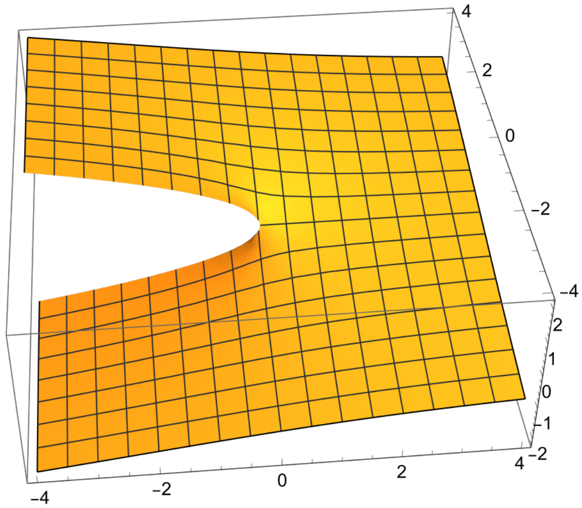
Problem 1 :

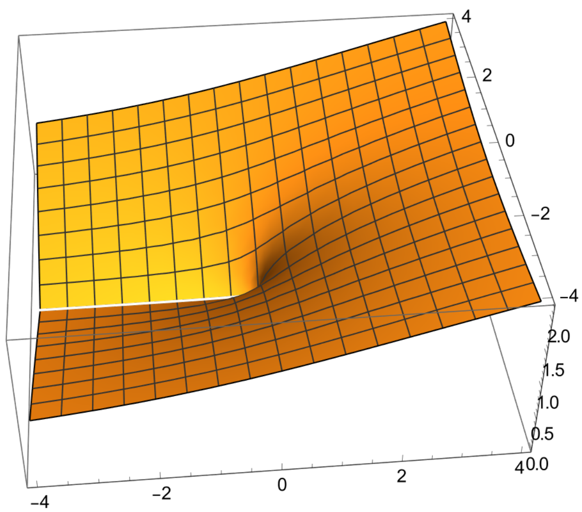
In[∘]:=
```
Plot3D[Im[Sqrt[x + I y]], {x, -4, 4}, {y, -4, 4}]
```

Out[∘]=



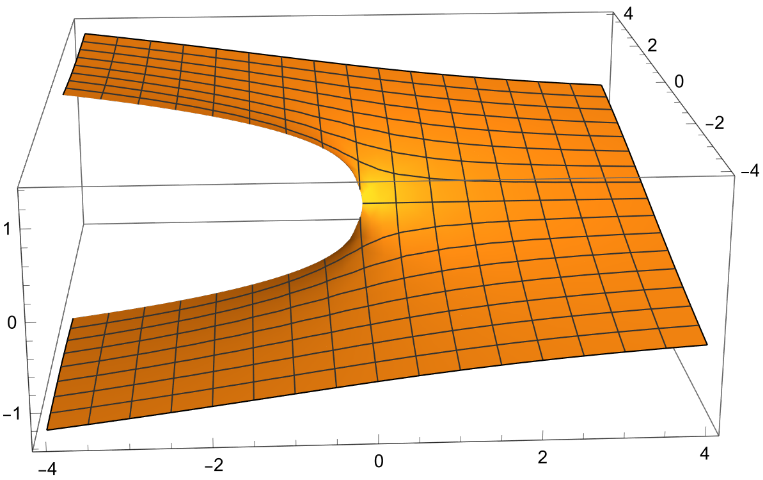
In[∘]:=
```
Plot3D[Re[Sqrt[x + I y]], {x, -4, 4}, {y, -4, 4}]
```

Out[∘]=



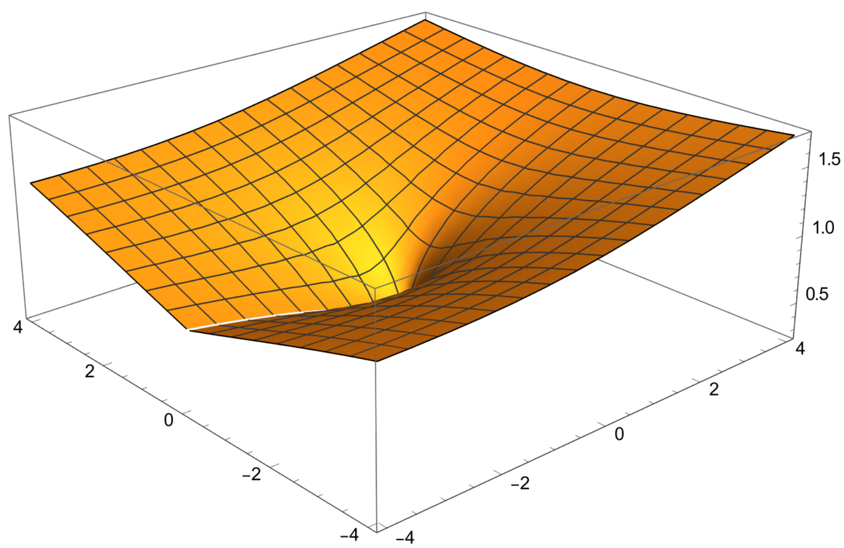
In[∘]:= `Plot3D[Im[(x + I y)^(1/3)], {x, -4, 4}, {y, -4, 4}]`

