

Physics 4321:
Homework #11:
Prof. Olness
Spring 2026

PROBLEM #1: Consider the metric:

$$g_{\mu\nu} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

and vector:

$$v = \begin{pmatrix} x \\ y \end{pmatrix}$$

- a) Compute the length-squared of the vector v .
- b) Write down a rotation matrix R .

$$R = \begin{pmatrix} c & -s \\ +s & c \end{pmatrix}$$

- c) Show the length of v is invariant under a rotation. That is, the length of $v' = Rv$ squared is: $v'.g.v'$ and show this is equal to $v.g.v$.

PROBLEM #2: Consider the metric:

$$g_{\mu\nu} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

and vector:

$$v = \begin{pmatrix} t \\ x \end{pmatrix}$$

- a) Compute the length-squared of the vector v .
- b) Write down a Boost matrix B .

$$B = \begin{pmatrix} ch & +sh \\ +sh & ch \end{pmatrix}$$

- c) Show the length of v is invariant under a Boost. That is, the length of $v' = Bv$ squared is: $v'.g.v'$ and show this is equal to $v.g.v$.

PROBLEM #3: Consider the metric:

$$g_{\mu\nu} = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

and vector:

$$v = \begin{pmatrix} x \\ y \end{pmatrix}$$

- a) Compute the length-squared of the vector v .
- b) Can you think of a physical example where this is the appropriate metric to use to measure distance???. (There are more than one answer, but the more natural the better.)