
Problem 1 :

```
In[ ]:= Clear["Global`*"]
```

```
In[ ]:= 
$$\frac{(2 + 3 I)}{(4 + 5 I)} (6 + 7 I)$$

```

```
Out[ ]:= 
$$\frac{124}{41} + \frac{173 i}{41}$$

```

```
In[ ]:= Exp[4 π / 3 I] // ExpToTrig
```

```
Out[ ]:= 
$$-\frac{1}{2} - \frac{i \sqrt{3}}{2}$$

```

```
In[ ]:= Exp[4 π / 3] // ExpToTrig
```

```
Out[ ]:= 
$$\text{Cosh}\left[\frac{4 \pi}{3}\right] + \text{Sinh}\left[\frac{4 \pi}{3}\right]$$

```

Problem 2 :

```
In[ ]:= Clear["Global`*"]
```

```
In[ ]:= vol =  $\frac{4}{3} \pi r^3$ ;
```

```
D[vol, r]
```

```
Out[ ]:=  $4 \pi r^2$ 
```

```
In[ ]:= Integrate[ $4 \pi r^2$ , r]
```

```
Out[ ]:=  $\frac{4 \pi r^3}{3}$ 
```

```
In[ ]:= vol =  $a r^{36}$ ;
```

```
D[vol, r]
```

```
Out[ ]:=  $36 a r^{35}$ 
```

```
In[ ]:= Integrate[ $b r^{42-1}$ , r]
```

```
Out[ ]:=  $\frac{b r^{42}}{42}$ 
```

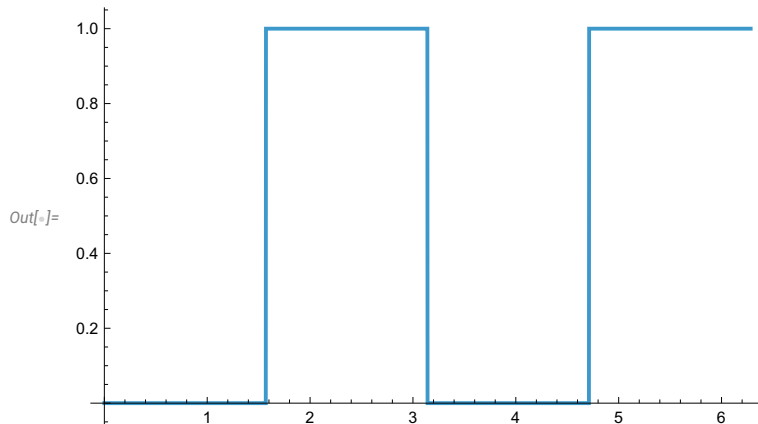
Problem 3: $f[x_]=...$ for $x=[0, 2 \pi]$ NEW

```
In[ ]:= Clear["Global`*"]
```

Part a)

```
In[ ]:= Clear[f];
f[x_]:=0 /; 0 < x <= N[Pi/2];
f[x_]:=1 /; N[Pi/2] < x <= N[Pi];
f[x_]:=0 /; N[Pi] < x <= N[3 Pi/2];
f[x_]:=1 /; N[3 Pi/2] < x <= N[2Pi];
```

```
In[ ]:= Plot[f[x],{x,0,2 Pi}]
```



Compute the Fourier coefficients

```
In[ ]:= a[n_] = 1/π (Integrate[Cos[n x], {x, π/2, π}] + Integrate[Cos[n x], {x, 3 π/2, 2 π}]) // Simplify
```

```
Out[ ]:= 
$$\frac{-\sin\left[\frac{n\pi}{2}\right] + \sin[n\pi] - \sin\left[\frac{3n\pi}{2}\right] + \sin[2n\pi]}{n\pi}$$

```

```
In[ ]:= Table[a[i], {i, 1, 10, 1}]
```

```
Out[ ]:= {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}
```

```
In[ ]:= Limit[a[i], i → 0]
```

```
Out[ ]:= 1
```

