4321 Final Exam

Tuesday 12 May at 11:30am

#		POINTS	SCORE
1	Misc. Re/Im, δ, Trig	20	
2	Higher Dimensions V, A	20	
3	Fourier Transform	20	
4	Lagrange Eqs.	20	
5	Relativity	20	
6	Lagrange Multiplier	20	
7	Calculus of Variations	20	
8	Oscillations	20	
TOTAL		160	

PROBLEM #1:

a) Compute the Re and Im part of: $\dfrac{2+3i}{4+5i}\ (6+7i)$

b) Compute: $\delta[a(x^2-b^2)]$

c) Convert Exp[4π/3 I] to Sin and Cos.

d) Convert Exp[4π/3] to Sinh and Cosh.

Hint: $\sinh x = \frac{e^x - e^{-x}}{2}$ and $\cosh x = \frac{e^x + e^{-x}}{2}$.

PROBLEM #2:

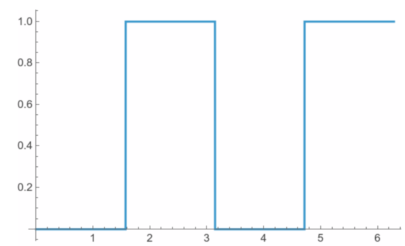
a) In 36 dimensions, the volume of the unit sphere is "a." Compute the surface area of this sphere.
Hint: consider 2D and 3D examples and remember the pattern.

b) In 42 dimensions, the surface area of the unit sphere is "b." Compute the volume of this sphere.

PROBLEM #3:

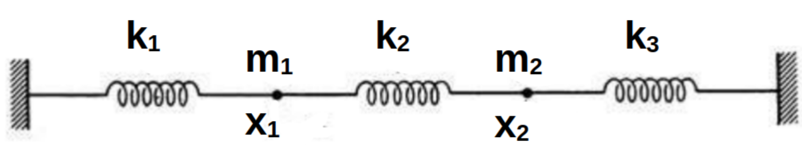
On the interval [0,2π], consider function f(x) = 1 for x=[π/2,π] and x=[3π/2,2π], and zero elsewhere. Compute a[0] and b[n]

$$a_n = \frac{1}{\pi}\int_0^{2\pi} dx\ \cos(nx) f(x), \quad b_n = \frac{1}{\pi}\int_0^{2\pi} dx\ \sin(nx) f(x),$$



PROBLEM #4:

a) Compute T, V, and L.
b) Then compute the Lagrange equations for x_1 and x_2. **DO NOT SOLVE.**
NOTE: *Assume the masses and spring constants are different, so keep the subscripts.*



PROBLEM #5:

The Electron Ion Collider (EIC) can be converted into a Muon Ion Collider (MuIC) with a muon beam energy of 1000 GeV and a proton energy of 275 GeV. Hint: You can ignore both the muon and proton mass, so that p_{muon} =(e,0,0,e), and p_{proton}=(p,0,0,-p).

a) Compute the total mass-energy (√s) available to produce final state particles.

b) Consider the case where the muon has an energy 1000 GeV, and the proton is at rest. p_{proton}=(m_p,0,0,0). Compute the total mass-energy (√s) available to produce final state particles. Take the proton mass m_p=1 GeV.

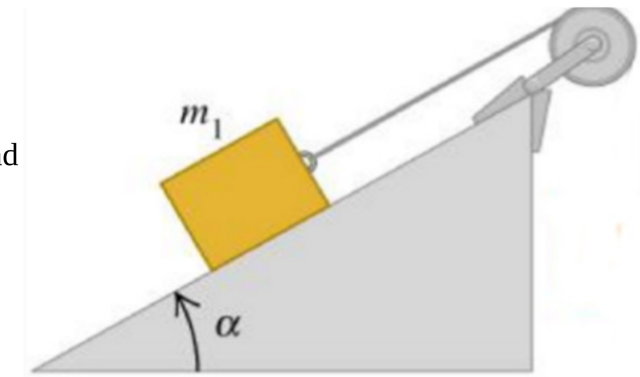
PROBLEM #6:

Mass $\mathbf{m_1}$ slides on a frictionless ramp of angle **α**, and it is tied to **MASSIVE** pulley of mass $\mathbf{m_2}$, radius **r**, and moment of inertia $\mathbf{I=\beta\, m_2 r^2}$. The string wound around the pulley does not slip. For coordinates, measure the distance of $\mathbf{m_1}$ along the ramp to be **x**, and the rotation of the pulley to be **θ**.

a) Using coordinates **{x,x',θ,θ'}**, compute the Lagrangian **L**.

b) Using a Lagrange multiplier **λ** and a constraint equation **x-r θ=0**, find the equations of motion.

c) Solve for the acceleration of $\mathbf{m_1}$ and **λ**.