Out[∘]=



In[∘]:= `Plot3D[Re[(x + I y)^(1/3)], {x, -4, 4}, {y, -4, 4}]`

Out[∘]=



Cube Root

In[∘]:= $(r\,\mathrm{Exp}[I\,\theta])^{1/3}$ // PowerExpand

Out[∘]= $e^{\frac{i\theta}{3}}\,r^{1/3}$

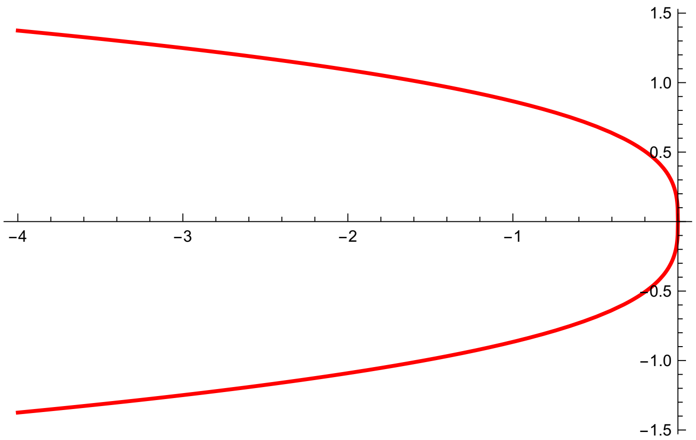
In[∘]:= ϵ = 0.001;
$(r\,\mathrm{Exp}[I\,\theta])^{1/3}$ /. {r → 1, θ → {π − ϵ, −π + ϵ}}

Out[∘]= {0.500289 + 0.865859 ⅈ, 0.500289 − 0.865859 ⅈ}

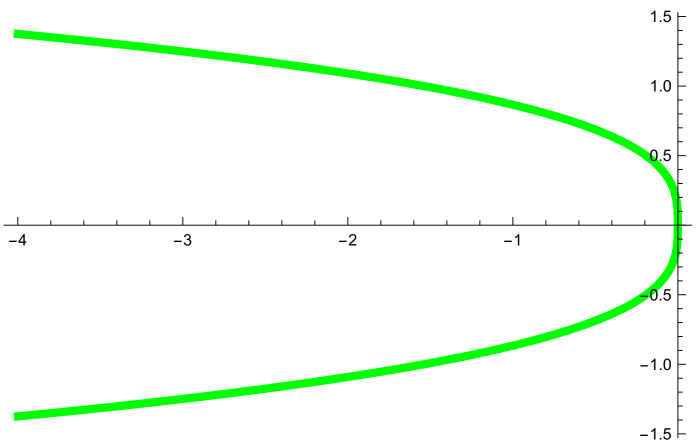
In[∘]:= `Sin[π / 3] // N`

Out[∘]= 0.866025

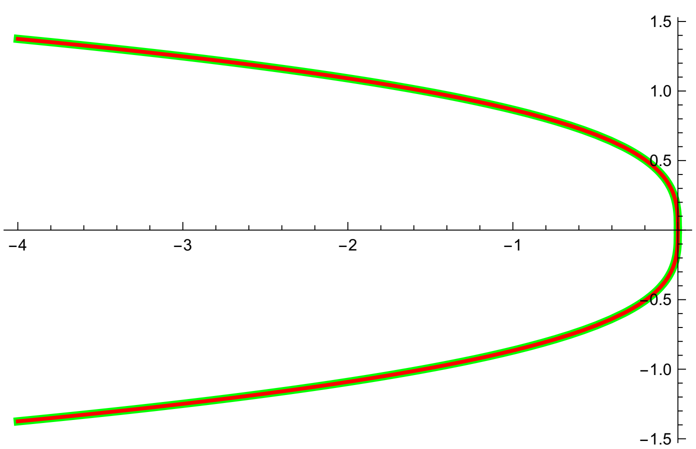
In[∘]:=
```
p1 = Plot[{ ∛Abs[r] Sin[π/3], - ∛Abs[r] Sin[π/3]} // Re , {r, -4, 0}, PlotStyle → Red]
```

Out[∘]=


In[∘]:=
```
p2 = Plot[{ ∛r Exp[I π/3], - ∛r Exp[I π/3]} // Im , {r, -4, 0}, PlotStyle → {Green, Thickness[0.012]}]
```

Out[∘]=


In[∘]:=
```
Show[p2, p1, PlotRange → All]
```

Out[∘]=


General Root

In[∘]:= $\left(r\, \text{Exp}[I\, \theta]\right)^{1/n}$ // PowerExpand

Out[∘]= $e^{\frac{i\theta}{n}}\, r^{\frac{1}{n}}$

In[∘]:=
```
{n → Table[i, {i, 2, 10, 1}]}
```

Out[∘]=
```
{n → {2, 3, 4, 5, 6, 7, 8, 9, 10}}
```

In[∘]:=
```
ϵ = 0.001;
tab = Table[i, {i, 2, 10, 1}]
(r Exp[I θ])^(1/n) /. {r → 1, θ → {π - ϵ, -π + ϵ}} /. {n → tab} // Transpose //
  TableForm[#, TableHeadings → {tab, None}] &
```

Out[∘]=
```
{2, 3, 4, 5, 6, 7, 8, 9, 10}
```

Out[∘]//TableForm=

2	0.0005 + 1. ⅈ	0.0005 - 1. ⅈ
3	0.500289 + 0.865859 ⅈ	0.500289 - 0.865859 ⅈ
4	0.707284 + 0.70693 ⅈ	0.707284 - 0.70693 ⅈ
5	0.809135 + 0.587623 ⅈ	0.809135 - 0.587623 ⅈ
6	0.866109 + 0.499856 ⅈ	0.866109 - 0.499856 ⅈ
7	0.901031 + 0.433755 ⅈ	0.901031 - 0.433755 ⅈ
8	0.923927 + 0.382568 ⅈ	0.923927 - 0.382568 ⅈ
9	0.939731 + 0.341916 ⅈ	0.939731 - 0.341916 ⅈ
10	0.951087 + 0.308922 ⅈ	0.951087 - 0.308922 ⅈ