Check

```
In[ ]:= 1/π (Integrate[1, {x, π/2, π}] + Integrate[1, {x, 3 π/2, 2 π}])
```

```
Out[ ]:= 1
```

```
In[ ]:= b[n_] = 1/π (Integrate[Sin[n x], {x, 0, π/2}] + Integrate[Sin[n x], {x, π, 3 π/2}]) // Simplify
```

```
Out[ ]:= 
$$\frac{4\left(1 + \cos\left[\frac{n\pi}{2}\right] + \cos[n\pi]\right)\sin\left[\frac{n\pi}{4}\right]^2}{n\pi}$$

```

```
In[ ]:= b[n_] = 1/π (Integrate[Sin[n x], {x, π/2, π}] + Integrate[Sin[n x], {x, 3 π/2, 2 π}]) //
FullSimplify[#, n ∈ Integers] &
Out[ ]:= 
$$\frac{-1 - (-1)^n + \cos\left[\frac{n\pi}{2}\right] + \cos\left[\frac{3n\pi}{2}\right]}{n\pi}$$

```

```
In[ ]:= Table[b[i], {i, 1, 10, 1}]
Out[ ]:= 
$$\left\{0, -\frac{2}{\pi}, 0, 0, 0, -\frac{2}{3\pi}, 0, 0, 0, -\frac{2}{5\pi}\right\}$$

```

Compute the Fourier coefficients

```
In[ ]:= (* Note the integration limits *)
(* Note, my function here is just "x" You will need to change in other problems. *)
(* Note, we often need to do the zero term separately *)
```

```
c[0] = 1/(2 Pi) ( Integrate[ 1 ,{x, π/2,π}] + Integrate[ 1 ,{x, 3π/2,2π} ] )
Out[ ]:= 
$$\frac{1}{2}$$

```

```
In[ ]:= c[m_] = 1/(2 Pi) ( Integrate[ 1 Exp[- I m x] ,{x, π/2,π}] + Integrate[ 1 Exp[- I m x] ,{x, 3π/2,2π} ] )
Out[ ]:= 
$$\frac{2 e^{-\frac{5}{4} i m \pi} \cos\left[\frac{m\pi}{2}\right] \sin\left[\frac{m\pi}{4}\right]}{m\pi}$$

```

```
In[ ]:= Integrate[ 1 Exp[- I m x], x]
Out[ ]:= 
$$\frac{i e^{-i m x}}{m}$$

```

```
In[ ]:= Integrate[ 1 Sin[ m x], x]
Out[ ]:= 
$$-\frac{\cos[m x]}{m}$$

```

```
In[ ]:= Integrate[ 1 Sin[ m x], {x, π/2, π}] + Integrate[ 1 Sin[ m x], {x, 3 π/2, 2 π}] // FullSimplify
Out[ ]:= 
$$\frac{\cos\left[\frac{m\pi}{2}\right] - \cos[m\pi] + \cos\left[\frac{3m\pi}{2}\right] - \cos[2m\pi]}{m}$$

```

```
In[ ]:= Table[c[i], {i, 0, 6}] // Simplify
Out[ ]:= 
$$\left\{\frac{1}{2}, 0, \frac{i}{\pi}, 0, 0, 0, \frac{i}{3\pi}\right\}$$

```

```
In[ ]:= term[0] = c[0];
```

```
In[ ]:= term[m_]= c[m] Exp[I m x] + c[-m] Exp[-I m x] //FullSimplify
```

$$\text{Out[]:= } \frac{2 \cos\left[m\left(\frac{5\pi}{4} - x\right)\right] \left(-\sin\left[\frac{m\pi}{4}\right] + \sin\left[\frac{3m\pi}{4}\right]\right)}{m\pi}$$

```
In[ ]:= series[n_]:= Sum[ term[m] ,{m,0,n}]
```

