Physics 4321 — Professor Olness

Exam 2 — Due: Tuesday 16 April, 2026

Each problem is 20 points.

PROBLEM		SCORE
1	Eigenvectors	
2	Lagrange Equations	
3	Brachistochrone Problem	
4	Lagrange Multiplier	
5	Delta Function	
6	Convolutions	
7	Laplace Equation	
Total	output of 140	

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Problem 1:

a) the given matrix, M find the eigenvalue and eigenvectors.

b) Normalize the eigenvectors.

Hint: Before you normalize the math should NOT be too complicated.

After you normalize, there is a radical denominator,

but you can factor that out when you check the diagonalization properties.

$$\begin{pmatrix} 7 & 3 \\ 3 & -1 \end{pmatrix}$$

Problem 2:

Consider 3 masses that can slide freely on a circular wire of radius R.

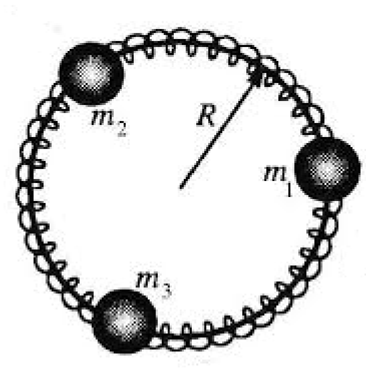
You may wish to label their positions $x_i = R\,\theta_i$, and thus $v_i = x_i' = R\,\theta_i'$,

a) Write down the kinetic energy T and potential energy V for the system in terms of θ_i and θ_i'. Assume all springs have equal k.

b) Using the Lagrange Equations, compute the 3 equations of motion.

c) Derive the T and V matrices.

[You should be alble to get the factors of 2 correctly.]



$$\frac{\partial L}{\partial \mathbf{r}_k} - \frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial \dot{\mathbf{r}}_k} + \sum_{i=1}^{C} \lambda_i \frac{\partial f_i}{\partial \mathbf{r}_k} = 0,$$

Problem 3:

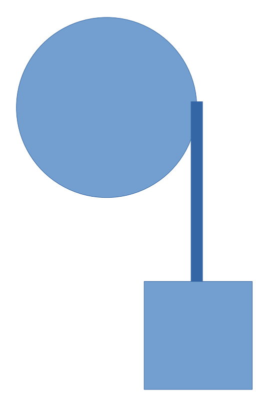
Set-up the solution of the Brachistochrone problem. Reduce it to a first-order differential equation (not a 2nd order DEQ). You do NOT need to make the trig substitutions to integrate the solution.

Problem 4:

Consider a mass m_1 at position x hanging from a pulley of mass m_2 at angle θ and moment of inertia $I = \alpha\, m_2 R^2$.

a) Using Lagrange multipliers, compute the equation of motion for x and θ with constraint equation $x - R\,\theta = 0$.

b) Solve for the acceleration x.



Problem 5: (Delta Functions)
Evaluate the delta functions

a) $\int_0^\infty \delta(x-2)$

b) $\int_0^\infty \delta(2x-2)$

c) $\int_0^\infty (5x-2)\,\delta(x-2)$

d) $\int_0^\infty \phi(x)\,\delta(x^2-a^2)$

e) $\int_{-\pi/2}^{\pi/2} \cos(x)\,\delta(\sin(x))$

f) $\int_0^{2\pi} \delta(\cos(x))$

g) $\int_{-\infty}^{\infty} \phi(x)\,\delta'(x)$

Problem 6: (Convolutions)

$$g(x) = e^{-x} - e^{-2x},$$

$$h(y) = e^{-y}$$

$$g \otimes h = \int_0^1 dx \int_0^1 dy\; g(x)\, h(y)\, \delta(x+y-z)$$

you may assume $1 > z > 0$

Problem 7: (Laplace Equation)

Consider a 2D potential problem for V(x,y). The boundary conditions are V=0 for x=0 and x=π. For y=0 V(0,y) = V_0, a constant. For V(∞,y)→0. Write the general form of the potential V(x,y)=$\sum_n$c(n)... for integer n which satisfied the boundary conditions, and then compute c(n).