In[∘]:=
```
Table[Sin[π / n] // N, {n, 2, 10, 1}]
```

Out[∘]=
```
{1., 0.866025, 0.707107, 0.587785, 0.5, 0.433884, 0.382683, 0.34202, 0.309017}
```

In[∘]:=
```
Exp[I π / 4] // Im
```

Out[∘]= $\frac{1}{\sqrt{2}}$

In[∘]:=
```
Sin[π / 4]
```

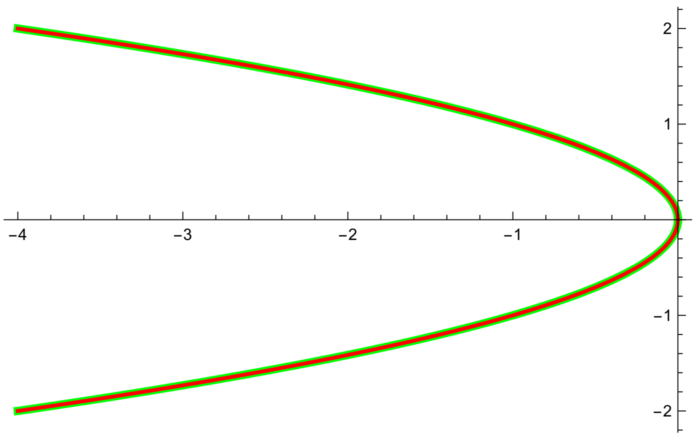
Out[∘]= $\frac{1}{\sqrt{2}}$

In[∘]:=
```
Clear[p1, p2]
```
p1[n_] := Plot$\left[\left\{\sqrt[n]{\text{Abs}[r]}\ \text{Sin}\left[\frac{\pi}{n}\right], -\sqrt[n]{\text{Abs}[r]}\ \text{Sin}\left[\frac{\pi}{n}\right]\right\}\right.$ // Re , {r, -4, 0}, PlotStyle → Red]

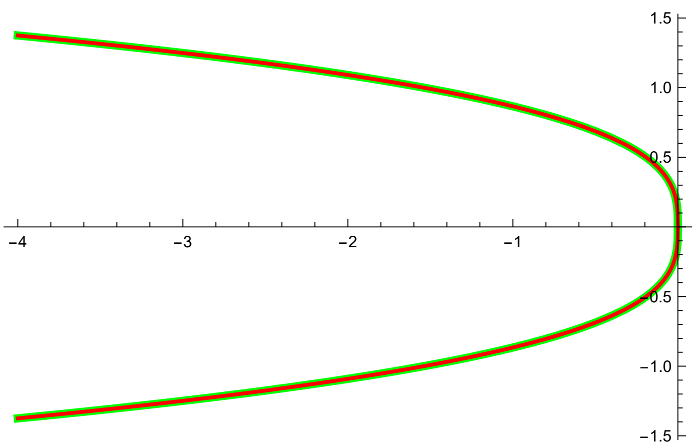
p2[n_] := Plot$\left[\left\{\sqrt[n]{\text{Abs}[r]}\ \text{Exp}\left[I\,\frac{\pi}{n}\right], -\sqrt[n]{\text{Abs}[r]}\ \text{Exp}\left[I\,\frac{\pi}{n}\right]\right\}\right.$ // Im ,
  {r, -4, 0}, PlotStyle → {Green, Thickness[0.012]}]