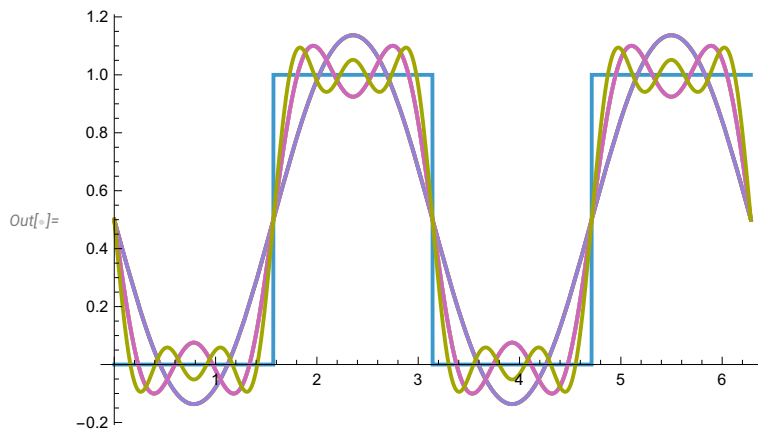
```
In[ ]:=
```

```
series[2]
```

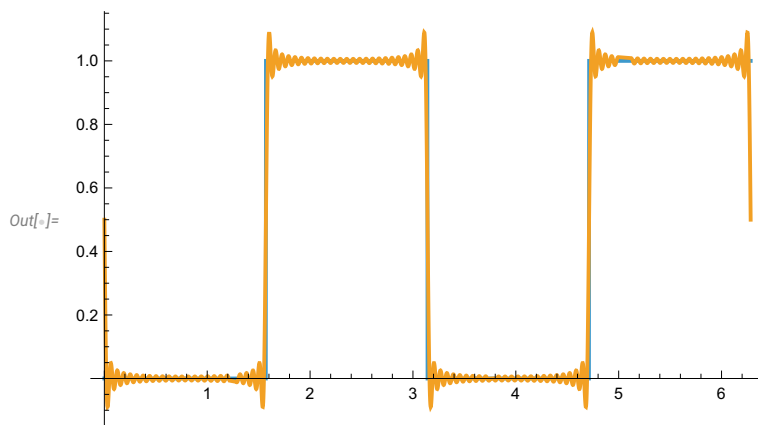
$$\text{Out[]:= } \frac{1}{2} - \frac{2 \cos\left[2\left(\frac{5\pi}{4} - x\right)\right]}{\pi}$$

Plot the series with different numbers of terms

```
In[ ]:= Plot[ Join[{f[x]},Table[series[i],{i,2,10,1}]] //Evaluate
, {x,0,2 Pi},PlotStyle->colors]
```



```
In[ ]:= Plot[ {f[x], series[100]} //Evaluate ,{x,0,2 Pi}]
```

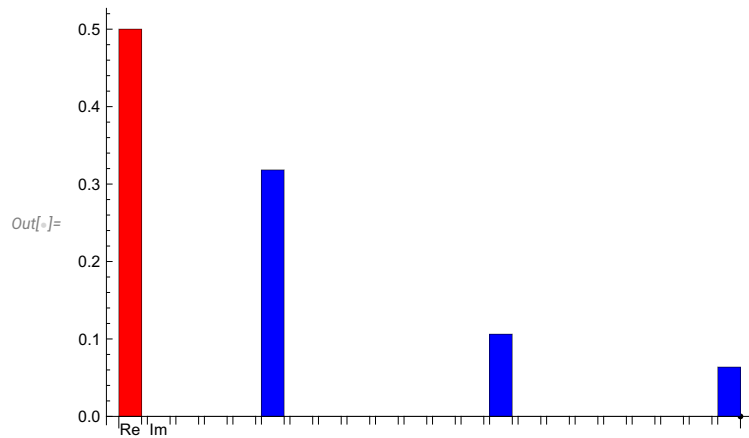


Make a "frequency domain" plot of the coefficients

```
In[ ]:= coeffs = Table[{Re[c[i]], Im[c[i]]}, {i, 0, 10}] // Flatten
```

```
Out[ ]:= { $\frac{1}{2}$ , 0, 0, 0, 0,  $\frac{1}{\pi}$ , 0, 0, 0, 0, 0, 0, 0,  $\frac{1}{3\pi}$ , 0, 0, 0, 0, 0, 0,  $\frac{1}{5\pi}$ }
```

```
In[ ]:= BarChart[coeffs, ChartStyle -> {Red, Blue}, ChartLabels -> {"Re", "Im"}]
```



