Homework #6: Phys 4321: Prof. Olness Spring 2025

Warm-up: watch these two videos on YouTube:

Convolutions | Why X+Y in probability is a beautiful mess
https://youtu.be/IaSGqQa5O-M?si=a12BqM0CUtXWXhZe

But what is a convolution?
https://youtu.be/KuXjwB4LzSA?si=tZqb1DKfFg4PCsZi

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1) a) Compute the area of a circle using a double integral in polar coordinates.

$$A = \int_0^R dr \int_0^{2\pi} r d\phi$$

Repeat, but insert a delta function:

$$C = \int_0^R dr \int_0^{2\pi} r d\phi \; \delta(r - R)$$

b) Repeat for a 3-D sphere:

$$V = \int_0^R dr \int_0^{2\pi} r d\phi \; \int_0^{\pi} \sin(\theta)\, r d\theta \qquad A = \int_0^R dr \int_0^{2\pi} r d\phi \; \int_0^{\pi} \sin(\theta)\, r d\theta \; \delta(r - R)$$

c) Repeat for a 4-D sphere. Obtain the 4-volume and the 3-surface formulas.

- The volume element in 4-dimensional spherical coordinates is given by $dV = r^3 \sin^2(\psi) \sin(\theta) dr d\psi d\theta d\phi$.
- The limits of integration for a 4-dimensional sphere are:
  - $0 \le r \le R$
  - $0 \le \psi \le \pi$
  - $0 \le \theta \le \pi$
  - $0 \le \phi \le 2\pi$

#2)

is in the vicinity of 0. Use part (c).

21. Make use of the operator equations (11.19) and previous equations to evaluate the following integrals.

(a) $\displaystyle\int_0^3 (5x - 2)\delta(2 - x)\, dx$ (b) $\displaystyle\int_0^\infty \phi(x)\delta(x^2 - a^2)\, dx$

(c) $\displaystyle\int_{-1}^1 \cos x \,\delta(-2x)\, dx$ (d) $\displaystyle\int_{-\pi/2}^{\pi/2} \cos x \,\delta(\sin x)\, dx$

3) Following my notes on convolution: Compute the result of flipping a coin 3 times.
That is, compute: $f \otimes f \otimes f$ [Hint: I did 2 times in my notes. Start from there.]

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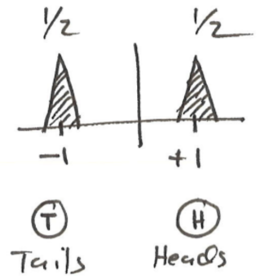
Example: Coin Flip:

$f(x) = g(y) = \frac{1}{2}\left[\delta(x-1) + \delta(x+1)\right] =$



Double Coin Flip: $f \oplus g$

$$f \oplus g = \int dx \int dy \, f(x)\, g(y)\, \delta[a - (x+y)]$$

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4) Following my notes on convolution: Compute the result of flipping a coin 2 times,
but the coin is not balanced and comes up 2/3'rds heads and 1/3'rd tails.

$$f(x) = \frac{2}{3}\delta(x-1) + \frac{1}{3}\delta(x+1)$$