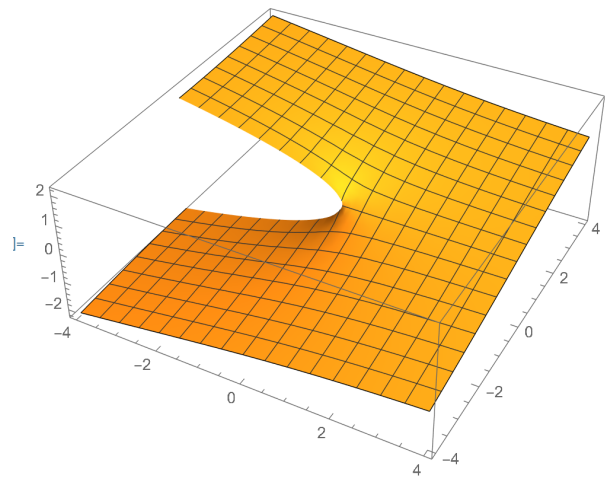


PROBLEM		SCORE
1	Complex Variables	
2	Eigenvectors	
3	Coupled motion	
4	Fourier Transform	
5	Group Theory	
6	Oscillations	
Total	out of 120	

Problem 1:

We found that the complex square root function (\sqrt{z}) has a discontinuity along the negative axis (with the conventional definition). This is also the case for the n-th root ($z^{1/n}$).

```
:= Plot3D[Im[Sqrt[x + I y]], {x, -4, 4}, {y, -4, 4}]
```



- a) Let's start with the cube root ($z^{1/3}$). Let's assume the branch cut is along the negative axis. Compute the discontinuity across the branch cut as a function of r.
- b) Sketch this for $r = [-4, 0]$

c) **GRADS ONLY:** In general, compute the discontinuity for the n-th root ($z^{1/n}$).

Problem 2:

- a) For the given matrix, M find the eigenvalue and eigenvectors.
- b) Normalize the eigenvectors.
- c) **GRADS ONLY:** Construct the matrix B from the normalized eigenvectors. Show that $B.M.B^T$ diagonalizes M.

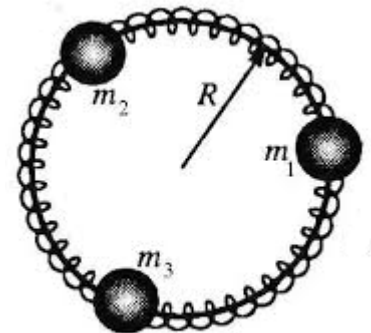
$$\begin{pmatrix} 14 & -3 \\ -3 & 6 \end{pmatrix}$$

Hint: Before you normalize the math should NOT be too complicated. After you normalize, there is a radical denominator, but you can factor that out when you check the diagonalization properties.

Problem 3:

Consider 3 masses that can slide freely on a circular wire of radius R. You may wish to label their positions $x_i = R \theta_i$, and thus $v_i = x_i' = R \theta_i'$,

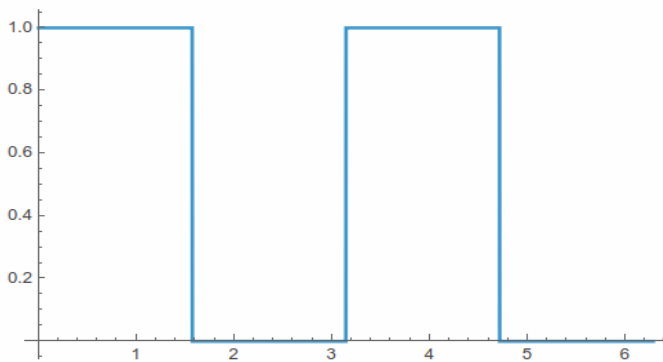
- a) Write down the kinetic energy T and potential energy V for the system in terms of θ_i and θ_i' . Assume all springs have equal k.
- b) Determine the T and V matrices for the system.



Problem 4:

On the interval $[0, 2\pi]$, consider function $f(x) = 1$ for $x \in [0, \pi/2]$ and $x \in [\pi, 3\pi/2]$, and zero elsewhere. Compute $a[0]$ and $b[n]$

Plot[f[x], {x, 0, 2 Pi}]



$$a_n = \frac{1}{\pi} \int_0^{2\pi} dx \cos(nx) f(x)$$

$$b_n = \frac{1}{\pi} \int_0^{2\pi} dx \sin(nx) f(x)$$

Problem 5: Group Theory:

Key properties of groups are i) closure, ii) associativity, iii) identity element, iv) inverse element.

a) On the Rubik's cube, if R represents a $1/4$ turn of the yellow face clockwise, we found this generates a cyclic group of order 4: $\{e, R, R^2, R^3, R^4=e\}$. A valid 2-element sub-group is $\{e, R^2\}$.

QUESTION: Why is $\{R, R^3\}$ NOT a valid 2-element subgroup.

b) For the $SU(2)$ Pauli matrices, we found the commutator was: $[\sigma_j, \sigma_k] = 2i \epsilon_{jkl} \sigma_l$.

For $SU(3)$, there is a similar relation. We won't compute in general, but will only do one case.

Compute $[\lambda_1, \lambda_4] = c \lambda_{\#}$, and determine which lambda matrix it yields ($\lambda_{\#}$) and the coefficient "c."

