda \rightarrow 1\}, \{\lambda \rightarrow 6\}\}
```

Find Eigenvectors: $\lambda = \{6, 1\}$

```
In[1]:= vec = {x, y};
eq1 = mat.vec == \lambda vec

Out[1]= \{5 x - 2 y, -2 x + 2 y\} == \{x \lambda, y \lambda\}

In[2]:= eq2 = eq1 // Thread
Out[2]= \{5 x - 2 y == x \lambda, -2 x + 2 y == y \lambda\}

In[3]:= Solve[eq2 /. {\lambda \rightarrow 6}, {x, y}]
Out[3]= \{y \rightarrow -\frac{x}{2}\}

In[4]:= eq3 = Join[eq2, {x^2 + y^2 == 1}]
Out[4]= \{5 x - 2 y == x \lambda, -2 x + 2 y == y \lambda, x^2 + y^2 == 1\}
```

... Solve: Equations may not give solutions for all "solve" variables. [i](#)

```
In[1]:= Solve[eq3 /. {λ → 6}, {x, y}]
Out[1]= {{x → -2/Sqrt[5], y → 1/Sqrt[5]}, {x → 2/Sqrt[5], y → -1/Sqrt[5]}}
In[2]:= Solve[eq3 /. {λ → 1}, {x, y}]
Out[2]= {{x → -1/Sqrt[5], y → -2/Sqrt[5]}, {x → 1/Sqrt[5], y → 2/Sqrt[5]}}
```

Examine

```
In[1]:= eval = Eigenvalues[mat]
Out[1]= {6, 1}

In[2]:= evec = Eigenvectors[mat]
Out[2]= {{-2, 1}, {1, 2}}

In[3]:= mat.evec[[1]] // Simplify
Out[3]= {-12, 6}

In[4]:= mat.evec[[2]] // Simplify
Out[4]= {1, 2}

In[5]:= evec = Normalize /@ evec
Out[5]= {{-2/Sqrt[5], 1/Sqrt[5]}, {1/Sqrt[5], 2/Sqrt[5]}}
In[6]:= Transpose[evec].mat.evec // Simplify // MatrixForm
Out[6]//MatrixForm=

$$\begin{pmatrix} 6 & 0 \\ 0 & 1 \end{pmatrix}$$

```

Boas : Ch3 Example 3 p.165

```
In[1]:= Clear["Global`*"]
```

Follow example

```
In[1]:= pot = 1/2 k x^2 + 1/2 k (x - y)^2 + 1/2 k y^2 // Simplify
Out[1]= k (x^2 - x y + y^2)
```

```

In[]:= D[pot, x]
D[pot, y]

Out[=] k (2 x - y)

Out[=] k (-x + 2 y)

In[=] mat = {{2, -1}, {-1, 2}};
mat // MatrixForm

Out[=]//MatrixForm=

$$\begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix}$$


In[=] lam = \[Lambda] DiagonalMatrix[{1, 1}];
lam // MatrixForm

Out[=]//MatrixForm=

$$\begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix}$$


In[=] mat - lam // MatrixForm

Out[=]//MatrixForm=

$$\begin{pmatrix} 2 - \lambda & -1 \\ -1 & 2 - \lambda \end{pmatrix}$$


In[=] eq0 = mat - lam // Det

Out[=] 3 - 4 \[Lambda] + \[Lambda]^2

In[=] Solve[eq0 == 0, \[Lambda]]

Out[=] {{\lambda \rightarrow 1}, {\lambda \rightarrow 3}}

```

Find Eigenvectors: $\lambda = \{1, 3\}$

```

In[=] vec = {x, y};
eq1 = mat.vec == \[Lambda] vec

Out[=] {2 x - y, -x + 2 y} == {x \[Lambda], y \[Lambda]}

In[=] eq2 = eq1 // Thread

Out[=] {2 x - y == x \[Lambda], -x + 2 y == y \[Lambda]}

In[=] Solve[eq2 /. {\lambda \rightarrow 1}, {x, y}]

Out[=] {{y \rightarrow x} }

In[=] eq3 = Join[eq2, {x^2 + y^2 == 1}]

Out[=] {2 x - y == x \[Lambda], -x + 2 y == y \[Lambda], x^2 + y^2 == 1}

```

```
In[1]:= Solve[eq3 /. {λ → 1}, {x, y}]
Out[1]= {{x → -1/Sqrt[2], y → -1/Sqrt[2]}, {x → 1/Sqrt[2], y → 1/Sqrt[2]}}
```



```
In[2]:= Solve[eq3 /. {λ → 3}, {x, y}]
Out[2]= {{x → -1/Sqrt[2], y → 1/Sqrt[2]}, {x → 1/Sqrt[2], y → -1/Sqrt[2]}}
```

Examine

```
In[3]:= eval = Eigenvalues[mat]
Out[3]= {3, 1}
```



```
In[4]:= evec = Eigenvectors[mat]
Out[4]= {{-1, 1}, {1, 1}}
```



```
In[5]:= mat.evec[[1]] // Simplify
Out[5]= {-3, 3}
```



```
In[6]:= mat.evec[[1]] == 3 evec[[1]] // Simplify
Out[6]= True
```



```
In[7]:= mat.evec[[2]] // Simplify
Out[7]= {1, 1}
```



```
In[8]:= mat.evec[[2]] == 1 evec[[2]] // Simplify
Out[8]= True
```



```
In[9]:= evec = Normalize /@ evec
Out[9]= {{-1/Sqrt[2], 1/Sqrt[2]}, {1/Sqrt[2], 1/Sqrt[2]}}
```



```
In[10]:= Transpose[evec].mat.evec // Simplify // MatrixForm
Out[10]//MatrixForm=
(3 0
 0 1)
```

Change coordinates

```
In[1]:= eq1 = {z1 == (x + y)/Sqrt[2], z2 == (x - y)/Sqrt[2]}
Out[1]= {z1 == (x + y)/Sqrt[2], z2 == (x - y)/Sqrt[2]}
```

```
In[]:= sol = Solve[eq1, {x, y}][[1]] // Simplify
```

$$\text{Out}[]= \left\{ x \rightarrow \frac{z1 + z2}{\sqrt{2}}, y \rightarrow \frac{z1 - z2}{\sqrt{2}} \right\}$$

```
In[]:= pot
```

$$\text{Out}[]= k(x^2 - x y + y^2)$$

```
In[]:= pot /. sol // Simplify
```

$$\text{Out}[]= \frac{1}{2} k(z1^2 + 3 z2^2)$$