Physics 4321:
Homework #9:
Prof. Olness
Spring 2025

PROBLEM #1: Consider the metric:

$$g_{\mu\nu} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

and vector:

$$v = \begin{pmatrix} x \\ y \end{pmatrix}$$

.

a) Compute the length-squared of the vector v.
b) Write down a rotation matrix R.
c) Show the length of v is invariant under a rotation.
d) Using Mathematica, make a ContourPlot of the length of the vector in the {x,y} space. Comment about the result.

PROBLEM #2: Consider the metric:

$$g_{\mu\nu} = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

and vector:

$$v = \begin{pmatrix} x \\ y \end{pmatrix}$$

.

a) Compute the length-squared of the vector v.
b) Using Mathematica, make a ContourPlot of the length of the vector in the {x,y} space. Comment about the result.
c) Can you think of a physical example where this is the appropriate metric to use to measure distance??? *(There are more than one answer, but the more natural the better.)*

PROBLEM #3: Consider the metric:

$$g_{\mu\nu} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

and vector:

$$v = \begin{pmatrix} t \\ x \end{pmatrix}$$

.

a) Compute the length-squared of the vector v.
b1) Write down a boost matrix B in terms of β and γ.
c1) Show the length of v is invariant under a boost.
b1) Write down a boost matrix B in terms of coshand sinh.
c1) Show the length of v is invariant under a boost in terms of coshand sinh.
d) Using Mathematica, make a ContourPlot of the length of the vector in the {t,x} space. Comment about the result.

PROBLEM #4: For the general mass case of $p_1 + p_2 \to p_{12}$ compute the components of all 4-vectors in terms of invariants of the problem: $\{m_1^2, m_2^2, s\}$. Assume the 3-momentum lies along the z-axis. Hint: The z-component of the momentum should be proportional to: $\Delta(s, m_1^2, m_2^2)$, where $\Delta(a,b,c) = \sqrt{a^2 + b^2 + c^2 - 2(ab + bc + ca)}$.

PROBLEM #5: For the zero mass case of $p_1 + p_2 \to p_3 + p_4$ compute the components of all 4-vectors in terms of invariants of the problem: $\{s, t, u\}$. Assume the 3-momentum of $\{p_1, p_2\}$ lies along the z-axis.

PROBLEM #6: Consider the reaction: $p\bar{p} \to 3X$ where X has a rest mass of $140 GeV$. Compute the threshold beam energy for

a) colliding beams, and

b) for a fixed target experiment.

PROBLEM #7: Consider the rapidity y and the pseudo-rapidity η:

$$y = \frac{1}{2} \ln \left(\frac{E + P_L}{E - P_L} \right)$$

$$\eta = -\ln \left[\tan \left(\frac{\theta}{2} \right) \right]$$

a) Make a parametric plot of $\{y, \eta\}$as a function of m/E where m is the mas of the particle.

b) Show that in the limit $m \to 0$ that $y \to \eta$.

c) Make a table of η for $\theta = [0^\circ, 180^\circ]$ in steps of 5 degrees.

c) Make a table of θ for $\eta = [0, 10]$ in steps of 1.

PROBLEM #8: For one-particle phase space, show the folowing equality (*with all the steps*):

$$\frac{d^3 \vec{P}}{(2\pi)^3 2E} = (2\pi)\delta(P^2 - m^2)\frac{d^4 P}{(2\pi)^4}$$

PROBLEM #9: In a Tevatron detector, consider two particles traveling in the transverse direction:

$$\begin{aligned} p_1^\mu &= \{E, 100, 0, 1\} \\ p_2^\mu &= \{E, 100, 1, 0\} \end{aligned}$$

where the componets are expresedn in GeV units. E is defined such that the particles are massless.

a) Compute E.

b) For each particle, compute the pseudorapidity η and azimuthal angle ϕ.

c) Explain how the above exercise justifies the correct jet radius definition to be:

$$R = \sqrt{\eta^2 + \phi^2}$$

In particular, why is the above correct and $R = \sqrt{\eta^2 + 2\phi^2}$, for example, incorrect.

PROBLEM #10: Consider the reaction: $pp \to pp$ $(12 \to 34)$ with **CMS** scattering angle θ. The CMS energy is $\sqrt{s} = 2\,TeV$.

a) Compute the boost from the CMS frame to the rest frame of #2 (lab frame)

b) Compute the energy of #1 in the lab frame.

c) Compute the scattering angle θ_{lab} as a function of the CMS θ and invariants.

PROBLEM #11: Write the 4×4 EM Field Strength tensor $F^{\mu\nu}$ in terms of $\{E, B\}$ fields.

Now apply a boost (Lorentz transformation) along the z axis and compute the transformed $F^{\mu\nu}$.

Examine how the $\{E, B\}$ fields transform and check against your EM textbook results.