PROBLEM #7:

The famous brachistochrone problem (from the Greek: brachistos = shortest, chronos = time, as in chronometer) is to find the shape of a wire joining two given points so that a bead will slide down under gravity from one point to the other (without friction) in the shortest time. Here we must minimize dt. If ds is an element of arc length, then the velocity of the particle is v = ds/dt.

a) Set up the problem and find the Lagrange equation. **DO NOT SOLVE.**
Hint: *I will accept EITHER a first or second order differential equation, but it needs to be correct.*
I think it is easier to use the first order, but ...

$$x''[t] + 2\gamma x'[t] + \omega_0^2 x[t] == Q_0 \, Exp[i\omega_D t]$$

<u>Problem 8</u>: Given the above notation, examine the figure below, and provide as much detail as possible about what parameters were used to generate the plots below. I want to know about the initial conditions, the relative size of the pieces, and any other information you can glean from the plot.
(Hint: the first two plots have many parameters in common.)

FIGURE A

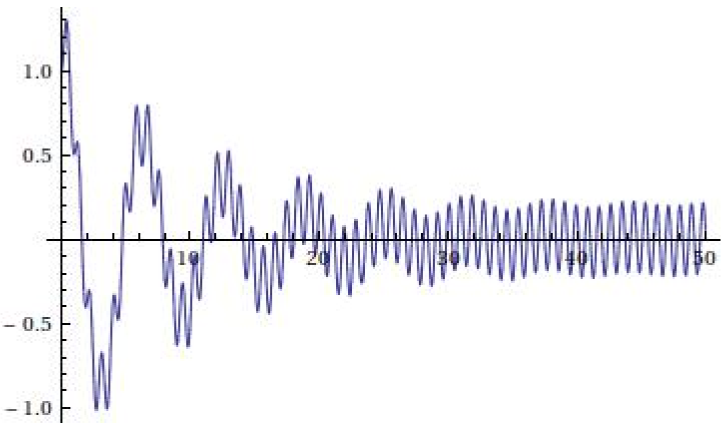


FIGURE B

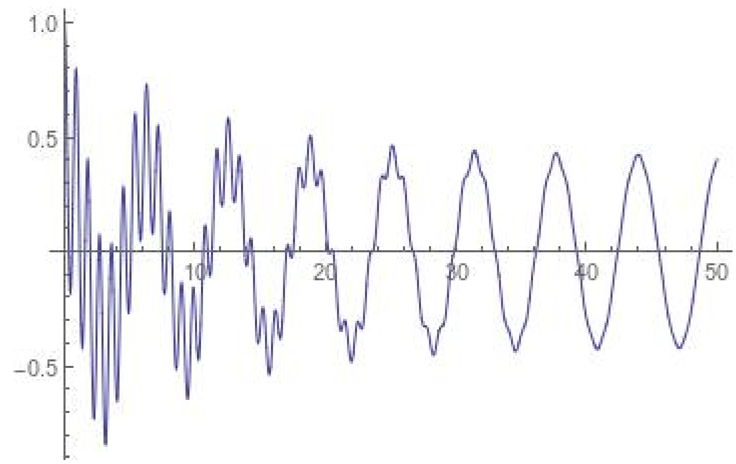


FIGURE C

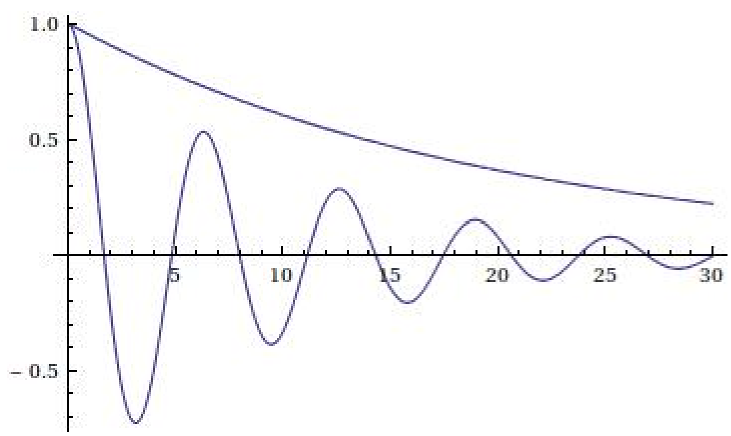


FIGURE D