Re-write the Exp series as a Trig series

```
In[ ]:= term2[0] = term[0] //ExpToTrig //Simplify
```

```
Out[ ]:=  $\frac{1}{2}$ 
```

```
In[ ]:= term2[m_] = term[m] //ExpToTrig //Simplify
```

```
Out[ ]:= 
$$\frac{2 \cos\left[m\left(\frac{5\pi}{4} - x\right)\right] \left(-\sin\left[\frac{m\pi}{4}\right] + \sin\left[\frac{3m\pi}{4}\right]\right)}{m\pi}$$

```

```
In[ ]:= tmp = term[m] //ExpToTrig // FullSimplify[#, Element[m, Integers]] &
```

```
Out[ ]:= 
$$\frac{2 \cos\left[\frac{5m\pi}{4} - mx\right] \left(-\sin\left[\frac{m\pi}{4}\right] + \sin\left[\frac{3m\pi}{4}\right]\right)}{m\pi}$$

```

```
In[ ]:= series2[n_] := Sum[term2[m], {m, 0, n}]
```

```
In[ ]:= series[5]
```

```
Out[ ]:= 
$$\frac{1}{2} - \frac{2 \cos\left[2\left(\frac{5\pi}{4} - x\right)\right]}{\pi}$$

```

```
In[ ]:= series[10] //ExpToTrig //TrigReduce
Out[ ]:= 
$$\frac{15 \pi - 60 \sin[2 x] - 20 \sin[6 x] - 12 \sin[10 x]}{30 \pi}$$

```

```
In[ ]:= series2[10] // TrigReduce
Out[ ]:= 
$$\frac{15 \pi - 60 \sin[2 x] - 20 \sin[6 x] - 12 \sin[10 x]}{30 \pi}$$

```

```
In[ ]:= 
$$\frac{2 \text{series2[40]}}{4} // \text{TrigReduce} // \text{Expand}$$

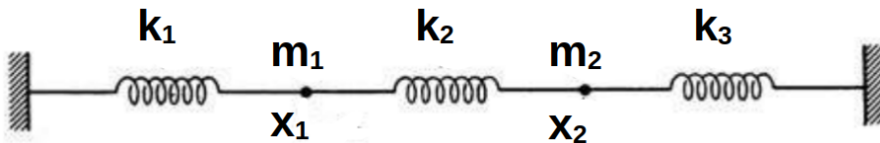
Out[ ]:= 
$$\frac{1}{4} \frac{\sin[2 x]}{\pi} - \frac{\sin[6 x]}{3 \pi} - \frac{\sin[10 x]}{5 \pi} - \frac{\sin[14 x]}{7 \pi} - \frac{\sin[18 x]}{9 \pi} - \frac{\sin[22 x]}{11 \pi} - \frac{\sin[26 x]}{13 \pi} - \frac{\sin[30 x]}{15 \pi} - \frac{\sin[34 x]}{17 \pi} - \frac{\sin[38 x]}{19 \pi}$$

```

Verify that they are identical

```
In[ ]:= series2[10] - series[10] // FullSimplify
Out[ ]:= 0
```

Problem 4:



```
In[ ]:= Clear["Global`*"]
```

```
In[ ]:= tkin = 
$$\frac{1}{2} m1 x1'[t]^2 + \frac{1}{2} m2 x2'[t]^2$$

```

```
Out[ ]:= 
$$\frac{1}{2} m1 x1'[t]^2 + \frac{1}{2} m2 x2'[t]^2$$

```

```
In[ ]:= vpot = 
$$\frac{1}{2} k1 x1[t]^2 + \frac{1}{2} k2 (x1[t] - x2[t])^2 + \frac{1}{2} k3 x2[t]^2$$

```

```
Out[ ]:= 
$$\frac{1}{2} k1 x1[t]^2 + \frac{1}{2} k2 (x1[t] - x2[t])^2 + \frac{1}{2} k3 x2[t]^2$$

```

```
In[ ]:= lag = tkin - vpot
```

```
Out[ ]:= 
$$-\frac{1}{2} k1 x1[t]^2 - \frac{1}{2} k2 (x1[t] - x2[t])^2 - \frac{1}{2} k3 x2[t]^2 + \frac{1}{2} m1 x1'[t]^2 + \frac{1}{2} m2 x2'[t]^2$$