Matrices [\[edit\]](#)

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}$$

$$\lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad \lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \quad \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}.$$

Problem 6: Oscillations:

Give me as much information as you can about $\{x_0, v_0, \beta, w_0, w_D, Q_0\}$

Figure a)

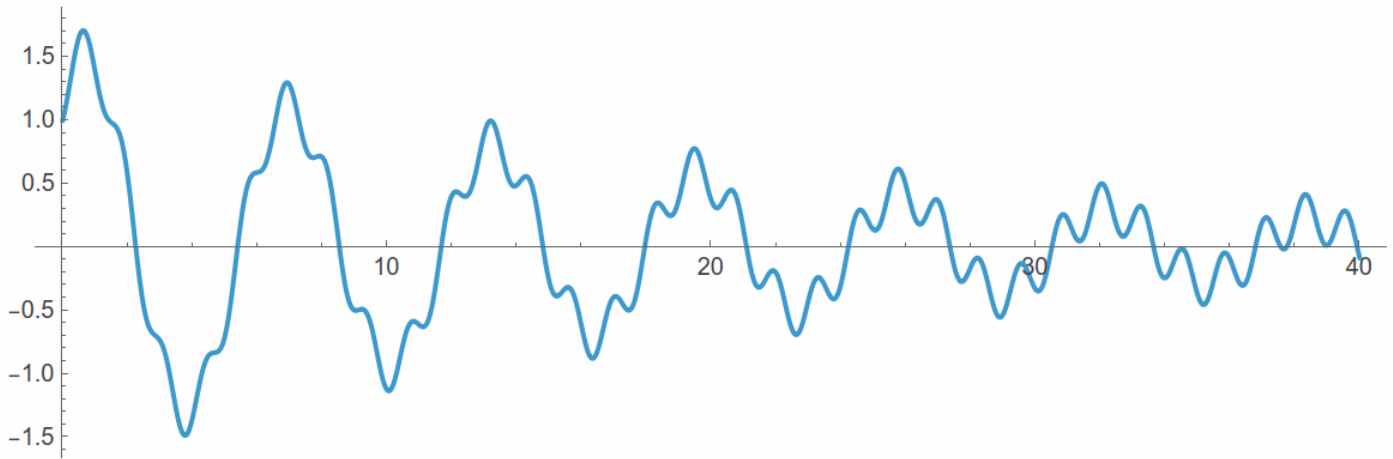


Figure b)

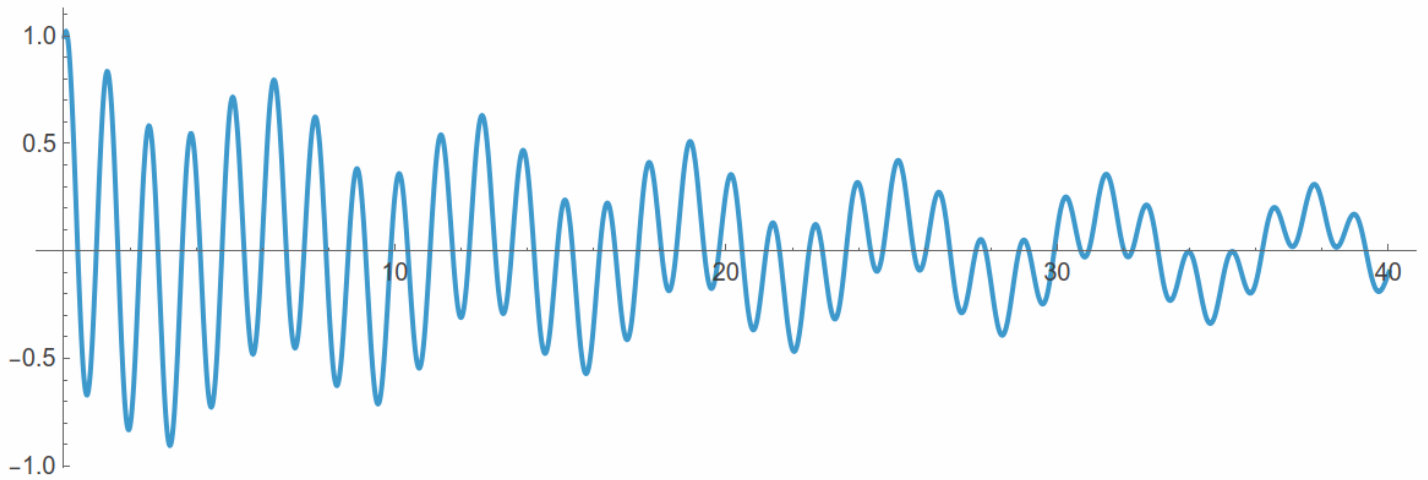


Figure c)

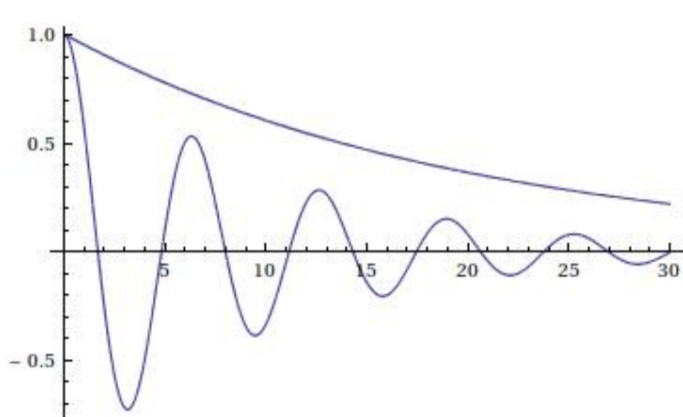


Figure d)