In[∘]:=
```
Do[
 Print[n];
 Show[p2[n], p1[n]] // Print
 , {n, 2, 10}]
```

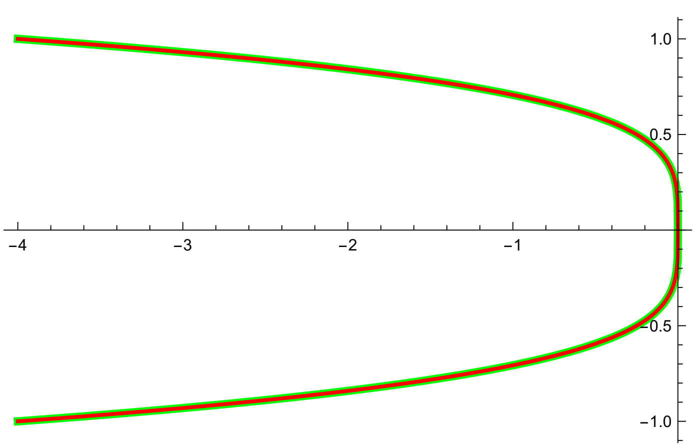
2



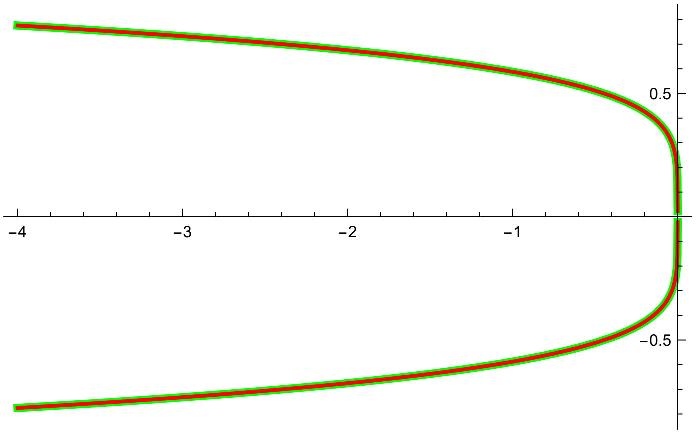
3



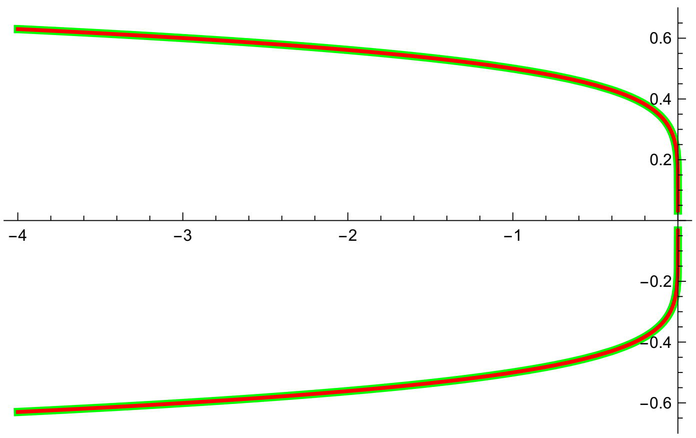
4



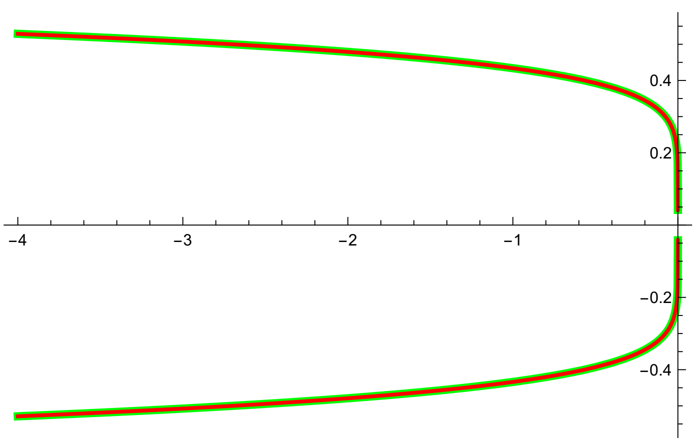
5



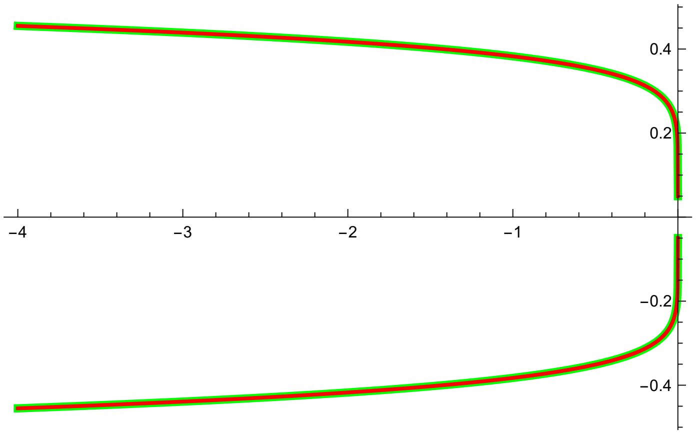
6



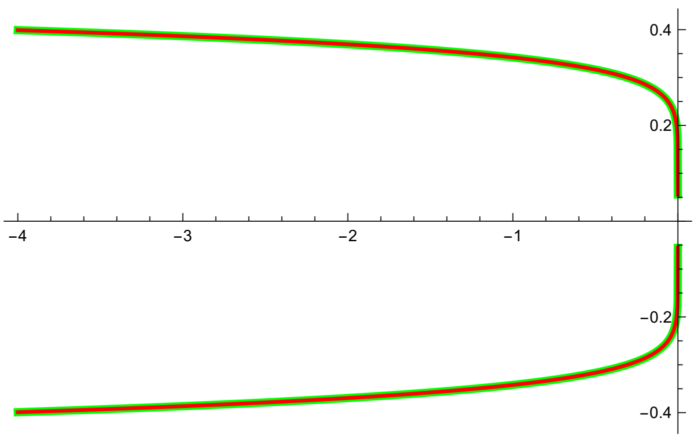
7



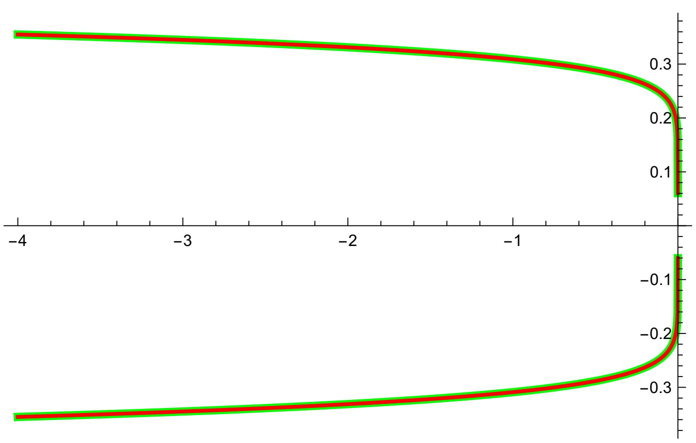
8



9



10



Problem 2 :