```

```
In[ ]:= D[lag, x1[t]] - D[D[lag, x1'[t]], t] // Simplify
```

```
Out[ ]:= -((k1 + k2) x1[t]) + k2 x2[t] - m1 x1''[t]
```

```
In[ ]:= D[lag, x2[t]] - D[D[lag, x2'[t]], t] // Simplify
```

```
Out[ ]:= k2 x1[t] - (k2 + k3) x2[t] - m2 x2''[t]
```

```
In[ ]:= vv = {{-k1 - k2, +k2}, {k2, -k2 - k3}};
vv // MatrixForm
```

```
Out[ ]//MatrixForm=

$$\begin{pmatrix} -k1 - k2 & k2 \\ k2 & -k2 - k3 \end{pmatrix}$$

```

Problem 5 :

```
In[ ]:= Clear["Global`*"]
```

```
In[ ]:= g = DiagonalMatrix[{1, -1, -1, -1}];
```

a) CMS

```
In[ ]:= pmuon = {e, 0, 0, e};
```

```
pproton = {p, 0, 0, -p};
```

```
In[ ]:= p12 = pmuon + pproton
```

```
Out[ ]:= {e + p, 0, 0, e - p}
```

```
In[ ]:= s = p12.g.p12 // Simplify
```

```
Out[ ]:= 4 e p
```

```
In[ ]:=  $\sqrt{s}$  /. {e → 1000, p → 275} // N
```

```
Out[ ]:= 1048.81
```

a) Proton at rest

```
In[ ]:= pmuon = {e, 0, 0, e};
```

```
pproton = {mp, 0, 0, 0};
```

```
In[ ]:= p12 = pmuon + pproton
```

```
Out[ ]:= {e + mp, 0, 0, e}
```

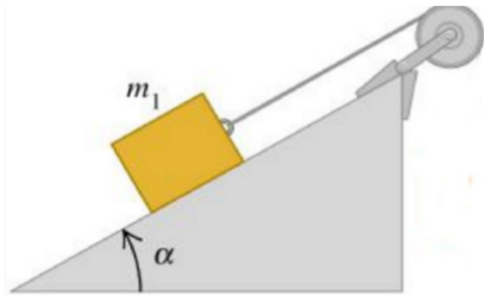
```
In[ ]:= s = p12.g.p12 // Simplify
```

```
Out[ ]:= mp (2 e + mp)
```

```
In[ ]:=  $\sqrt{s}$  /. {e → 1000, p → 275, mp → 1} // N
```

```
Out[ ]:= 44.7325
```

Problem 6 :



```
In[ ]:= Clear["Global`*"]
```

```
In[ ]:= tkin =  $\frac{1}{2} m_1 x'[t]^2 + \frac{1}{2} \alpha m_2 r^2 \theta'[t]^2$ 
```

```
Out[ ]:=  $\frac{1}{2} m_1 x'[t]^2 + \frac{1}{2} m_2 r^2 \alpha \theta'[t]^2$ 
```

```
In[ ]:= vpot = - m1 g Sin[alpha] x[t]
```

```
Out[ ]:= -g m1 Sin[alpha] x[t]
```

```
In[ ]:= lag = tkin - vpot
```

```
Out[ ]:=  $g m_1 \text{Sin}[\alpha] x[t] + \frac{1}{2} m_1 x'[t]^2 + \frac{1}{2} m_2 r^2 \alpha \theta'[t]^2$ 
```

```
In[ ]:= f = x[t] - r theta[t];
```

```
{D[f, x[t]], D[f, theta[t]]}
```

```
Out[ ]:= {1, -r}
```

```
In[ ]:= eqx = D[lag, x[t]] - D[D[lag, x'[t]], t] - lambda D[f, x[t]]
```

```
Out[ ]:= -lambda + g m1 Sin[alpha] - m1 x''[t]
```

```
In[ ]:= eqtheta = D[lag, theta[t]] - D[D[lag, theta'[t]], t] - lambda D[f, theta[t]]
```

```
Out[ ]:= r lambda - m2 r^2 alpha theta''[t]
```

```
In[ ]:= eqf = D[f, {t, 2}]
```

```
Out[ ]:= x''[t] - r theta''[t]
```

```
In[ ]:= eqs = {eqx, eqtheta, eqf} == 0 // Thread
```

```
Out[ ]:= {-lambda + g m1 Sin[alpha] - m1 x''[t] == 0, r lambda - m2 r^2 alpha theta''[t] == 0, x''[t] - r theta''[t] == 0}
```

```
In[ ]:= Solve[eqs, {x''[t], theta''[t], lambda}]
```

```
Out[ ]:=  $\left\{ \left\{ x''[t] \rightarrow \frac{g m_1 \text{Sin}[\alpha]}{m_1 + m_2 \alpha}, \theta''[t] \rightarrow \frac{g m_1 \text{Sin}[\alpha]}{r (m_1 + m_2 \alpha)}, \lambda \rightarrow \frac{g m_1 m_2 \alpha \text{Sin}[\alpha]}{m_1 + m_2 \alpha} \right\} \right\}$ 
```

