Homework #10: Phys 4321: Prof. Olness Spring 2026

1) a) For Cartesian Coordinates, compute the 3D Laplacian of f=f[x] fy[y] fz[z] and separate the variables so you have three 1D differential equations.
b) Solve each of the three 1D equations. [Hint, they are not all identical.]
c) For the periodic functions, find both the normalization and the orthogonality relations.

2) a) For Cylindrical Coordinates, compute the 3D Laplacian of f=f[r] fθ[θ] fz[z] and separate the variables so you have three 1D differential equations.
b) Solve each of the three 1D equations. [Hint, they are not all identical.]
c) For the Bessel functions, find both the normalization and the orthogonality relations.
Demonstrate both the normalization and the orthogonality for 3 example functions.
Hint: See inset.

General Orthogonality Formula

For a fixed order n and distinct zeros α_{ni} and α_{nj}:

$$\int_0^1 x J_n(\alpha_{ni}x) J_n(\alpha_{nj}x)\,dx = \begin{cases} 0 & i \neq j \\ \frac{1}{2}\left[J_{n+1}(\alpha_{ni})\right]^2 & i = j \end{cases}$$

3) a) For Spherical Coordinates, compute the 3D Laplacian of f=f[r] fθ[θ] fφ[φ] and separate the variables so you have three 1D differential equations.
b) Solve each of the three 1D equations. [Hint, they are not all identical.]
c) For the Legendre functions, find both the normalization and the orthogonality relations.
Demonstrate both the normalization and the orthogonality for 3 example functions.
Hint: check out wiki.

4) Boas: Chpt 13: Section 2:
Laplace's equation; steady-state temperature in a rectangular plate:
Make a 3D plot of the solution for a finite number of terms and verify that it satisfies the boundary condition.

5) Consider a 2D potential problem for V(x,y). The boundary conditions are V=0 for x=0 and x=π. For y=0 V(0,y) = y. For V(∞,y)→0. Find the potential at all interior points, and make a 3-D plot of the solution for different orders of the expansion.

6) For an infinite cylindrical well the wave functions will be combinations of radial BesselJ functions. They can be finite at r=0, but must vanish at r=1. For the first 5 BesselJ[n, k_n r] functions (n={1...5}) find k_n that satisfies the boundary condition.

FOR GRADS ONLY:

7) Pick a coordinate system of your choice. Separate variables and solve the differential equations (as best you can.) Be creative.