In[∘]:=
```
Clear["Global`*"]
```

```
In[∘]:= mat = {{14, -3}, {-3, 6}};
        mat // MatrixForm
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} 14 & -3 \\ -3 & 6 \end{pmatrix}$$

```
In[∘]:=
        Eigenvalues[mat] // Simplify
        Eigenvectors[mat]

Out[∘]= {15, 5}

Out[∘]= {{-3, 1}, {1, 3}}

In[∘]:= Tr[mat]

Out[∘]= 20

In[∘]:= Det[mat]

Out[∘]= 75

In[∘]:= Eigenvalues[mat] // Simplify

Out[∘]= {15, 5}

In[∘]:= b = Eigenvectors[mat]

Out[∘]= {{-3, 1}, {1, 3}}

In[∘]:= b = Normalize /@ b
```

Out[∘]= $\left\{\left\{-\frac{3}{\sqrt{10}}, \frac{1}{\sqrt{10}}\right\}, \left\{\frac{1}{\sqrt{10}}, \frac{3}{\sqrt{10}}\right\}\right\}$

```
In[∘]:= b.Transpose[b]

Out[∘]= {{1, 0}, {0, 1}}

In[∘]:= test = b.mat.Transpose[b]  // Simplify // MatrixForm
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} 15 & 0 \\ 0 & 5 \end{pmatrix}$$

Problem 3 :

```
In[∘]:= Clear["Global`*"]
```

In[∘]:= tmat = $\frac{1}{2}$ m r^2 (θ1'[t]^2 + θ2'[t]^2 + θ3'[t]^2)

Out[∘]= $\frac{1}{2} m r^2 \left(\theta 1'[t]^2 + \theta 2'[t]^2 + \theta 3'[t]^2\right)$

In[∘]:=
```
tt = 1/2 m r^2 DiagonalMatrix[{1, 1, 1}];
tt // MatrixForm
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} \frac{m r^2}{2} & 0 & 0 \\ 0 & \frac{m r^2}{2} & 0 \\ 0 & 0 & \frac{m r^2}{2} \end{pmatrix}$$

In[∘]:=
```
vmat = k/2 r^2 ((θ1 - θ2)^2 + (θ2 - θ3)^2 + (θ3 - θ1)^2)
```

Out[∘]= $\frac{1}{2} k r^2 \left((\theta1 - \theta2)^2 + (\theta2 - \theta3)^2 + (-\theta1 + \theta3)^2\right)$

In[∘]:=
```
vmat // Expand
```

Out[∘]= $k r^2 \theta1^2 - k r^2 \theta1 \theta2 + k r^2 \theta2^2 - k r^2 \theta1 \theta3 - k r^2 \theta2 \theta3 + k r^2 \theta3^2$

In[∘]:=
```
test1 = vmat/(k r^2) // Expand
```

Out[∘]= $\theta1^2 - \theta1 \theta2 + \theta2^2 - \theta1 \theta3 - \theta2 \theta3 + \theta3^2$

In[∘]:=
```
D[test1, θ1]
```

Out[∘]= $2 \theta1 - \theta2 - \theta3$

In[∘]:=
```
vv = {{2, -1, -1}, {-1, 2, -1}, {-1, -1, 2}};
vv // MatrixForm
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix}$$

In[∘]:=
```
Det[{{2, -1, -1}, {-1, 2, -1}, {-1, -1, 2}}]
```

Out[∘]= 0

In[∘]:=
```
vec = {θ1, θ2, θ3};
```

In[∘]:=
```
MatrixForm /@ {vec, vv, vec}
```

Out[∘]= $\left\{\begin{pmatrix} \theta1 \\ \theta2 \\ \theta3 \end{pmatrix}, \begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix}, \begin{pmatrix} \theta1 \\ \theta2 \\ \theta3 \end{pmatrix}\right\}$

In[∘]:=
```
test2 = 1/2 vec.vv.vec // Expand
```

Out[∘]= $\theta1^2 - \theta1 \theta2 + \theta2^2 - \theta1 \theta3 - \theta2 \theta3 + \theta3^2$

In[∘]:=
```
test1 == test2
```

Out[∘]= True

```
In[∘]:= D[test1, θ1]
Out[∘]= 2 θ1 - θ2 - θ3
```

```
In[∘]:= vvmat = k r^2 vv;
        vvmat // MatrixForm
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} 2\,k\,r^2 & -k\,r^2 & -k\,r^2 \\ -k\,r^2 & 2\,k\,r^2 & -k\,r^2 \\ -k\,r^2 & -k\,r^2 & 2\,k\,r^2 \end{pmatrix}$$

```
In[∘]:= tt (-2 w2) // MatrixForm
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} -m\,r^2\,w2 & 0 & 0 \\ 0 & -m\,r^2\,w2 & 0 \\ 0 & 0 & -m\,r^2\,w2 \end{pmatrix}$$

```
In[∘]:= mat1 = vvmat + tt (-2 w2)
```

Out[∘]= $\{\{2\,k\,r^2 - m\,r^2\,w2, -k\,r^2, -k\,r^2\}, \{-k\,r^2, 2\,k\,r^2 - m\,r^2\,w2, -k\,r^2\}, \{-k\,r^2, -k\,r^2, 2\,k\,r^2 - m\,r^2\,w2\}\}$

```
In[∘]:= Det[mat1]
```