Problem 7

In[]:= `Clear["Global`*"]`

In[]:= `term = $\sqrt{\frac{x'[y]^2 + 1}{y}}$`

Out[]:= $\sqrt{\frac{1 + x'[y]^2}{y}}$

In[]:= `D[term, x[y]]`

Out[]:= 0

In[]:= `D[D[term, x'[y]], y]`

Out[]:=
$$-\frac{x'[y]}{y^2 \sqrt{\frac{1+x'[y]^2}{y}}} + \frac{x''[y]}{y \sqrt{\frac{1+x'[y]^2}{y}}} - \frac{x'[y] \left(-\frac{1+x'[y]^2}{y^2} + \frac{2 x'[y] x''[y]}{y} \right)}{2 y \left(\frac{1+x'[y]^2}{y} \right)^{3/2}}$$

In[]:= `tmp2 = D[term, x'[y]] // PowerExpand`

Out[]:= $\frac{x'[y]}{\sqrt{y} \sqrt{1 + x'[y]^2}}$

In[]:= `eq = tmp2^2 == $\frac{1}{2 a}$`

Out[]:= $\frac{x'[y]^2}{y (1 + x'[y]^2)} == \frac{1}{2 a}$

In[]:= `sol = Solve[eq, x'[y]]`

Out[]:= $\left\{ \left\{ x'[y] \rightarrow -\frac{\sqrt{y}}{\sqrt{2 a - y}} \right\}, \left\{ x'[y] \rightarrow \frac{\sqrt{y}}{\sqrt{2 a - y}} \right\} \right\}$

In[]:= `tmp4 = x'[y] /. sol[[2]]`

Out[]:= $\frac{\sqrt{y}}{\sqrt{2 a - y}}$

In[]:= `jacobian = D[a (1 - Cos[θ]), θ]`

Out[]:= $a \text{Sin}[\theta]$

In[]:= `tmp5 = tmp4 jacobian /. {y → a (1 - Cos[θ])}`

Out[]:= $\frac{a \sqrt{a (1 - \text{Cos}[\theta])} \text{Sin}[\theta]}{\sqrt{2 a - a (1 - \text{Cos}[\theta])}}$

```
In[ ]:= tmp6 =  $\sqrt{\text{tmp5}^2}$  // FullSimplify // PowerExpand
```

```
Out[ ]:=  $2 a \sin\left[\frac{\theta}{2}\right]^2$ 
```

```
In[ ]:= tmp7 = tmp6 // TrigExpand // FullSimplify
```

```
Out[ ]:=  $a - a \cos[\theta]$ 
```

```
In[ ]:= tmp8 = Integrate[tmp7,  $\theta$ ]
```

```
Out[ ]:=  $a \theta - a \sin[\theta]$ 
```

```
In[ ]:= y[ $\theta$ ] = -tmp7
```

```
x[ $\theta$ ] = tmp8
```

```
Out[ ]:=  $-a + a \cos[\theta]$ 
```

```
Out[ ]:=  $a \theta - a \sin[\theta]$ 
```

```
In[ ]:= ParametricPlot[{x[ $\theta$ ], y[ $\theta$ ]} /. {a -> 1}, { $\theta$ , 0, 6  $\pi$ }]
```

