Problem #1 :

In[∘]:=
```
Clear["Global`*"]
```

In[∘]:=
```
g = DiagonalMatrix[{1, -1, -1, -1}];
g // MatrixForm
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

In[∘]:=
```
p1 = {e1, 0, 0, +pz};
p2 = {e2, 0, 0, -pz};
p12 = p1 + p2;
```

In[∘]:=
```
p2.g.p2
```

Out[∘]= $e2^2 - pz^2$

In[∘]:=
```
p12.g.p12
```

Out[∘]= $(e1 + e2)^2$

In[∘]:=
```
eqs = {p1.g.p1 == m12, p2.g.p2 == m22, p12.g.p12 == s}
```

Out[∘]= $\{e1^2 - pz^2 == m12,\ e2^2 - pz^2 == m22,\ (e1 + e2)^2 == s\}$

In[∘]:=
```
sol = Solve[eqs, {e1, e2, pz}] // Simplify;
sol[[4]]
```

Out[∘]= $\left\{e1 \to \frac{m12 - m22 + s}{2\sqrt{s}},\ e2 \to \frac{-m12 + m22 + s}{2\sqrt{s}},\ pz \to \frac{\sqrt{m12^2 + (m22 - s)^2 - 2\, m12\, (m22 + s)}}{2\sqrt{s}}\right\}$

In[∘]:=
```
tmp1 = pz /. sol[[4]]
```

Out[∘]= $\frac{\sqrt{m12^2 + (m22 - s)^2 - 2\, m12\, (m22 + s)}}{2\sqrt{s}}$

In[∘]:=
```
tmp2 = (4 s) tmp1^2 // Expand
```

Out[∘]= $m12^2 - 2\, m12\, m22 + m22^2 - 2\, m12\, s - 2\, m22\, s + s^2$

In[∘]:=
```
del[a_, b_, c_] = a^2 + b^2 + c^2 - 2 (a b + b c + c a);
```

In[∘]:=
```
del[m12, m22, s] == tmp2 // Simplify
```

Out[∘]= True

Problem #2 :

a) CMS System

In[∘]:=
```
Clear["Global`*"]
```

In[∘]:=
```
g = DiagonalMatrix[{1, -1, -1, -1}];
g // MatrixForm
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

```
(* BEFORE *)
p1 = {e1, 0, 0, +pz};
p2 = {e2, 0, 0, -pz};
p12 = p1 + p2;
e1 = e2 = e; (* set energies equal *)
```

In[∘]:=
```
sBefore = p12.g.p12
```

Out[∘]= $4\,e^2$

In[∘]:=
```
(* AFTER *)
p3 = {m, 0, 0, 0};
p4 = {m, 0, 0, 0};
p5 = {m, 0, 0, 0};
p6 = {m, 0, 0, 0};
pAFTER = (p3 + p4 + p5 + p6)
e1 = e2 = e; (* set energies equal *)
```

Out[∘]= {4 m, 0, 0, 0}

In[∘]:=
```
sAfter = pAFTER.g.pAFTER
```

Out[∘]= $16\,m^2$

In[∘]:=
```
eq = sBefore == sAfter
```

Out[∘]= $4\,e^2 == 16\,m^2$

In[∘]:=
```
Solve[eq, e]
```

Out[∘]= $\{\{e \to -2\,m\}, \{e \to 2\,m\}\}$

a) Fixed Target System

In[∘]:=
```
Clear["Global`*"]
```

```
In[]:= g = DiagonalMatrix[{1, -1, -1, -1}];
g // MatrixForm
```

Out[]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

```
In[]:= (* BEFORE *)
p1 = {e, 0, 0, +pz};
p2 = {m, 0, 0, 0};
p12 = p1 + p2;
```

```
In[]:= sBefore = p12.g.p12
```

Out[]= $(e + m)^2 - pz^2$

```
In[]:= eq1 = p1.g.p1 == m^2
```

Out[]= $e^2 - pz^2 == m^2$

```
In[]:= sol = Solve[eq1, pz]
```

Out[]= $\left\{\left\{pz \to -\sqrt{e^2 - m^2}\right\}, \left\{pz \to \sqrt{e^2 - m^2}\right\}\right\}$

```
In[]:= sBefore = sBefore /. sol[[2]] // Simplify
```

Out[]= $2\,m\,(e + m)$

```
In[]:= (* AFTER *)
p3 = {m, 0, 0, 0};
p4 = {m, 0, 0, 0};
p5 = {m, 0, 0, 0};
p6 = {m, 0, 0, 0};
pAFTER = (p3 + p4 + p5 + p6)
e1 = e2 = e; (* set energies equal *)
```