Out[∘]= $-9\,k^2\,m\,r^6\,w2 + 6\,k\,m^2\,r^6\,w2^2 - m^3\,r^6\,w2^3$

```
In[∘]:= Det[mat1] /. {w2 → 0}
Out[∘]= 0
```

```
In[∘]:= Solve[Det[mat1] == 0, w2]
```

Out[∘]= $\{\{w2 \to 0\}, \{w2 \to \frac{3\,k}{m}\}, \{w2 \to \frac{3\,k}{m}\}\}$

Problem 2: f[x_]=... for x=[0,2 π]

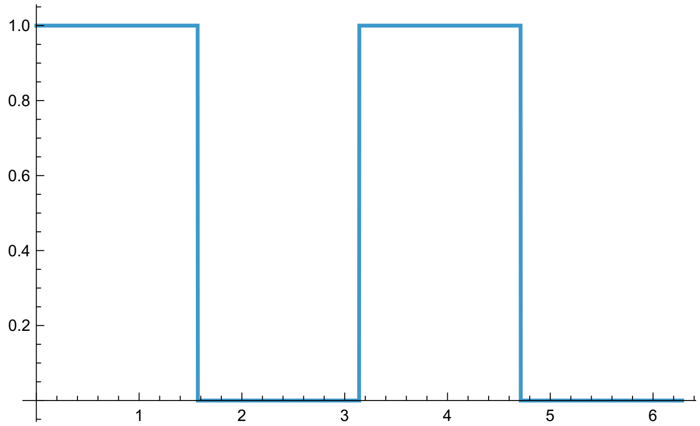
```
In[∘]:= Clear["Global`*"]
```

Part a)

```
In[∘]:= Clear[f];
        f[x_]:=1 /; 0< x <= N[Pi/2];
        f[x_]:=0 /; N[Pi/2] < x <= N[Pi];
        f[x_]:=1 /; N[Pi] < x <= N[3 Pi/2];
        f[x_]:=0 /; N[3 Pi/2] < x <= N[2Pi];
```

In[•]:=
```
Plot[f[x],{x,0,2 Pi}]
```

Out[•]=



Compute the Fourier coefficients

In[•]:= a[n_] = $\frac{1}{\pi}$ (Integrate[Cos[n x], {x, 0, π/2}] + Integrate[Cos[n x], {x, π, 3 π/2}]) // Simplify

Out[•]= $$\frac{\text{Sin}\left[\frac{n\pi}{2}\right] - \text{Sin}[n\pi] + \text{Sin}\left[\frac{3n\pi}{2}\right]}{n\pi}$$

In[•]:= a[n_] = $\frac{1}{\pi}$ (Integrate[Cos[n x], {x, 0, π/2}] + Integrate[Cos[n x], {x, π, 3 π/2}]) //
Simplify[#, n ∈ Integers] &

Out[•]= $$\frac{\text{Sin}\left[\frac{n\pi}{2}\right] + \text{Sin}\left[\frac{3n\pi}{2}\right]}{n\pi}$$

In[•]:= Table[a[i], {i, 1, 10, 1}]

Out[•]= {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}

Check

In[•]:= $\frac{1}{\pi}$ (Integrate[1, {x, 0, π/2}] + Integrate[1, {x, π, 3 π/2}])

Out[•]= 1

In[•]:= b[n_] = $\frac{1}{\pi}$ (Integrate[Sin[n x], {x, 0, π/2}] + Integrate[Sin[n x], {x, π, 3 π/2}]) // Simplify

Out[•]= $$\frac{4\left(1 + \text{Cos}\left[\frac{n\pi}{2}\right] + \text{Cos}[n\pi]\right)\text{Sin}\left[\frac{n\pi}{4}\right]^2}{n\pi}$$

In[∘]:=
```
b[n_] = 1/π (Integrate[Sin[n x], {x, 0, π/2}] + Integrate[Sin[n x], {x, π, 3 π/2}]) //
  FullSimplify[#, n ∈ Integers] &
```

Out[∘]= $$\frac{1 + (-1)^n - 2 (-1)^n \mathrm{Cos}\left[\frac{n\pi}{2}\right]}{n\pi}$$

In[∘]:=
```
Table[b[i], {i, 1, 10, 1}]
```

Out[∘]= $$\left\{0, \frac{2}{\pi}, 0, 0, 0, \frac{2}{3\pi}, 0, 0, 0, \frac{2}{5\pi}\right\}$$

Compute the Fourier coefficients

In[∘]:=
```
(* Note the integration limits  *)
(* Note, my function here is just "x"  You will need to change in other problems. *)
(* Note, we often need to do the zero term separately *)

c[0]= 1/(2 Pi)(  Integrate[ 1  ,{x,0, Pi/2}]+   Integrate[ 1  ,{x,Pi, 3Pi/2}] )
```

Out[∘]= $$\frac{1}{2}$$

In[∘]:=
```
c[m_]= 1/(2 Pi)( Integrate[ 1 Exp[- I m x] ,{x,0, Pi/2}] +  Integrate[ 1 Exp[- I m x] ,{x,Pi, 3Pi/2
```

Out[∘]= $$\frac{2\, e^{-\frac{3}{4} i m \pi} \mathrm{Cos}\left[\frac{m\pi}{2}\right] \mathrm{Sin}\left[\frac{m\pi}{4}\right]}{m\pi}$$

In[∘]:=
```
Integrate[1 Exp[- I m x], x]
```

Out[∘]= $$\frac{i\, e^{-i m x}}{m}$$

In[∘]:=
```
Integrate[1 Sin[m x], x]
```

Out[∘]= $$-\frac{\mathrm{Cos}[m x]}{m}$$

In[∘]:=
```
Integrate[1 Sin[m x], {x, 0, π/2}] + Integrate[1 Sin[m x], {x, π, 3 π/2}] // FullSimplify
```

