Physics 4321:
Homework #12:
Prof. Olness
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PROBLEM #1: For the general mass case of $p_1 + p_2 \rightarrow p_{12}$ compute the components of all 4-vectors in terms of invariants of the problem: $\{m_1^2, m_2^2, s\}$. Assume the 3-momentum lies along the z-axis. Hint: The z-component of the momentum should be proportional to: $\Delta(s, m_1^2, m_2^2)$, where $\Delta(a,b,c) = \sqrt{a^2+b^2+c^2-2(ab+bc+ca)}$.

PROBLEM #2: Consider the reaction: (proton+proton) $pp \rightarrow pppp\bar{p}$ (3 proton, 1 anti-proton). Compute the threshold beam energy for

a) colliding beams, and

b) for a fixed target experiment.

PROBLEM #3: Consider the reaction: (proton+anti-proton) $p\bar{p} \rightarrow X$ where X has a rest mass of $125 GeV$. Compute the threshold beam energy for

a) colliding beams, and

b) for a fixed target experiment.

PROBLEM #4: For one-particle phase space, show the folowing equality (*with all the steps*):

$$\frac{d^3\vec{P}}{(2\pi)^3 2E} = (2\pi)\delta(P^2 - m^2)\frac{d^4P}{(2\pi)^4}$$

Hint: note that $d^4P = dE\, d^3\overrightarrow{P}$ and $(P^2 - m^2) = (E^2 - \overrightarrow{P}^2 - m^2) \equiv (E^2 - a^2)$, and then used your delta-function knowledge.

PROBLEM #5: Consider the reaction: $pp \rightarrow pp$ $(12 \rightarrow 34)$ with **CMS** scattering angle θ. The CMS energy is $\sqrt{s} = 2\,TeV$.

a) Compute the boost from the CMS frame to the rest frame of #2 (lab frame)

b) Compute the energy of #1 in the lab frame.

c) Compute the scattering angle θ_{lab} as a function of the CMS θ and invariants.

PROBLEM #6: Write the 4×4 EM Field Strength tensor $F^{\mu\nu}$ in terms of $\{E, B\}$ fields.

Now apply a boost (Lorentz transformation) along the z axis and compute the transformed $F^{\mu\nu}$.

Examine how the $\{E, B\}$ fields transform and check against your EM textbook results.

Hint: use the Mathematica starter file.

GRADS ONLY:

PROBLEM #7: Consider the reaction: $pp \rightarrow pp$ $(12 \rightarrow 34)$ with **CMS** scattering angle θ. The CMS energy is $\sqrt{s} = 2\,TeV$.

a) Compute the boost from the CMS frame to the rest frame of #2 (lab frame)

b) Compute the energy of #1 in the lab frame.

c) Compute the scattering angle θ_{lab} as a function of the CMS θ and invariants.

PROBLEM #8: Consider the rapidity y and the pseudo-rapidity η:

$$y = \frac{1}{2}\ln\left(\frac{E + P_L}{E - P_L}\right)$$

$$\eta = -\ln\left[\tan\left(\frac{\theta}{2}\right)\right]$$

a) Make a parametric plot of $\{y, \eta\}$as a function of m/E where m is the mas of the particle.

b) Show that in the limit $m \rightarrow 0$ that $y \rightarrow \eta$.

c) Make a table of η for $\theta = [0°, 180°]$ in steps of 5 degrees.

c) Make a table of θ for $\eta = [0, 10]$ in steps of 1.