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```
Clear["Global`*"]
```

Example 5. Let's consider a model of a linear triatomic molecule in which we approximate the forces between the atoms by forces due to springs (Figure 12.2).

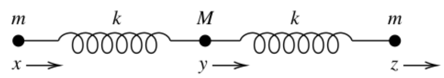

Figure 12.2

Follow example

In[ ]:=
```
pot = + 1/2 k (x - y)^2 + 1/2 k (y - z)^2 // Simplify
```

Out[ ]= $\frac{1}{2} k \left((x - y)^2 + (y - z)^2\right)$

In[ ]:=
```
pot // Expand // Factor
```

Out[ ]= $\frac{1}{2} k \left(x^2 - 2\, x\, y + 2\, y^2 - 2\, y\, z + z^2\right)$

In[ ]:=
```
{D[pot, x], D[pot, y], D[pot, z]}
```

Out[ ]= $\left\{k\,(x - y),\ \frac{1}{2} k\,(-2\,(x - y) + 2\,(y - z)),\ -k\,(y - z)\right\}$

In[ ]:=
```
eqs = -{D[pot, x], D[pot, y], D[pot, z]} == -{m ω2 x, M ω2 y, m ω2 z} // Thread;
eqs // Expand // TableForm
```

Out[ ]//TableForm=

$-k\,x + k\,y == -m\,x\,\omega 2$

$k\,x - 2\,k\,y + k\,z == -M\,y\,\omega 2$

$k\,y - k\,z == -m\,z\,\omega 2$

In[ ]:=
```
eqs = -1/k {D[pot, x], D[pot, y], D[pot, z]} == -1/k {m ω2 x, M ω2 y, m ω2 z} // Thread;
eqs // Expand // TableForm
```

Out[ ]//TableForm=

$-x + y == -\frac{m\,x\,\omega 2}{k}$

$x - 2\,y + z == -\frac{M\,y\,\omega 2}{k}$

$y - z == -\frac{m\,z\,\omega 2}{k}$

In[ ]:= `eqs[[2]] = eqs[[2]]/(M / m) // Thread[#, Equal] & // Expand`

Out[ ]= $\frac{m\,x}{M} - \frac{2\,m\,y}{M} + \frac{m\,z}{M} == -\frac{m\,y\,\omega 2}{k}$

In[ ]:= `eqs // Expand // TableForm`

Out[ ]//TableForm=

$-x + y == -\frac{m\,x\,\omega 2}{k}$

$\frac{m\,x}{M} - \frac{2\,m\,y}{M} + \frac{m\,z}{M} == -\frac{m\,y\,\omega 2}{k}$

$y - z == -\frac{m\,z\,\omega 2}{k}$

In[ ]:= `mat = {{-1, 1, 0}, m/M {1, -2, 1}, {0, 1, -1}};`

`mat // MatrixForm`

Out[ ]//MatrixForm=

$$\begin{pmatrix} -1 & 1 & 0 \\ \frac{m}{M} & -\frac{2\,m}{M} & \frac{m}{M} \\ 0 & 1 & -1 \end{pmatrix}$$

In[ ]:= `lam = λ DiagonalMatrix[{1, 1, 1}];`

`lam // MatrixForm`

Out[ ]//MatrixForm=

$$\begin{pmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{pmatrix}$$

In[ ]:= `mat - lam // MatrixForm`

Out[ ]//MatrixForm=

$$\begin{pmatrix} -1-\lambda & 1 & 0 \\ \frac{m}{M} & -\frac{2\,m}{M}-\lambda & \frac{m}{M} \\ 0 & 1 & -1-\lambda \end{pmatrix}$$

In[ ]:= `eq0 = mat - lam // Det // Factor`

Out[ ]= $-\frac{\lambda\,(1+\lambda)\,(2\,m + M + M\,\lambda)}{M}$

In[ ]:= `sol = Solve[eq0 == 0, λ] // Simplify`

Out[ ]= $\left\{\{\lambda \to -1\}, \{\lambda \to 0\}, \left\{\lambda \to -1 - \frac{2\,m}{M}\right\}\right\}$

Find Eigenvectors: λ ={1,2,3}

In[ ]:= `eval = Eigenvalues[mat] // Simplify`

Out[ ]= $\left\{0, -1 - \frac{2\,m}{M}, -1\right\}$

In[ ]:=
```
emat = Eigenvectors[mat]
```
Out[ ]= $\left\{\{1, 1, 1\}, \left\{1, -\frac{2\,m}{M}, 1\right\}, \{-1, 0, 1\}\right\}$

In[ ]:=
```
sol
```
Out[ ]= $\left\{\{\lambda \to -1\}, \{\lambda \to 0\}, \left\{\lambda \to -1 - \frac{2\,m}{M}\right\}\right\}$

Find first eigenvector

In[ ]:=
```
vec = {x, y, z};
eq1 = mat.vec == λ vec //. sol[[1]] // Thread
```
Out[ ]= $\left\{-x + y == -x, \frac{m\,x}{M} - \frac{2\,m\,y}{M} + \frac{m\,z}{M} == -y, y - z == -z\right\}$

In[ ]:=
```
sol1 = Solve[eq1, {x, y, z}]
```
Solve : Equations may not give solutions for all "solve " variables .

Out[ ]= $\{\{y \to 0, z \to -x\}\}$

Find second eigenvector

In[ ]:=
```
vec = {x, y, z};
eq2 = mat.vec == λ vec /. sol[[2]] // Thread // Simplify
```
Out[ ]= $\left\{x == y, \frac{m\,(x - 2\,y + z)}{M} == 0, y == z\right\}$

In[ ]:=
```
sol2 = Solve[eq2[[{1, 2}]], {x, y, z}]
```
Solve : Equations may not give solutions for all "solve " variables .

Out[ ]= $\{\{y \to x, z \to x\}\}$

Find third eigenvector

In[ ]:=
```
vec = {x, y, z};
eq3 = mat.vec == λ vec /. sol[[3]] // Thread
```
Out[ ]= $\left\{-x + y == \left(-1 - \frac{2\,m}{M}\right) x, \frac{m\,x}{M} - \frac{2\,m\,y}{M} + \frac{m\,z}{M} == \left(-1 - \frac{2\,m}{M}\right) y, y - z == \left(-1 - \frac{2\,m}{M}\right) z\right\}$

In[ ]:=
```
Solve[eq3, {x, y, z}]
```
Solve : Equations may not give solutions for all "solve " variables .

Out[ ]= $\left\{\left\{y \to -\frac{2\,m\,x}{M}, z \to x\right\}\right\}$

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In[ ]:=
```
Clear["Global`*"]
```

Example 7. Find the characteristic frequencies and the characteristic modes of vibration for the system of masses and springs shown in Figure 12.3, where the motion is along a vertical line.

Follow example

In[ ]:=
```
tmat = {{4, 0}, {0, 1}};
tmat // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix}$$