Out[∘]= $$\frac{4 \left(1 + \mathrm{Cos}\left[\frac{m\pi}{2}\right] + \mathrm{Cos}[m\pi]\right) \mathrm{Sin}\left[\frac{m\pi}{4}\right]^2}{m}$$

In[∘]:=
```
Table[c[i], {i, 0, 6}] // Simplify
```

Out[∘]= $$\left\{\frac{1}{2}, 0, -\frac{i}{\pi}, 0, 0, 0, -\frac{i}{3\pi}\right\}$$

In[∘]:=
```
term[0]= c[0];
```

In[∘]:=
```
term[m_]= c[m] Exp[I m x] + c[-m]  Exp[-I m x]  //FullSimplify
```

Out[∘]=
$$\frac{2\,\mathrm{Cos}\left[m\left(\frac{3\pi}{4}-x\right)\right]\left(-\mathrm{Sin}\left[\frac{m\pi}{4}\right]+\mathrm{Sin}\left[\frac{3m\pi}{4}\right]\right)}{m\,\pi}$$

In[∘]:=
```
series[n_]:= Sum[ term[m] ,{m,0,n}]
```

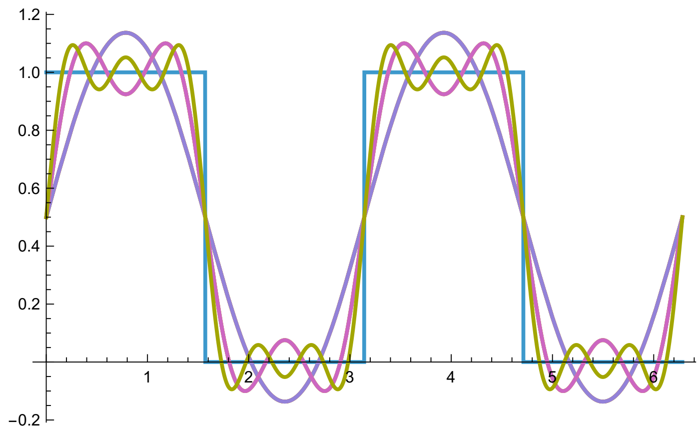
In[∘]:=
```
series[2]
```

Out[∘]=
$$\frac{1}{2}-\frac{2\,\mathrm{Cos}\left[2\left(\frac{3\pi}{4}-x\right)\right]}{\pi}$$

Plot the series with different numbers of terms

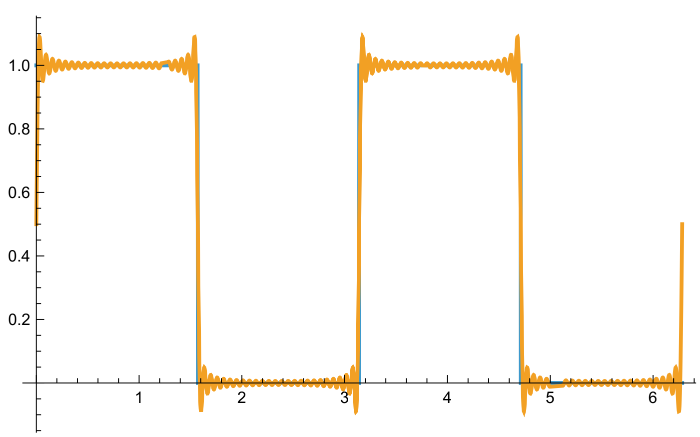
In[∘]:=
```
Plot[ Join[{f[x]},Table[series[i],{i,2,10,1}]] //Evaluate
  ,{x,0,2 Pi},PlotStyle->colors]
```

Out[∘]=



In[∘]:=
```
Plot[ {f[x], series[100] } //Evaluate  ,{x,0,2 Pi}]
```

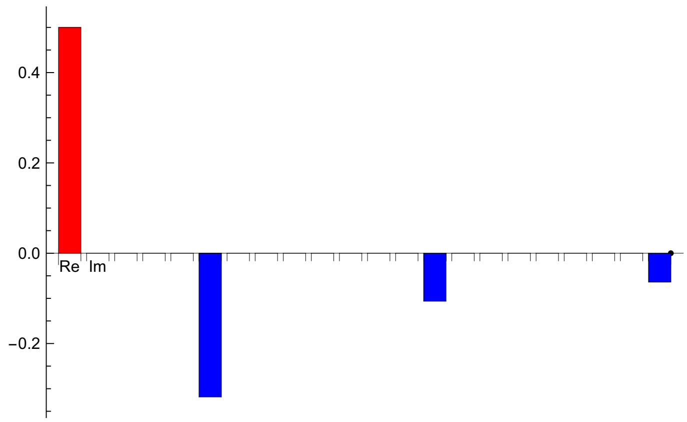
Out[∘]=



Make a "frequency domain" plot of the coefficients

In[∘]:=
```
coeffs = Table[{Re[c[i]], Im[c[i]]}, {i, 0, 10}] // Flatten
```
Out[∘]= $\left\{\frac{1}{2}, 0, 0, 0, 0, -\frac{1}{\pi}, 0, 0, 0, 0, 0, 0, 0, -\frac{1}{3\pi}, 0, 0, 0, 0, 0, 0, 0, -\frac{1}{5\pi}\right\}$