Out[]= $\{4\,m, 0, 0, 0\}$

```
In[]:= sAfter = pAFTER.g.pAFTER
```

Out[]= $16\,m^2$

```
In[]:= eq = sBefore == sAfter
```

Out[]= $2\,m\,(e + m) == 16\,m^2$

```
In[]:= Solve[eq, e]
```

Out[]= $\{\{e \to 7\,m\}\}$

Problem #3 :

a) CMS System

In[∘]:=
```
Clear["Global`*"]
```

In[∘]:=
```
g = DiagonalMatrix[{1, -1, -1, -1}];
g // MatrixForm
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

In[∘]:=
```
(* BEFORE *)
p1 = {e1, 0, 0, +pz};
p2 = {e2, 0, 0, -pz};
p12 = p1 + p2;
e1 = e2 = e; (* set energies equal *)
```

In[∘]:=
```
sBefore = p12.g.p12
```

Out[∘]= $4\, e^2$

In[∘]:=
```
(* AFTER *)
p3 = {mX, 0, 0, 0};
```

In[∘]:=
```
sAfter = p3.g.p3
```

Out[∘]= mX^2

In[∘]:=
```
eq = sBefore == sAfter
```

Out[∘]= $4\, e^2 == mX^2$

In[∘]:=
```
msol = Solve[eq, e]
```

Out[∘]= $\left\{\left\{e \to -\frac{mX}{2}\right\}, \left\{e \to \frac{mX}{2}\right\}\right\}$

In[∘]:=
```
e /. msol[[2]] /. {m → 1, mX → 125} // N
```

Out[∘]= 62.5

a) Fixed Target System

In[∘]:=
```
Clear["Global`*"]
```

In[∘]:=
```
g = DiagonalMatrix[{1, -1, -1, -1}];
g // MatrixForm
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

In[∘]:=
```
(* BEFORE *)
p1 = {e, 0, 0, +pz};
p2 = {m, 0, 0, 0};
p12 = p1 + p2;
```

In[∘]:=
```
sBefore = p12.g.p12
```

Out[∘]= $(e + m)^2 - pz^2$

In[∘]:=
```
eq1 = p1.g.p1 == m^2
```

Out[∘]= $e^2 - pz^2 == m^2$

In[∘]:=
```
sol = Solve[eq1, pz]
```

Out[∘]= $\left\{\left\{pz \to -\sqrt{e^2 - m^2}\right\}, \left\{pz \to \sqrt{e^2 - m^2}\right\}\right\}$

In[∘]:=
```
sBefore = sBefore /. sol[[2]] // Simplify
```

Out[∘]= $2\, m\, (e + m)$

In[∘]:=
```
(* AFTER *)
p3 = {mX, 0, 0, 0}
```

Out[∘]= {mX, 0, 0, 0}

In[∘]:=
```
sAfter = p3.g.p3
```

Out[∘]= mX^2

In[∘]:=
```
eq = sBefore == sAfter
```

Out[∘]= $2\, m\, (e + m) == mX^2$

In[∘]:=
```
msol = Solve[eq, e] // Simplify
```

Out[∘]= $\left\{\left\{e \to -m + \frac{mX^2}{2\, m}\right\}\right\}$

In[∘]:=
```
e /. msol /. {m → 1, mX → 125} // N
```

Out[∘]= {7811.5}

Problem #5 :

a) z-Boost:

In[∘]:=
```
Clear["Global`*"]
```

In[∘]:=
```
boostz = γ {{1, 0, 0, β}, {0, 1, 0, 0}, {0, 0, 1, 0}, {β, 0, 0, 1}};
boostz // MatrixForm
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} \gamma & 0 & 0 & \beta\gamma \\ 0 & \gamma & 0 & 0 \\ 0 & 0 & \gamma & 0 \\ \beta\gamma & 0 & 0 & \gamma \end{pmatrix}$$

In[∘]:=
```
(* CMS *)
p1 = {e, 0, 0, +pz};
p2 = {e, 0, 0, -pz};
```

In[∘]:=
```
boostz.p2
```

Out[∘]= $\{e\,\gamma - pz\,\beta\,\gamma,\ 0,\ 0,\ -pz\,\gamma + e\,\beta\,\gamma\}$