In[ ]:=
```
vmat = {{4, -1}, {-1, 1}};
vmat // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} 4 & -1 \\ -1 & 1 \end{pmatrix}$$

In[ ]:=
```
tinv = Inverse[tmat];
tinv // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} \frac{1}{4} & 0 \\ 0 & 1 \end{pmatrix}$$

In[ ]:=
```
tv = tinv.vmat;
tv // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} 1 & -\frac{1}{4} \\ -1 & 1 \end{pmatrix}$$

In[ ]:=
```
Eigensystem[tv]
```

Out[ ]=

$$\left\{\left\{\frac{3}{2}, \frac{1}{2}\right\}, \left\{\left\{-\frac{1}{2}, 1\right\}, \left\{\frac{1}{2}, 1\right\}\right\}\right\}$$

Change variables

In[ ]:=
```
change = {{1/2, 0}, {0, 1}};
change // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} \frac{1}{2} & 0 \\ 0 & 1 \end{pmatrix}$$

In[ ]:=
```
mat2 = change.vmat.change;
mat2 // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} 1 & -\frac{1}{2} \\ -\frac{1}{2} & 1 \end{pmatrix}$$

In[ ]:=
```
Eigensystem[mat2]
```

Out[ ]= $\left\{\left\{\frac{3}{2}, \frac{1}{2}\right\}, \{\{-1, 1\}, \{1, 1\}\}\right\}$

In[ ]:=
```
evecs = Normalize /@ Eigenvectors[mat2]
```

Out[ ]= $\left\{\left\{-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right\}, \left\{\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right\}\right\}$

In[ ]:=
```
Transpose[evecs].evecs // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$