In[∘]:=
```
BarChart[coeffs, ChartStyle → {Red, Blue}, ChartLabels → {"Re", "Im"}]
```
Out[∘]=



Re-write the Exp series as a Trig series

In[∘]:=
```
term2[0] = term[0] //ExpToTrig //Simplify
```
Out[∘]= $\frac{1}{2}$

In[∘]:=
```
term2[m_]= term[m] //ExpToTrig //Simplify
```
Out[∘]= $\frac{2\,\mathrm{Cos}\left[m\left(\frac{3\pi}{4}-x\right)\right]\left(-\mathrm{Sin}\left[\frac{m\pi}{4}\right]+\mathrm{Sin}\left[\frac{3m\pi}{4}\right]\right)}{m\pi}$

In[∘]:=
```
tmp= term[m] //ExpToTrig // FullSimplify[#, Element[m, Integers]] &
```
Out[∘]= $\frac{2\,\mathrm{Cos}\left[\frac{3m\pi}{4}-m\,x\right]\left(-\mathrm{Sin}\left[\frac{m\pi}{4}\right]+\mathrm{Sin}\left[\frac{3m\pi}{4}\right]\right)}{m\pi}$

In[∘]:=
```
series2[n_]:= Sum[ term2[m] ,{m,0,n}]
```

In[∘]:=
```
series[5]
```
Out[∘]= $\frac{1}{2}-\frac{2\,\mathrm{Cos}\left[2\left(\frac{3\pi}{4}-x\right)\right]}{\pi}$

In[∘]:=
```
series[10]  //ExpToTrig //TrigReduce
```
Out[∘]= $\frac{15\pi+60\,\mathrm{Sin}[2x]+20\,\mathrm{Sin}[6x]+12\,\mathrm{Sin}[10x]}{30\pi}$

In[∘]:= `series2[10] // TrigReduce`

Out[∘]= $\frac{15\,\pi + 60\,\text{Sin}[2\,x] + 20\,\text{Sin}[6\,x] + 12\,\text{Sin}[10\,x]}{30\,\pi}$

In[∘]:= $\frac{2\,\text{series2}[40]}{4}$ `// TrigReduce // Expand`

Out[∘]= $\frac{1}{4} + \frac{\text{Sin}[2\,x]}{\pi} + \frac{\text{Sin}[6\,x]}{3\,\pi} + \frac{\text{Sin}[10\,x]}{5\,\pi} + \frac{\text{Sin}[14\,x]}{7\,\pi} + \frac{\text{Sin}[18\,x]}{9\,\pi} + \frac{\text{Sin}[22\,x]}{11\,\pi} + \frac{\text{Sin}[26\,x]}{13\,\pi} + \frac{\text{Sin}[30\,x]}{15\,\pi} + \frac{\text{Sin}[34\,x]}{17\,\pi} + \frac{\text{Sin}[38\,x]}{19\,\pi}$

Verify that they are identical

In[∘]:= `series2[10] - series[10] // FullSimplify`

Out[∘]= 0

Problem

In[∘]:= `Clear["Global`*"]`

In[∘]:=
```
λ1 = {{0, 1, 0}, {1, 0, 0}, {0, 0, 0}};
λ2 = {{0, -I, 0}, {I, 0, 0}, {0, 0, 0}};
λ3 = {{1, 0, 0}, {0, -1, 0}, {0, 0, 0}};
λ4 = {{0, 0, 1}, {0, 0, 0}, {1, 0, 0}};
λ5 = {{0, 0, -I}, {0, 0, 0}, {I, 0, 0}};
λ6 = {{0, 0, 0}, {0, 0, 1}, {0, 1, 0}};
λ7 = {{0, 0, 0}, {0, 0, -I}, {0, I, 0}};
λ8 = 1 / Sqrt[3] {{1, 0, 0}, {0, 1, 0}, {0, 0, -2}};
```

In[∘]:= `MatrixForm /@ {λ1, λ2, λ3, λ4, λ5, λ6, λ7, λ8}`

Out[∘]= $\left\{\begin{pmatrix}0&1&0\\1&0&0\\0&0&0\end{pmatrix}, \begin{pmatrix}0&-i&0\\i&0&0\\0&0&0\end{pmatrix}, \begin{pmatrix}1&0&0\\0&-1&0\\0&0&0\end{pmatrix}, \begin{pmatrix}0&0&1\\0&0&0\\1&0&0\end{pmatrix}, \begin{pmatrix}0&0&-i\\0&0&0\\i&0&0\end{pmatrix}, \begin{pmatrix}0&0&0\\0&0&1\\0&1&0\end{pmatrix}, \begin{pmatrix}0&0&0\\0&0&-i\\0&i&0\end{pmatrix}, \begin{pmatrix}\frac{1}{\sqrt{3}}&0&0\\0&\frac{1}{\sqrt{3}}&0\\0&0&-\frac{2}{\sqrt{3}}\end{pmatrix}\right\}$

In[∘]:= `comm[a_, b_] = a.b - b.a`

Out[∘]= a.b - b.a

```
In[∘]:= tmp = comm[λ1, λ4];
        tmp // MatrixForm
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}$$

```
In[∘]:= tmp // MatrixForm
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}$$

```
In[∘]:= I λ7 // MatrixForm
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}$$

```
In[∘]:= comm[λ1, λ2] // MatrixForm
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} 2i & 0 & 0 \\ 0 & -2i & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

```
In[∘]:= λ1.λ4 // MatrixForm
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

```
In[∘]:= comm[λ1, λ4] == I λ7
Out[∘]= True
```