```
eq1 = boostz.p2 == {m, 0, 0, 0} // Thread
```

Out[∘]= $\{e\,\gamma - pz\,\beta\,\gamma == m,\ \text{True},\ \text{True},\ -pz\,\gamma + e\,\beta\,\gamma == 0\}$

In[∘]:=
```
eq1[[4]]
```

Out[∘]= $-pz\,\gamma + e\,\beta\,\gamma == 0$

In[∘]:=
```
bsol = Solve[eq1[[4]], β][[1]]
```

Out[∘]= $\left\{\beta \to \frac{pz}{e}\right\}$

Check the boost:

In[∘]:=
```
boostz.p2 /. bsol // Together
```

Out[∘]= $\left\{\frac{e^2\,\gamma - pz^2\,\gamma}{e},\ 0,\ 0,\ 0\right\}$

In[∘]:=
```
pzSol = Solve[e^2 - pz^2 == m^2, pz][[1]]
```

Out[∘]= $\left\{pz \to -\sqrt{e^2 - m^2}\right\}$

In[∘]:=
```
boostz.p2 /. bsol /. pzSol /. {γ → e / m} // Together
```

Out[∘]= {m, 0, 0, 0}

Boost p1

In[•]:=
```
tmp1 = boostz.p1
```
Out[•]= $\{e\,\gamma + pz\,\beta\,\gamma,\ 0,\ 0,\ pz\,\gamma + e\,\beta\,\gamma\}$

In[•]:=
```
tmp2 = tmp1[[1]] /. bsol /. pzSol /. {γ → e / m} // Together
```
Out[•]= $\frac{2\,e^2 - m^2}{m}$

In[•]:=
```
tmp2 /. {e → 2000, m → 1}
```
Out[•]= 7 999 999

In[•]:=
```
tmp2 /. {m^2 → 0} /. {e → 2000, m → 1}
```
Out[•]= 8 000 000

In[•]:=
```
tmp2 /. {m^2 → 0} /. {m → e / γ}
```
Out[•]= $2\,e\,\gamma$

In[•]:=
```
γ /. {γ → e / m} /. {e → 2000, m → 1}
```
Out[•]= 2000

Scattering angle

In[•]:=
```
eq = Tan[θlab] == 1/γ Tan[θcm/2]
```
Out[•]= $\text{Tan}[\theta\text{lab}] == \frac{\text{Tan}\left[\frac{\theta\text{cm}}{2}\right]}{\gamma}$

In[•]:=
```
eq /. {γ → 2000}
```
Out[•]= $\text{Tan}[\theta\text{lab}] == \frac{\text{Tan}\left[\frac{\theta\text{cm}}{2}\right]}{2000}$

In[•]:=
```
sol = Solve[eq , θlab][[1]]
```
Out[•]= $\left\{\theta\text{lab} \to \text{ArcTan}\left[\frac{\text{Tan}\left[\frac{\theta\text{cm}}{2}\right]}{\gamma}\right] + \pi\,c_1 \text{ if } c_1 \in \mathbb{Z}\right\}$

In[•]:=
```
θlab /. sol /. {γ → 2000, θcm → 90 Degree, c1 → 0} // N
```
Out[•]= 0.000809887

Problem #6 :

In[•]:=
```
Clear["Global`*"]
```

In[∘]:=
```
f = {{0, -ex, -ey, -ez}, {ex, 0, -bz, +by}, {ey, bz, 0, -bx}, {ez, -by, bx, 0}};
f // MatrixForm
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} 0 & -ex & -ey & -ez \\ ex & 0 & -bz & by \\ ey & bz & 0 & -bx \\ ez & -by & bx & 0 \end{pmatrix}$$

In[∘]:=
```
Transpose[f] + f
```

Out[∘]= {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}

In[∘]:=
```
boostz = {{1, 0, 0, β}, {0, 1, 0, 0}, {0, 0, 1, 0}, {β, 0, 0, 1}};
boostz // MatrixForm
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & \beta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \beta & 0 & 0 & 1 \end{pmatrix}$$

In[∘]:=
```
boostz.f.Transpose[boostz] // MatrixForm
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} 0 & -ex - by\,\beta & -ey + bx\,\beta & -ez + ez\,\beta^2 \\ ex + by\,\beta & 0 & -bz & by + ex\,\beta \\ ey - bx\,\beta & bz & 0 & -bx + ey\,\beta \\ ez - ez\,\beta^2 & -by - ex\,\beta & bx - ey\,\beta & 0 \end{pmatrix}$$

Problem #8 :

In[∘]:=
```
Clear["Global`*"]
```

In[∘]:=
```
pmu = {e, p Sin[θ], 0, p Cos[θ]}
```

Out[∘]= {e, p Sin[θ], 0, p Cos[θ]}

In[∘]:=
```
g = DiagonalMatrix[{1, -1, -1, -1}];
```

In[∘]:=
```
eq1 = pmu.g.pmu == m^2 // Simplify
```

Out[∘]= $e^2 == m^2 + p^2$

In[∘]:=
```
sol = Solve[eq1, p][[2]]
```

Out[∘]= $\{p \to \sqrt{e^2 - m^2}\}$

In[∘]:=
```
arg = (pmu[[1]] + pmu[[4]])/(pmu[[1]] - pmu[[4]]) /. sol /. {m → x e}
```

Out[∘]=

$$\frac{e + \sqrt{e^2 - e^2 x^2}\ \mathrm{Cos}[\theta]}{e - \sqrt{e^2 - e^2 x^2}\ \mathrm{Cos}[\theta]}$$

In[∘]:=
```
y[θ_, x_] = 1/2 Log[arg] /. {e → 1}
```
Out[∘]= $\frac{1}{2}\,\mathrm{Log}\left[\frac{1+\sqrt{1-x^2}\,\mathrm{Cos}[\theta]}{1-\sqrt{1-x^2}\,\mathrm{Cos}[\theta]}\right]$

In[∘]:=
```
η[θ_] = -Log[Tan[θ / 2]]
```
Out[∘]= $-\mathrm{Log}\left[\mathrm{Tan}\left[\frac{\theta}{2}\right]\right]$

In[∘]:=
```
tab = Table[
    ParametricPlot[{y[θ, x], η[θ]}, {θ, 0, Pi},
     PlotStyle → {Thick, Hue[x]}],
   {x, -1, 1, 0.1}];
```

In[∘]:=
```
z = 3;
Show[tab, PlotRange → {{-z, z}, {-z, z}}]
```
Out[∘]=